Lecture 8: Thu Sep 10, 2020

Reminder:

- HW3 due midnight tonight.

Lecture

- review RADAR problem: an application of convolution
  - ▹ cross correlation, autocorrelation function, and the matched filter
  - ▹ good radar signals: pulse compression
- eigensignals of LTI systems
- frequency response
- LTI systems: sinusoid in, sinusoid out

Reading Assignment

Handouts for ECE 3084

- syllabus
- 3084 book

(FINISH)

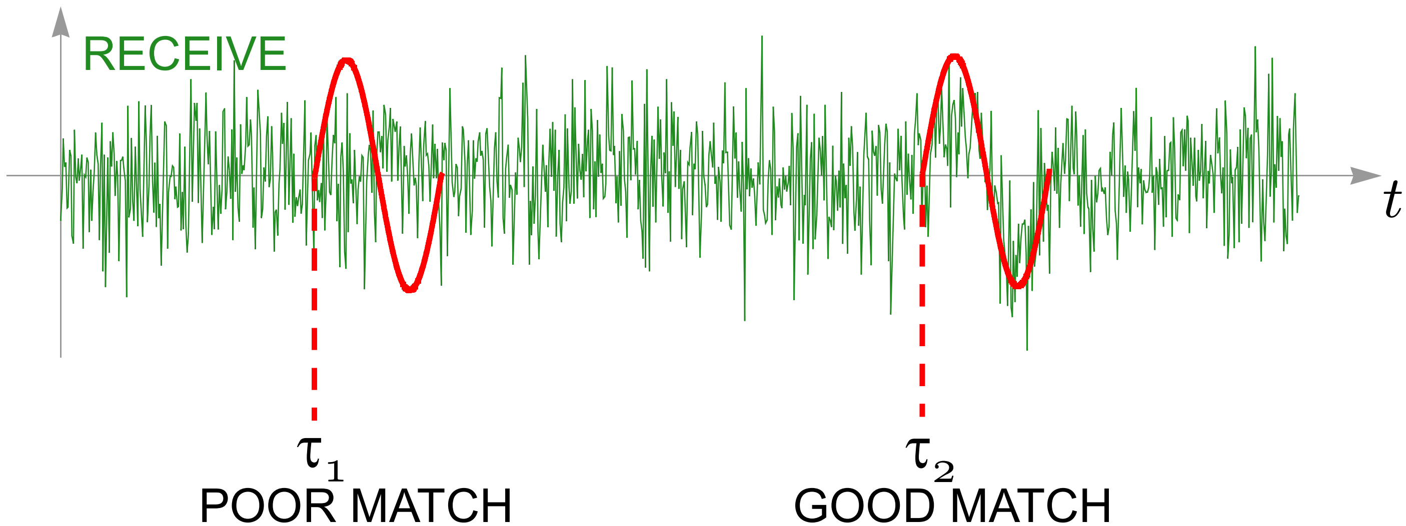
Coming Next: Frequency Response

(Chapter 10 of 2026 1st edition.)

10 Frequency Response 285

Radar: Look for Best Match

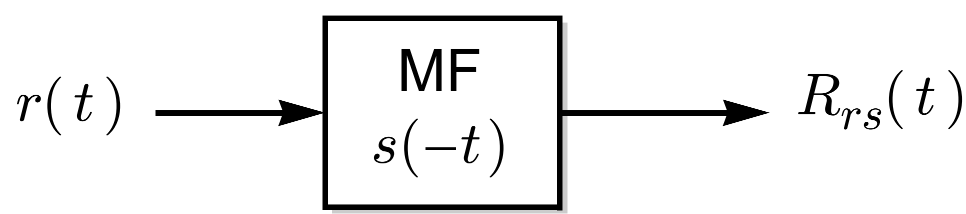
Conceptually we want to *slide* template $s(t-\tau)$ and look for "best" match



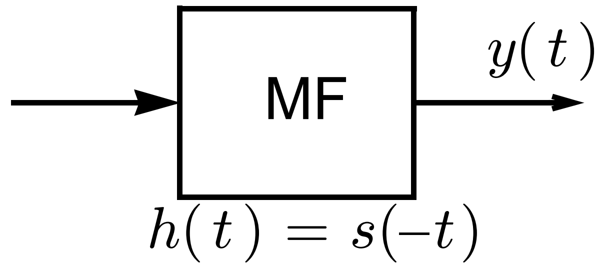
Last time: the best match maximizes the "crosscorrelation function"

$$R_{rs}(\tau) = \int_{-\infty}^{\infty} r(t)s(t-\tau)dt$$

Implementation:



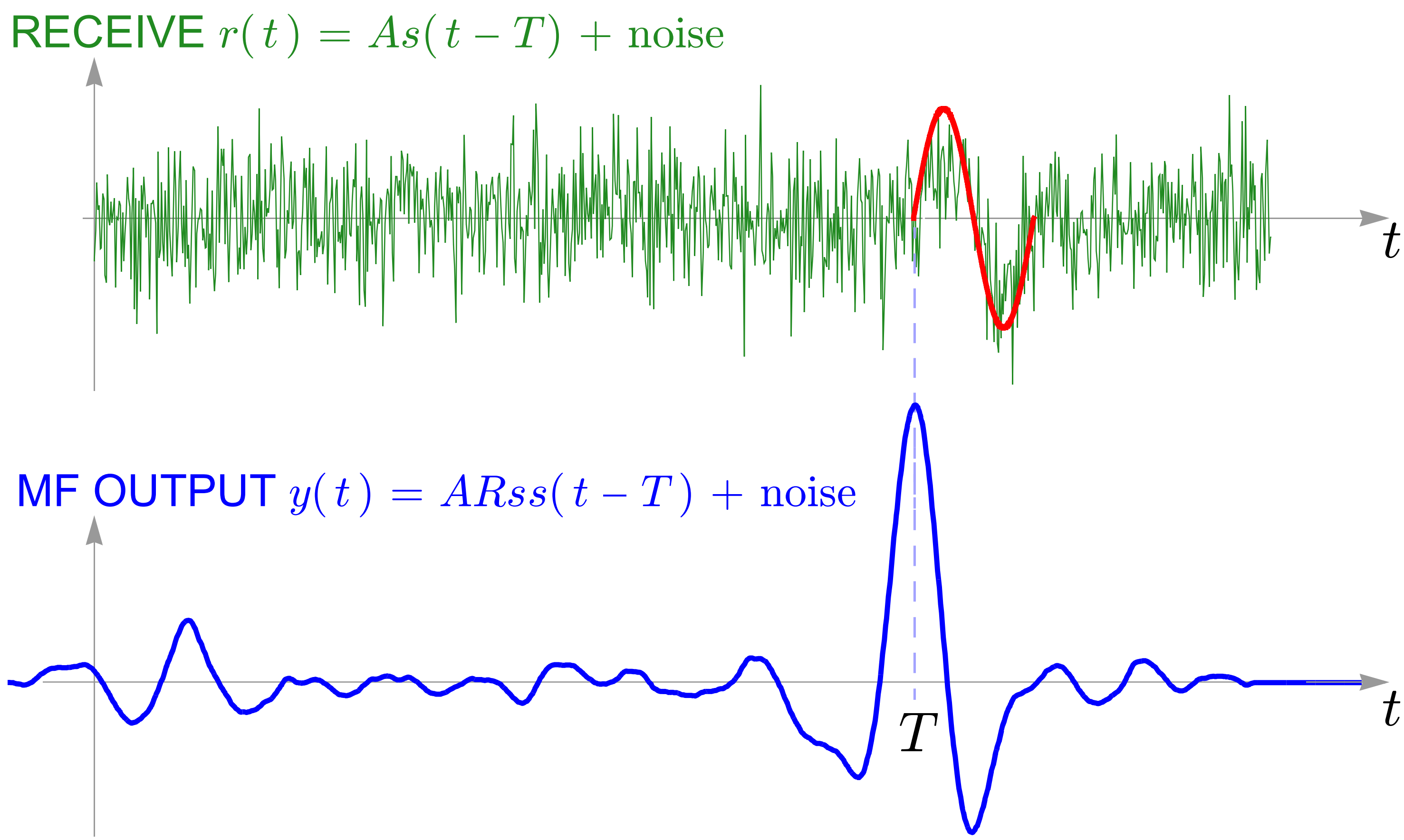
Two Common MF Inputs



❶ input is $s(t)$ $\Rightarrow$ output is $R_{ss}(t)$.

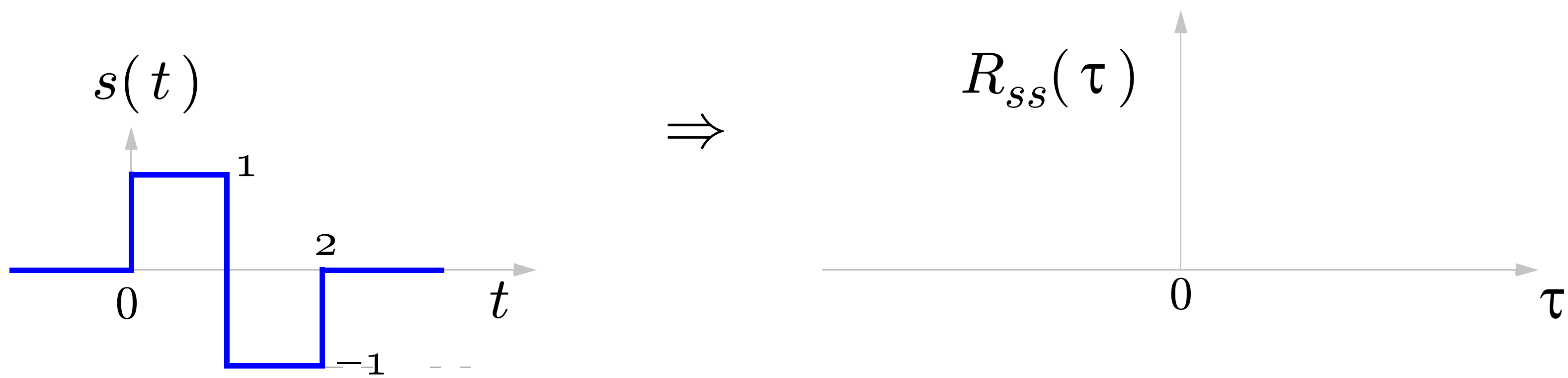
❷ input is $r(t) = As(t - T)$ $\Rightarrow$ output is $AR_{ss}(t - T)$.

With Noise

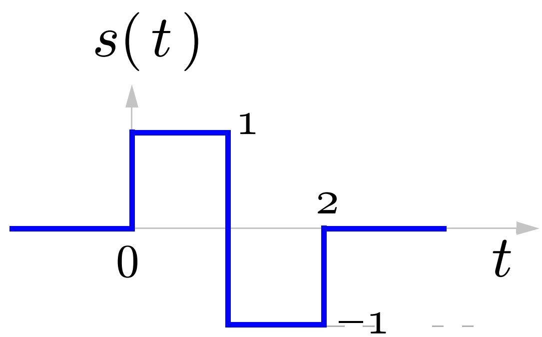


RECEIVE $r(t) = As(t-T) + \text{noise}$
t
MF OUTPUT $y(t) = ARss(t-T) + \text{noise}$
T
t

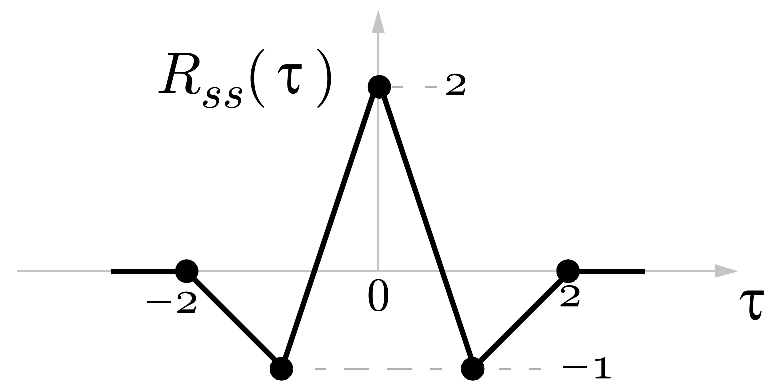
Binary Code Autocorrelation Examples



Binary Code Autocorrelation Examples



$\Rightarrow$



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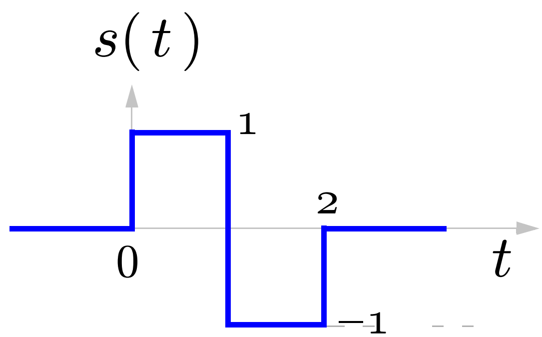

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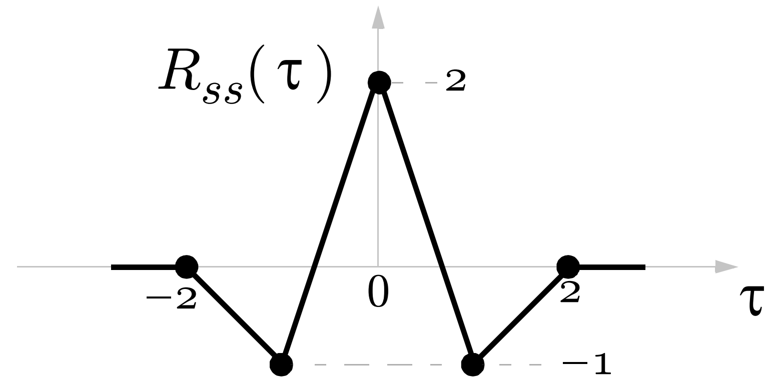
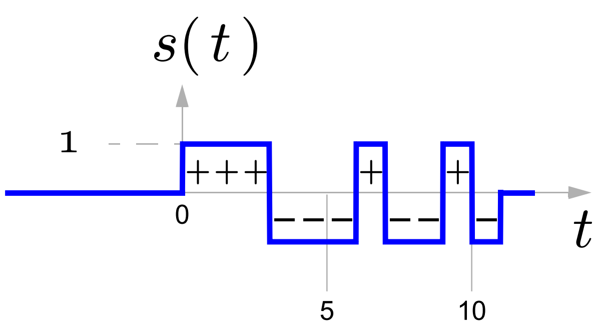
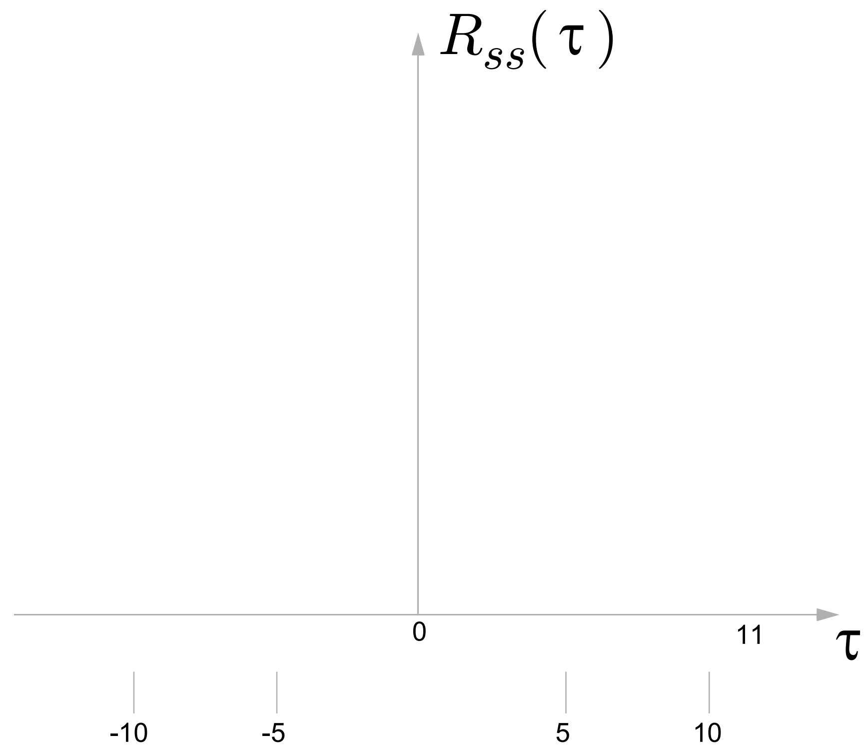


Length-11 Barker code:

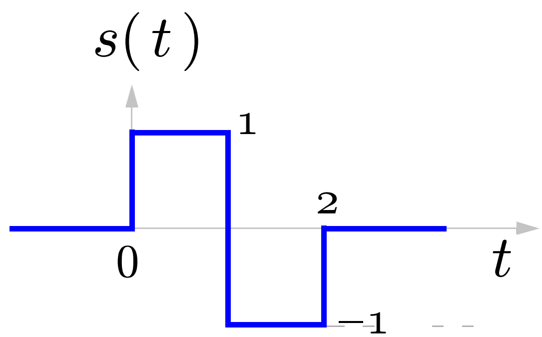
+++−−−+−−+−



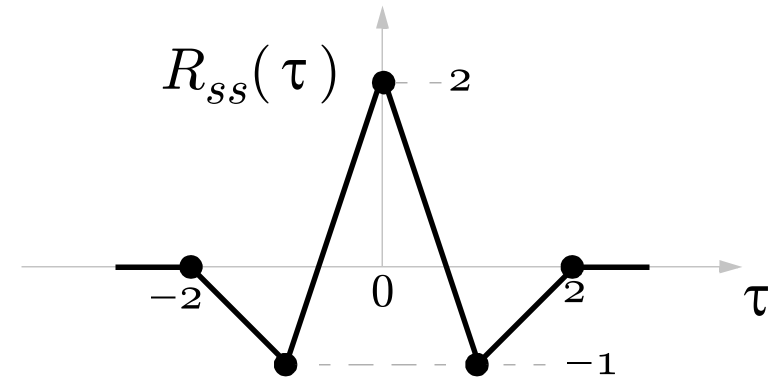
$\Rightarrow$



Binary Code Autocorrelation Examples



$\Rightarrow$



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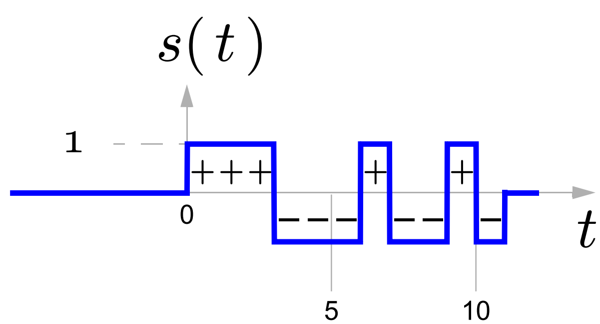

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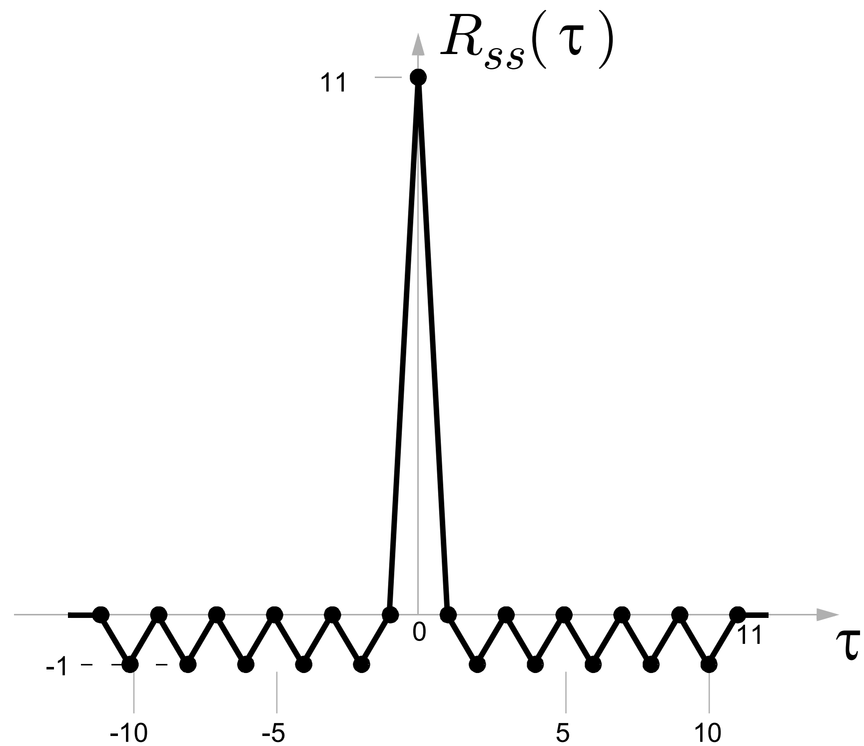

What is an Eigensignal?

$$x(t) \longrightarrow \boxed{} \longrightarrow y(t)$$

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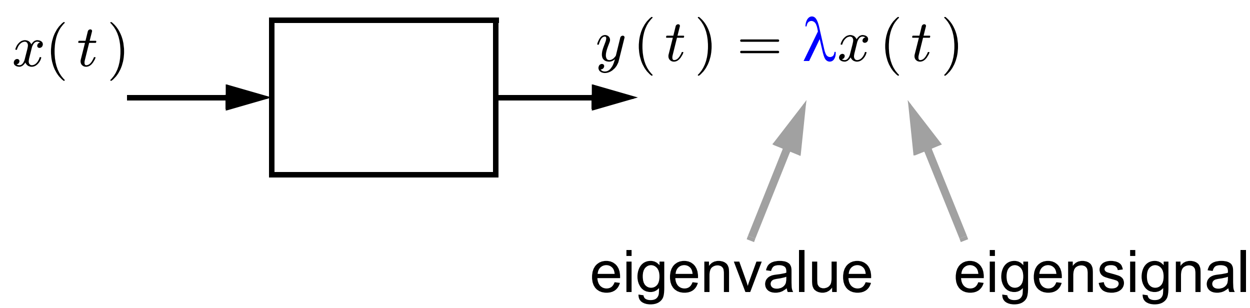
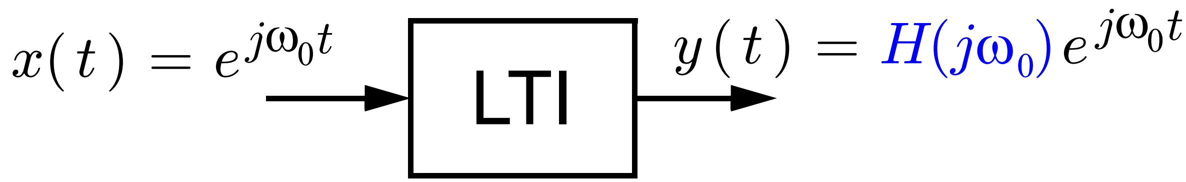


Complex Exponentials are Eigensignals of LTI Systems



Scaling constant (eigenvalue) is:

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$$ "frequency response"

evaluated at ω_0.

Hermitian (Conjugate) Symmetry

Fact: If $h(t)$ is real, then $H(-j\omega) = H^*(j\omega)$

Why?

Look at definition: $H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$.

▷ The LHS of equality replaces ω by $-\omega$ in integral.

▷ The RHS of equality replaces j by $-j$ in integral.

▷ Effect on the integral is the same $\Rightarrow$ LHS = RHS.

Response to Real Sinusoid

Response to *real* sinusoid

$$x(t) = A\cos(\omega_0 t + \theta) = \frac{A}{2}e^{j\theta}e^{j\omega_0 t} + \frac{A}{2}e^{-j\theta}e^{-j\omega_0 t}$$

is therefore (exploiting linearity)

$$\begin{aligned} y(t) &= \frac{A}{2}e^{j\theta}H(\omega_0)\,e^{j\omega_0 t} + \frac{A}{2}e^{-j\theta}H(-\omega_0)\,e^{-j\omega_0 t} \\ &= \left(\frac{A}{2}e^{j\theta}H(\omega_0)\,e^{j\omega_0 t}\right) + \left(\frac{A}{2}e^{j\theta}H(\omega_0)\,e^{j\omega_0 t}\right)^* \end{aligned}$$

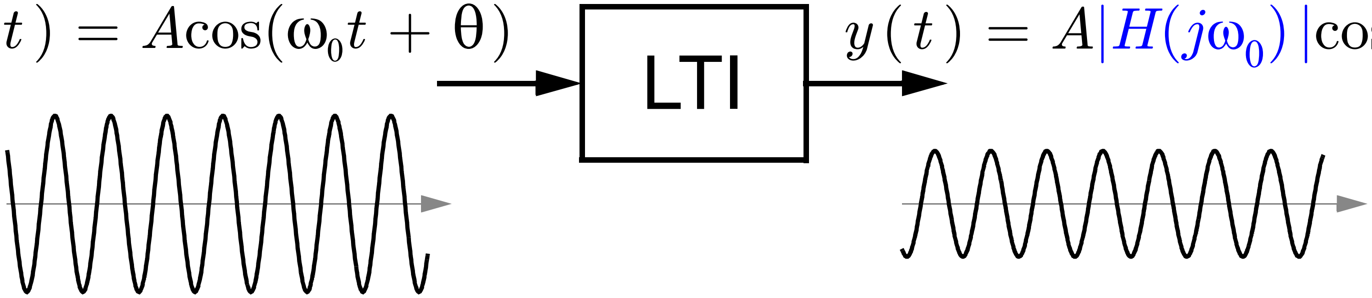
$$= 2\mathrm{Re}\{\ \cdot\ \}$$

$$= A|H(j\omega_0)|\cos(\omega_0 t + \theta + \varphi)$$

where $\varphi = \text{angle}\{H(j\omega_0)\}$.

Sinusoid-In, Sinusoid Out

$x(t) = A\cos(\omega_0 t + \theta)$ $\quad$ $y(t) = A|H(j\omega_0)|\cos(\omega_0 t + \theta + \varphi)$



An LTI system does 2 things to a sinusoid with frequency ω_0:

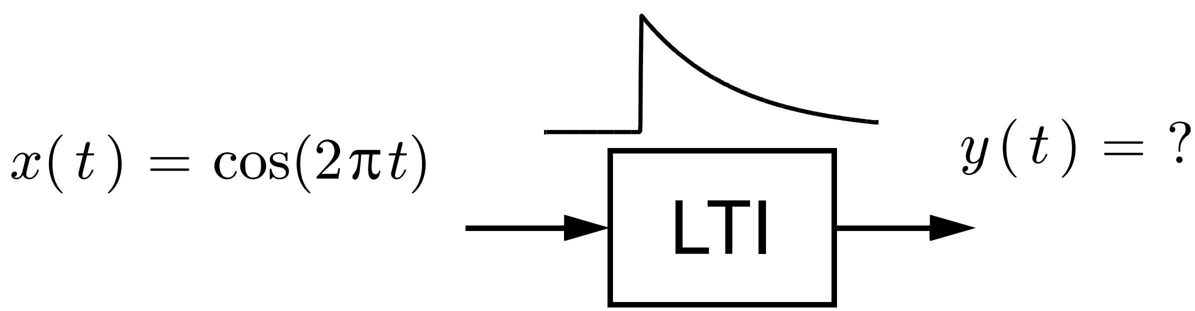
- ▷ it attenuates amplitude by $|H(j\omega_0)|$
- ▷ it shifts phase by $\varphi = \text{angle}\{H(j\omega_0)\}$

Notation:

- ▷ $H(j\omega)$ = frequency response
- ▷ $|H(j\omega)|$ = magnitude response
- ▷ $\text{angle}\{H(j\omega)\}$ = phase response

Numeric Example

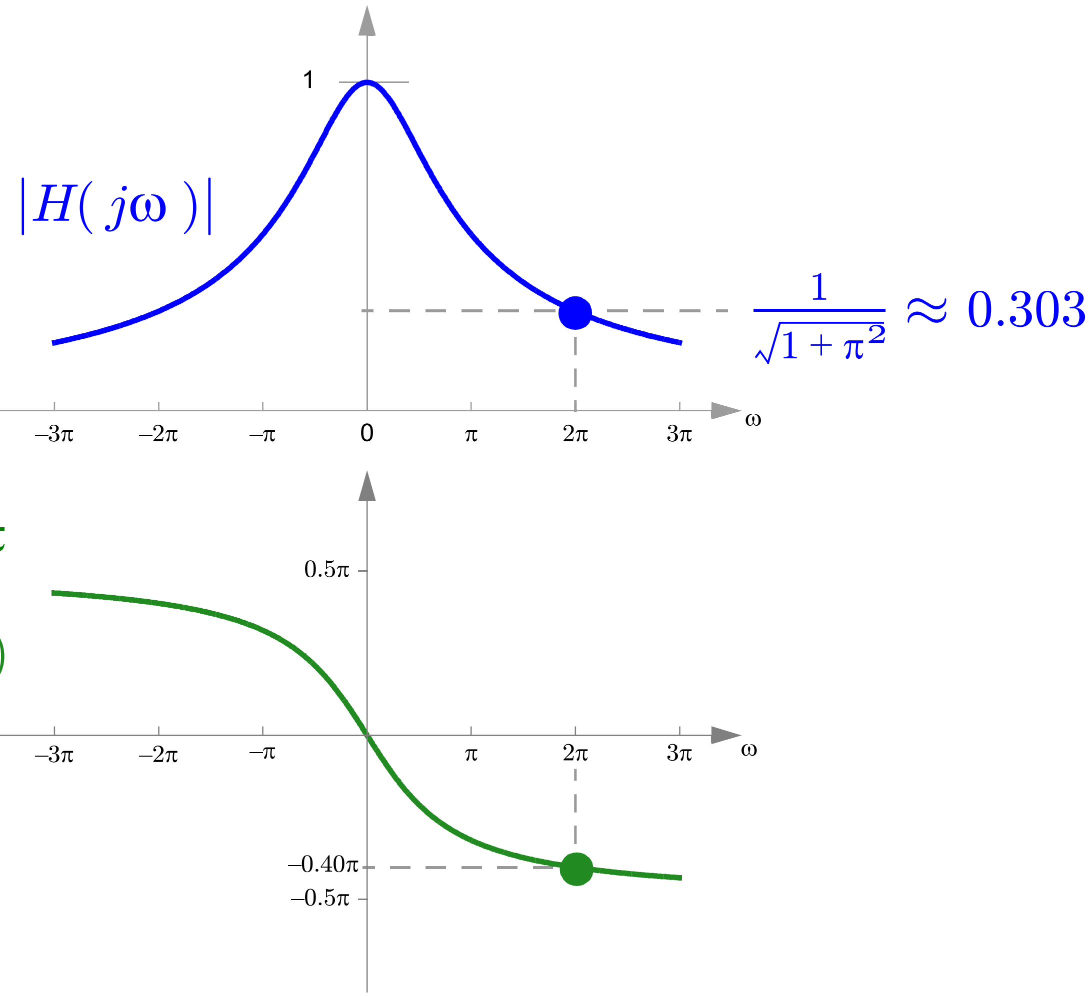
Find output $y(t)$ when $x(t) = \cos(2\pi t)$ is input to an LTI system with impulse response $h(t) = 2e^{-2t}u(t)$:



Evaluate at $\omega = 2\pi$

Integrate

$$\Rightarrow H(j\omega) = \frac{2}{2 + j\omega}:$$

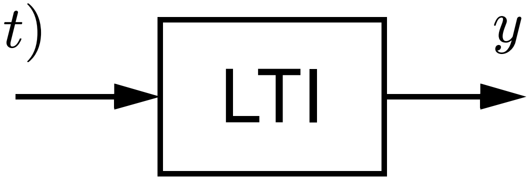


$$\Rightarrow H(j2\pi) = \frac{1}{1 + j\pi}$$

$$\approx 0.303e^{-j0.40\pi}$$

$\arg H(j\omega)$

$$x(t) = \cos(2\pi t)$$



$$y(t) = |H(j2\pi)|\cos(2\pi t + \varphi)$$

$$\approx 0.303\cos(2\pi t - 0.40\pi)$$