

# Lecture 8: Thu Sep 10, 2020

Reminder:

- HW3 due midnight tonight.

Lecture

- review RADAR problem: an application of convolution
  - ▷ cross correlation, autocorrelation function, and the matched filter
  - ▷ good radar signals: pulse compression
- eigensignals of LTI systems
- frequency response
- LTI systems: sinusoid in, sinusoid out

# Reading Assignment

## Handouts for ECE 3084

- [syllabus](#)
- [3084 book](#)

|                                                                                                     |           |
|-----------------------------------------------------------------------------------------------------|-----------|
| <b>1 What are Signals?</b>                                                                          | <b>1</b>  |
| 1.1 Convenient continuous-time signals . . . . .                                                    | 1         |
| 1.1.1 Unit step functions . . . . .                                                                 | 1         |
| 1.1.2 Delta “functions” . . . . .                                                                   | 3         |
| 1.1.3 Calculus with Dirac deltas and unit steps . . . . .                                           | 4         |
| 1.2 Shifting, flipping and scaling continuous-time signals in time . . . . .                        | 4         |
| 1.3 Under the hood: what professors don’t want to talk about . . . . .                              | 5         |
| <b>2 What are Systems?</b>                                                                          | <b>9</b>  |
| 2.1 System properties . . . . .                                                                     | 10        |
| 2.1.1 Linearity . . . . .                                                                           | 10        |
| 2.1.2 Time-invariance . . . . .                                                                     | 11        |
| 2.1.3 Causality . . . . .                                                                           | 11        |
| 2.1.4 Examples of systems and their properties . . . . .                                            | 12        |
| 2.2 Concluding thoughts . . . . .                                                                   | 13        |
| 2.2.1 Linearity and time-invariance as approximations . . . . .                                     | 13        |
| 2.2.2 Contemplations on causality . . . . .                                                         | 14        |
| 2.2.3 How these properties play out in practice in a typical “signals and systems” course . . . . . | 14        |
| <b>3 Why are LTI Systems so Important?</b>                                                          | <b>15</b> |
| 3.1 Review of convolution for discrete-time signals . . . . .                                       | 15        |
| 3.2 Convolution for continuous-time signals . . . . .                                               | 16        |
| 3.3 Review of frequency response of discrete-time systems . . . . .                                 | 16        |
| 3.4 Frequency response of continuous-time systems . . . . .                                         | 17        |
| 3.5 Connection to Fourier transforms . . . . .                                                      | 18        |
| 3.6 Finishing the picture . . . . .                                                                 | 19        |
| 3.7 A few observations . . . . .                                                                    | 19        |
| <b>4 More on Continuous-Time Convolution</b>                                                        | <b>21</b> |
| 4.1 The convolution integral . . . . .                                                              | 21        |
| 4.2 Properties of convolution . . . . .                                                             | 22        |
| 4.3 Convolution examples . . . . .                                                                  | 23        |
| 4.4 Some final comments . . . . .                                                                   | 28        |
| <b>5 Cross-Correlation and Matched Filtering</b>                                                    | <b>29</b> |
| 5.1 Cross-correlation properties . . . . .                                                          | 30        |
| 5.2 Cross-correlation examples . . . . .                                                            | 31        |
| 5.3 Matched filter implementation . . . . .                                                         | 31        |
| 5.4 Delay estimation . . . . .                                                                      | 32        |
| 5.5 Causal concerns . . . . .                                                                       | 33        |
| 5.6 A caveat . . . . .                                                                              | 34        |
| 5.7 Under the hood: squared-error metrics and correlation processing . . . . .                      | 34        |

(FINISH)

# Coming Next: Frequency Response

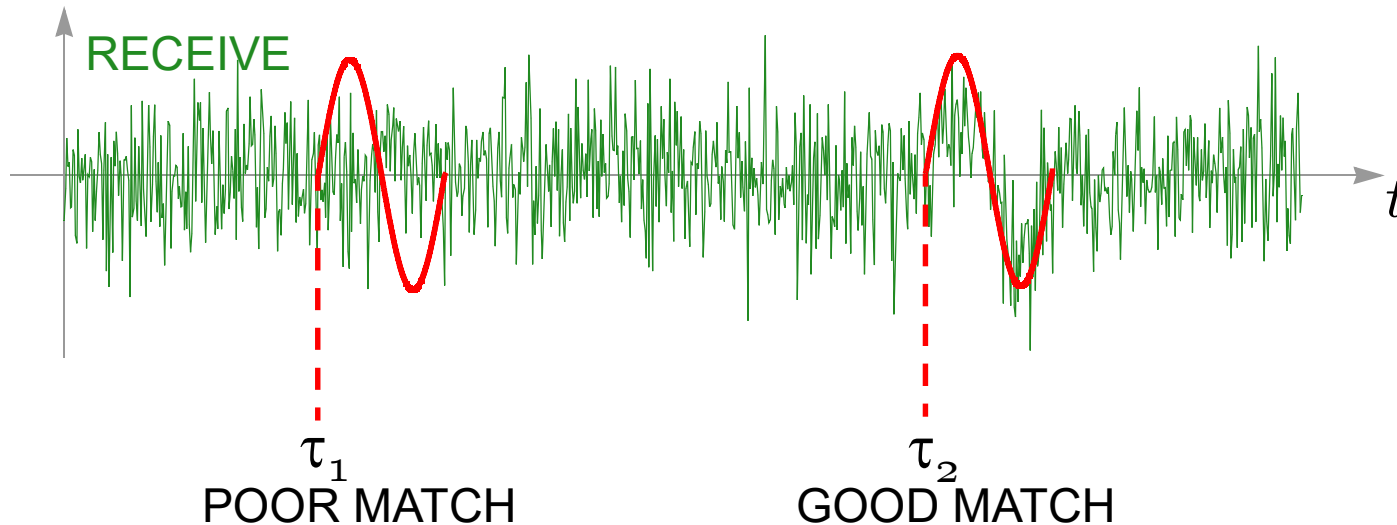
(Chapter 10 of 2026 1st edition.)

## 10 Frequency Response 285

|          |                                                           |     |
|----------|-----------------------------------------------------------|-----|
| 10-1     | The Frequency Response Function for LTI Systems . . . . . | 285 |
| 10-1.1   | Plotting the Frequency Response . . . . .                 | 287 |
| 10-1.1.1 | Logarithmic Plot . . . . .                                | 288 |
| 10-1.2   | Magnitude and Phase Changes . . . . .                     | 288 |
| 10-2     | Response to Real Sinusoidal Signals . . . . .             | 289 |
| 10-2.1   | Cosine Inputs . . . . .                                   | 290 |
| 10-2.2   | Symmetry of $H(j\omega)$ . . . . .                        | 290 |
| 10-2.3   | Response to a General Sum of Sinusoids . . . . .          | 293 |
| 10-2.4   | Periodic Input Signals . . . . .                          | 294 |
| 10-3     | Ideal Filters . . . . .                                   | 295 |
| 10-3.1   | Ideal Delay System . . . . .                              | 295 |
| 10-3.2   | Ideal Lowpass Filter . . . . .                            | 296 |
| 10-3.3   | Ideal Highpass Filter . . . . .                           | 297 |
| 10-3.4   | Ideal Bandpass Filter . . . . .                           | 297 |
| 10-4     | Application of Ideal Filters . . . . .                    | 298 |
| 10-5     | Time-Domain or Frequency-Domain? . . . . .                | 300 |
| 10-6     | Summary/Future . . . . .                                  | 301 |
| 10-7     | Problems . . . . .                                        | 302 |

# Radar: Look for Best Match

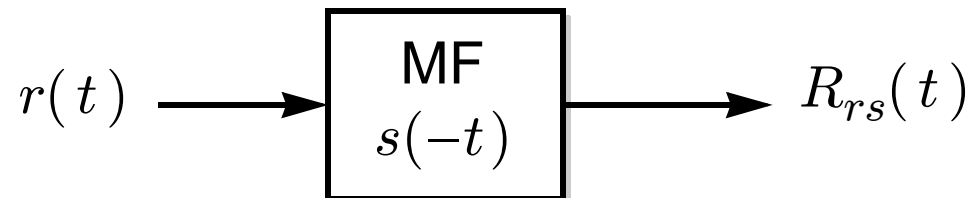
Conceptually we want to *slide* template  $s(t - \tau)$  and look for “best” match



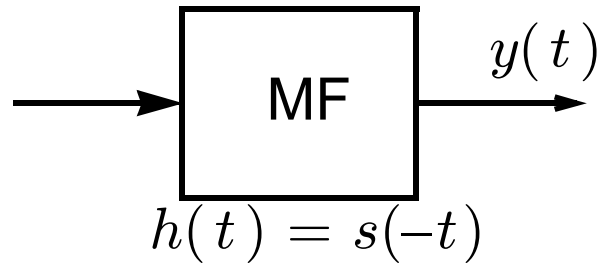
Last time: the best match maximizes the “crosscorrelation function”

$$R_{rs}(\tau) = \int_{-\infty}^{\infty} r(t) s(t - \tau) dt$$

Implementation:



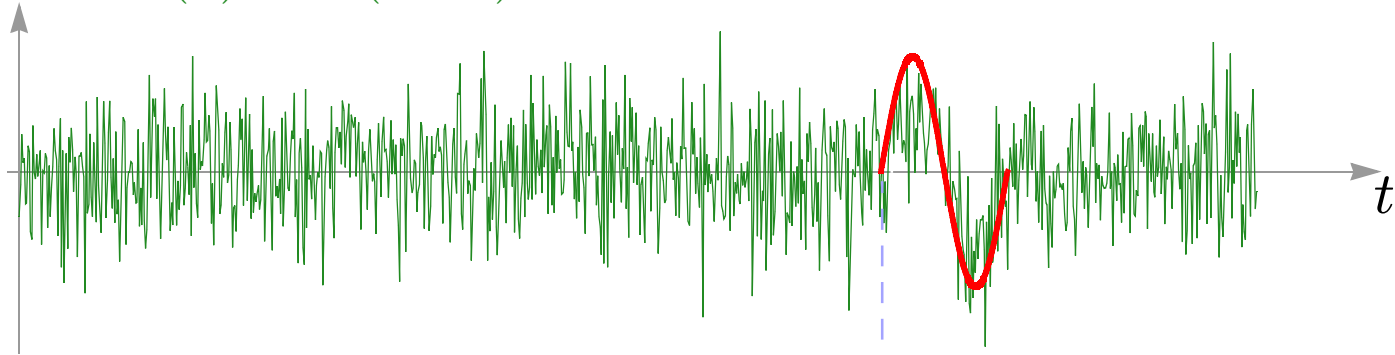
# Two Common MF Inputs



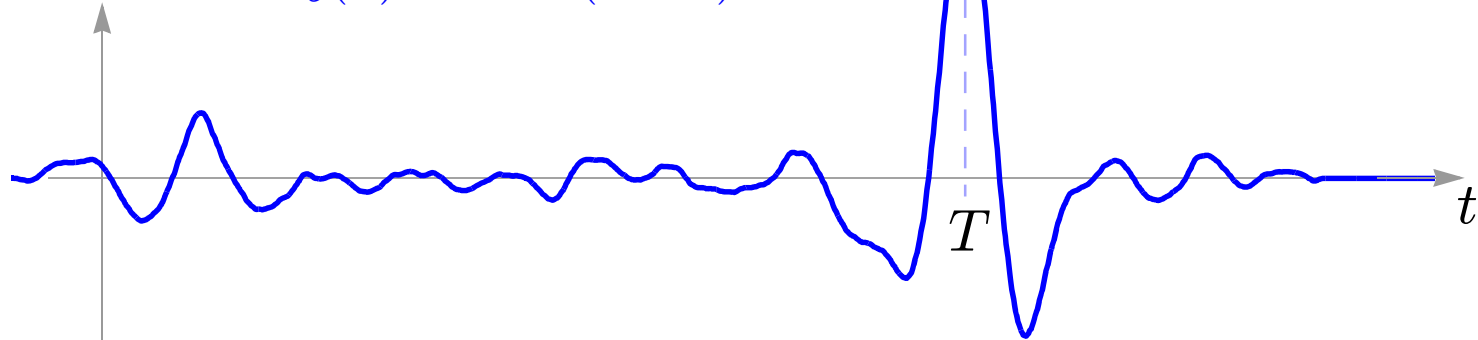
- ❶ input is  $s(t)$   $\Rightarrow$  output is  $R_{ss}(t)$ .
- ❷ input is  $r(t) = As(t - T)$   $\Rightarrow$  output is  $AR_{ss}(t - T)$ .

# With Noise

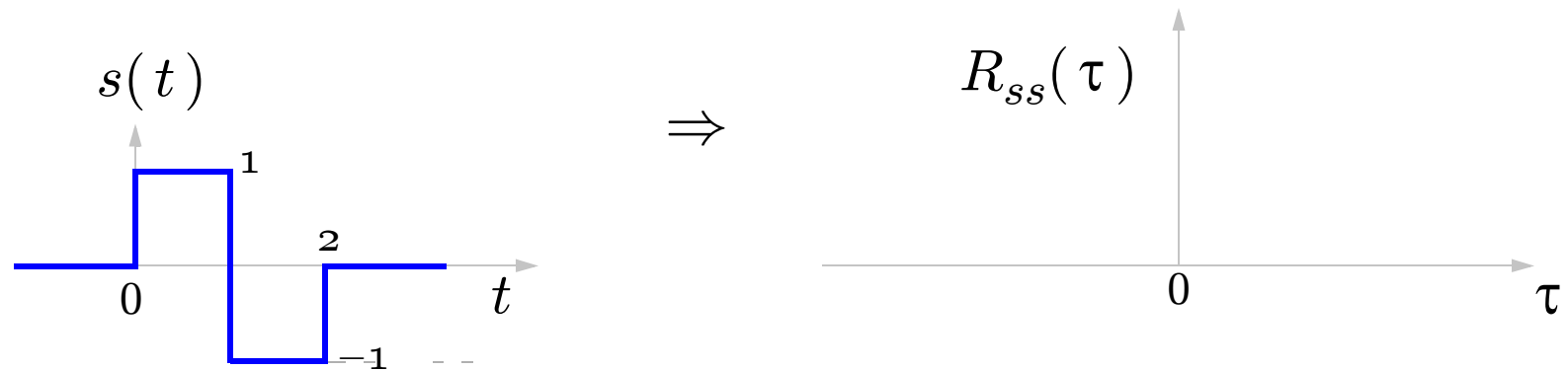
RECEIVE  $r(t) = As(t - T) + \text{noise}$



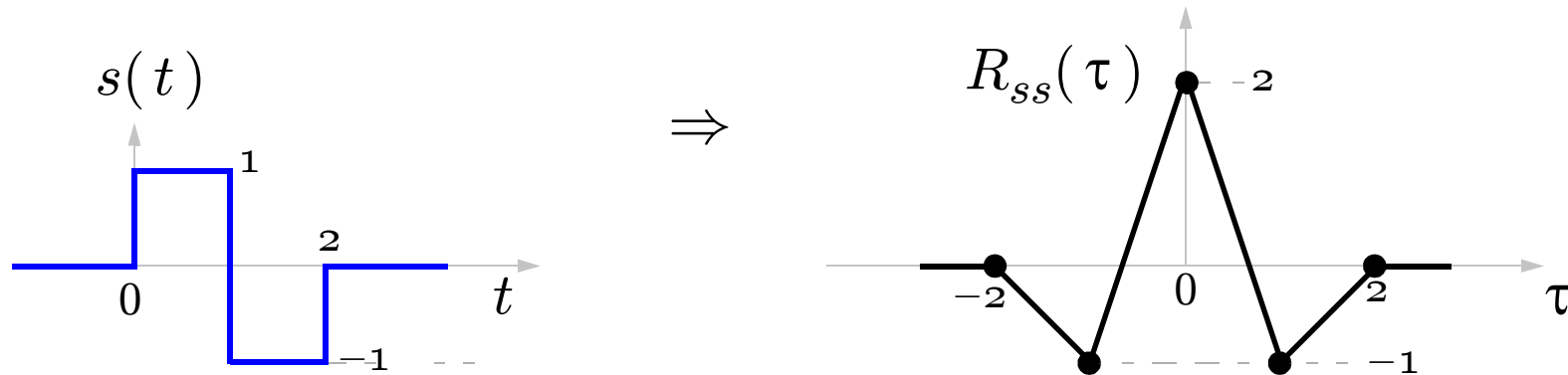
MF OUTPUT  $y(t) = ARss(t - T) + \text{noise}$



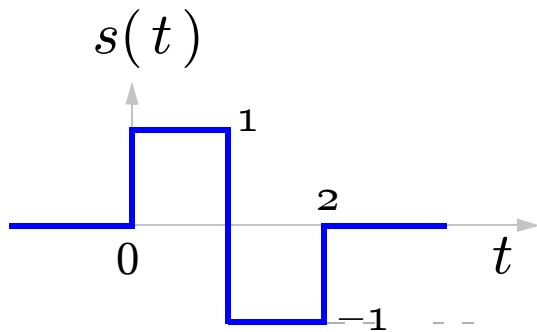
# Binary Code Autocorrelation Examples



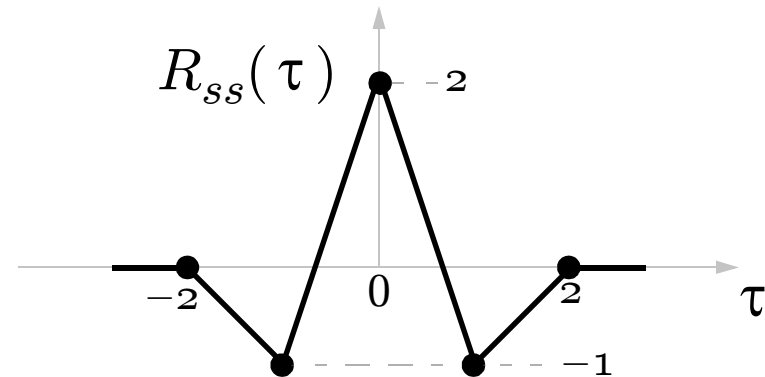
# Binary Code Autocorrelation Examples



# Binary Code Autocorrelation Examples

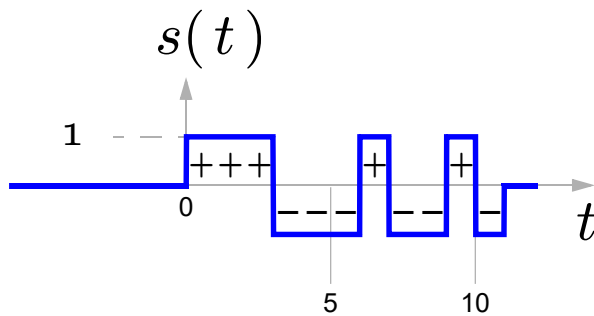


$\Rightarrow$

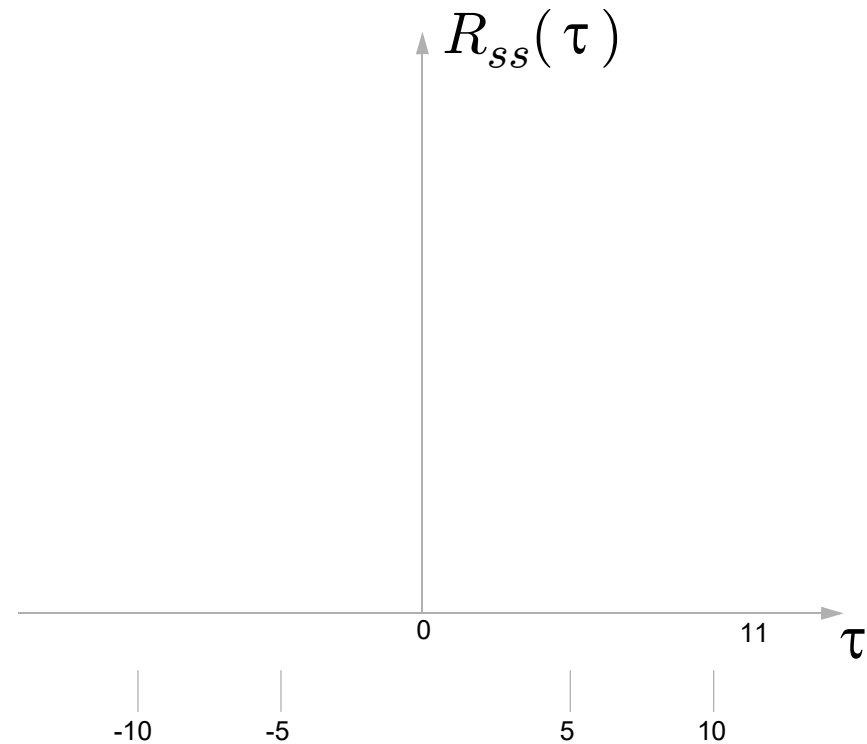


Length-11 Barker code:

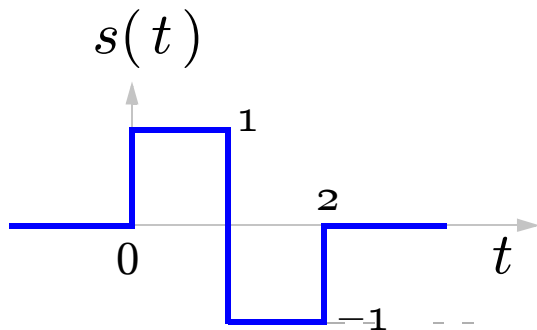
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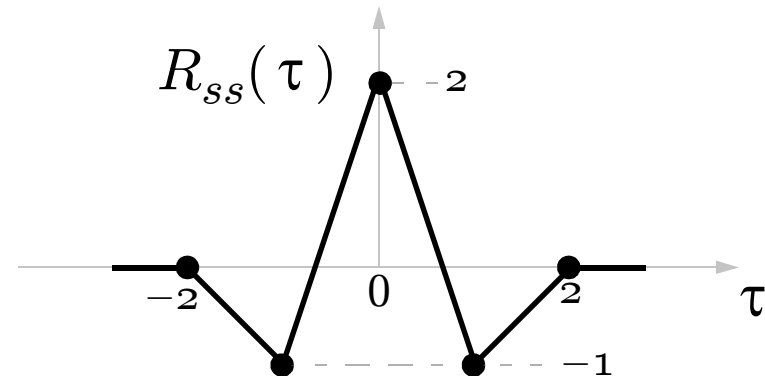
$\Rightarrow$



# Binary Code Autocorrelation Examples

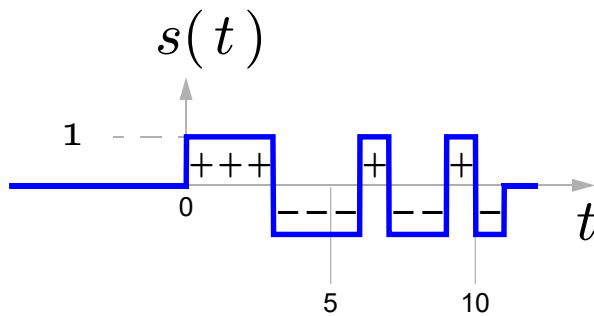


$\Rightarrow$

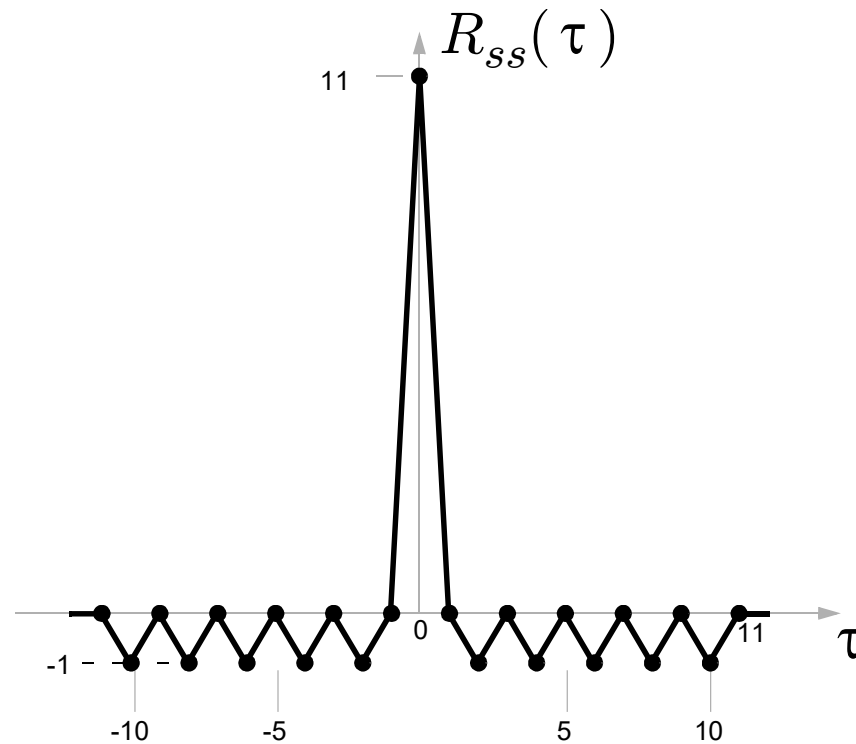


Length-11 Barker code:

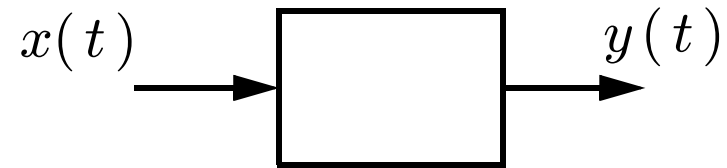
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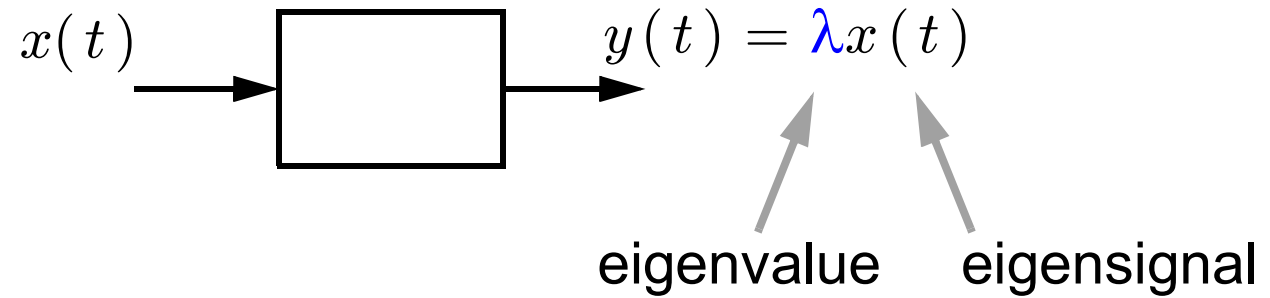
$\Rightarrow$



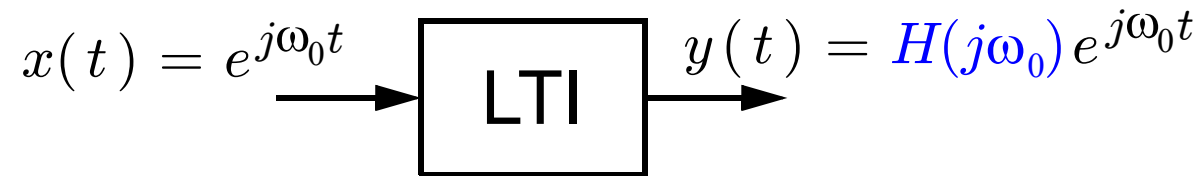
# What is an Eigensignal?



# What is an Eigensignal?



# Complex Exponentials are Eigensignals of LTI Systems



Scaling constant (eigenvalue) is:

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt \quad \text{“frequency response”}$$

evaluated at  $\omega_0$ .

# Hermitian (Conjugate) Symmetry

**Fact:** If  $h(t)$  is real, then  $H(-j\omega) = H^*(j\omega)$

Why?

Look at definition:  $H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt.$

- ▷ The LHS of equality replaces  $\omega$  by  $-\omega$  in integral.
- ▷ The RHS of equality replaces  $j$  by  $-j$  in integral.
- ▷ Effect on the integral is the same  $\Rightarrow$  LHS = RHS.

# Response to Real Sinusoid

Response to *real* sinusoid

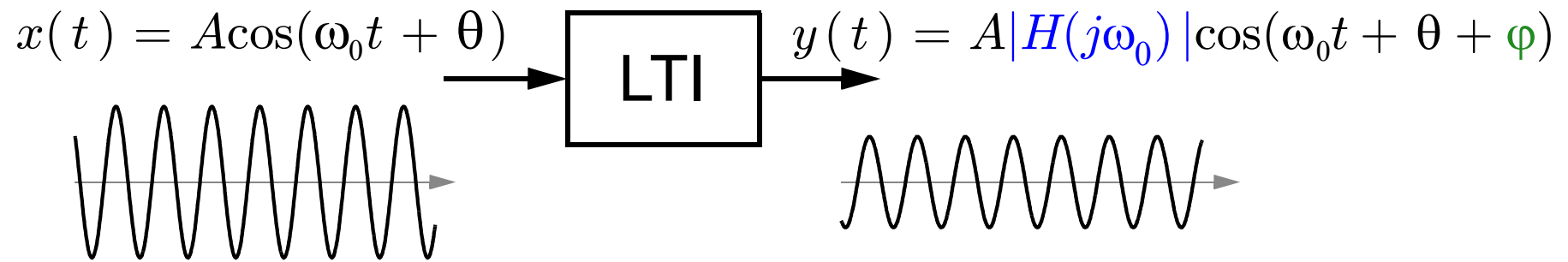
$$x(t) = A \cos(\omega_0 t + \theta) = \frac{A}{2} e^{j\theta} e^{j\omega_0 t} + \frac{A}{2} e^{-j\theta} e^{-j\omega_0 t}$$

is therefore (exploiting linearity)

$$\begin{aligned} y(t) &= \frac{A}{2} e^{j\theta} H(\omega_0) e^{j\omega_0 t} + \frac{A}{2} e^{-j\theta} H(-\omega_0) e^{-j\omega_0 t} \\ &= \left( \frac{A}{2} e^{j\theta} H(\omega_0) e^{j\omega_0 t} \right) + \left( \frac{A}{2} e^{j\theta} H(\omega_0) e^{j\omega_0 t} \right)^* \\ &= 2 \operatorname{Re} \{ \cdot \} \\ &= A |H(j\omega_0)| \cos(\omega_0 t + \theta + \varphi) \end{aligned}$$

where  $\varphi = \angle \{ H(j\omega_0) \}$ .

# Sinusoid-In, Sinusoid Out



An LTI system does 2 things to a sinusoid with frequency  $\omega_0$ :

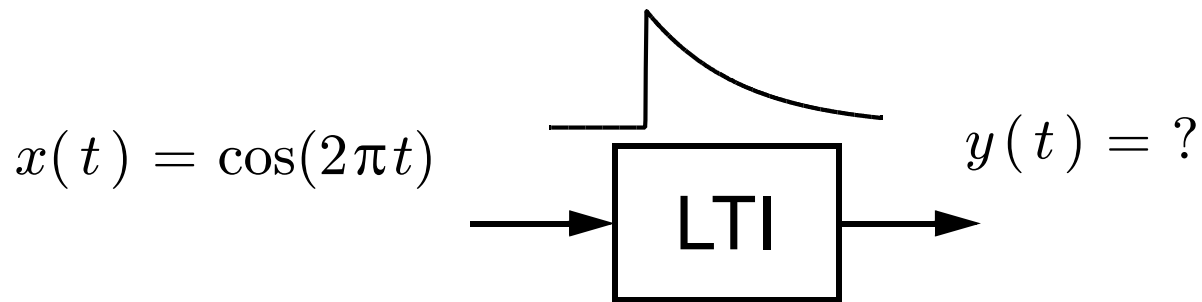
- ▷ it attenuates amplitude by  $|H(j\omega_0)|$
- ▷ it shifts phase by  $\varphi = \text{angle}\{H(j\omega_0)\}$

*Notation:*

- ▷  $H(j\omega)$  = frequency response
- ▷  $|H(j\omega)|$  = magnitude response
- ▷  $\text{angle}\{H(j\omega)\}$  = phase response

# Numeric Example

Find output  $y(t)$  when  $x(t) = \cos(2\pi t)$  is input to an LTI system with impulse response  $h(t) = 2e^{-2t}u(t)$ :

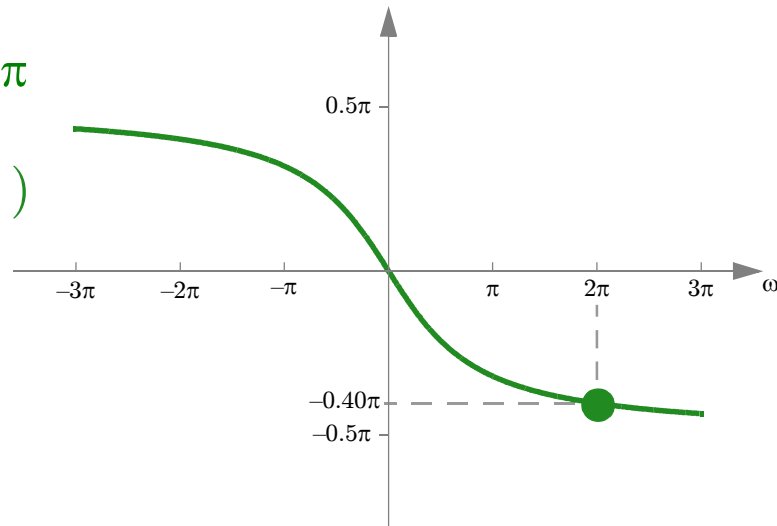
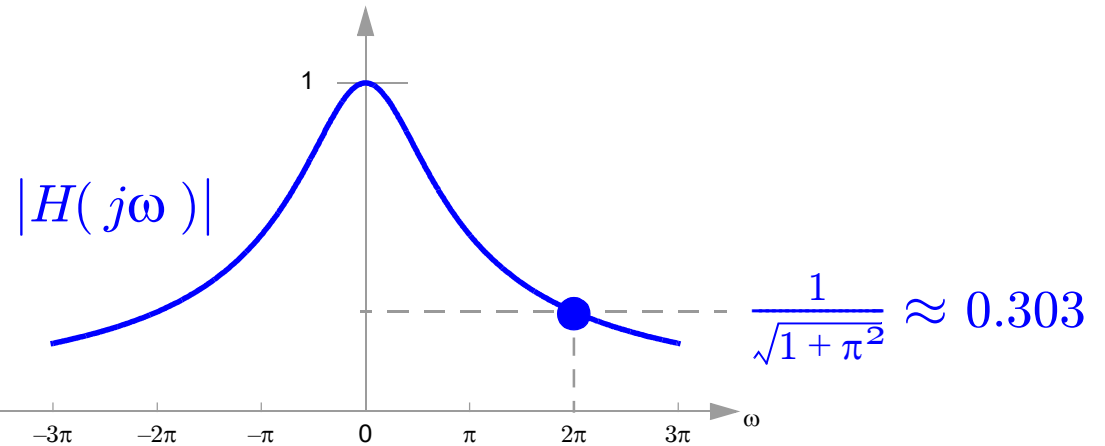


# Evaluate at $\omega = 2\pi$

Integrate

$$\Rightarrow H(j\omega) = \frac{2}{2 + j\omega}:$$

$$\begin{aligned}\Rightarrow H(j2\pi) &= \frac{1}{1 + j\pi} \\ &\approx 0.303e^{-j0.40\pi} \\ \arg H(j\omega) &\end{aligned}$$



$$\begin{aligned}x(t) = \cos(2\pi t) &\rightarrow \boxed{\text{LTI}} \rightarrow y(t) = |H(j2\pi)| \cos(2\pi t + \varphi) \\ &\approx 0.303 \cos(2\pi t - 0.40\pi)\end{aligned}$$