Lecture 6: Thu Sep 3, 2020

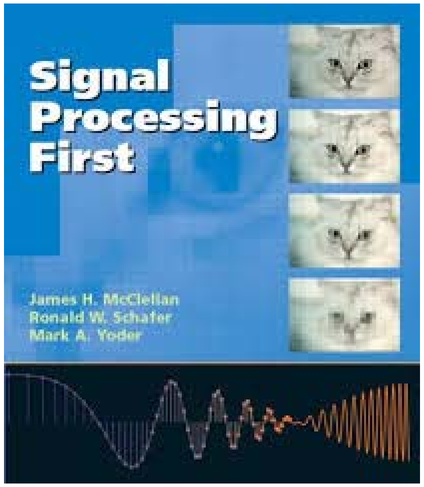
Reminder:

- HW2 due midnight tonight.
- HW3 posted later today.

Lecture

- convolution: more properties
- convolution examples

Reminder: Reading Assignment



(TODAY)

Reading Assignment

Handouts for ECE 3084

- syllabus
- 3084 book

(TODAY)

Reminder: Videos from Summer

https://www.youtube.com/playlist?list=PLOunECWxELQRYwsuj4BL4Hu1nvj9dxRQ6

Over 60 minutes of convolution examples:

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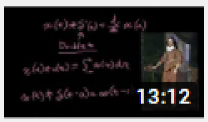
ECE3084 Lecture 10: Convolution: Flipping & Shifting (Signals & Systems, Summer 2020, Georgia Tech)
Lantertronics

11 

ECE3084 Lecture 11: Convolution with Impulses, Steps, & Doublets (Signals & Systems, 2020, GA Tech)
Lantertronics

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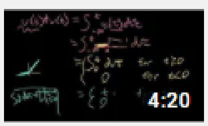
ECE3084 Lecture 12: Convolving Boxcars (Signals & Systems, Summer 2020, Georgia Tech)
Lantertronics

13 

ECE3084 Bonus Lecture 12A: Convolving Step Functions (Signals & Systems, Summer 2020, Georgia Tech)
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ECE3084 Bonus Lecture 12B: Multiplying Step Functions (Signals & Systems, 2020, Georgia Tech)
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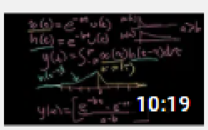
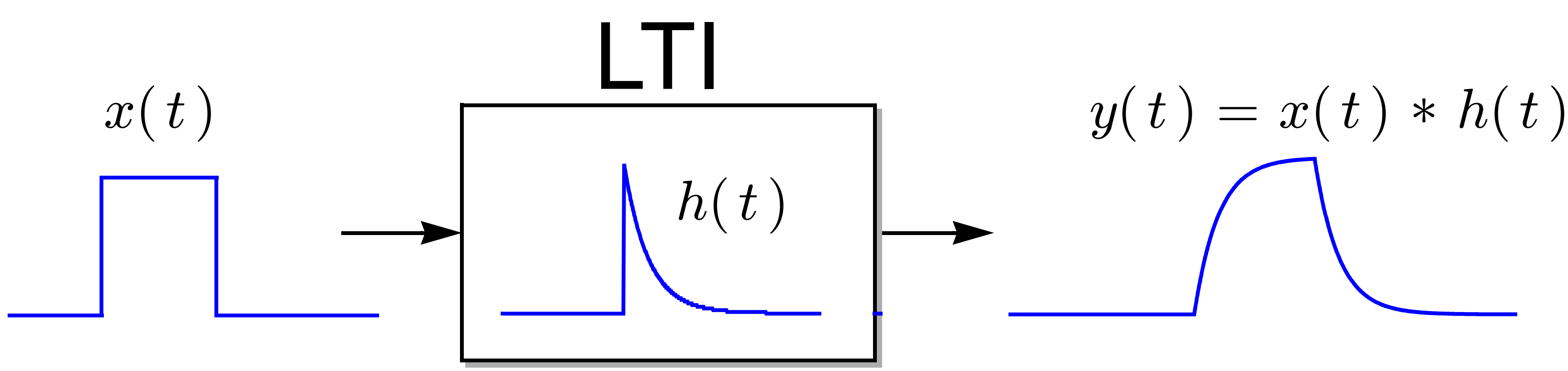
15 

ECE3084 Bonus Lecture 12C: Convolving Decaying Exponentials (Signals & Systems, 2020, Georgia Tech)
Lantertronics

Why Impulse Response is Important

An LTI system is *completely* characterized by $h(t)$.

Response to ***any*** input can be found by convolving it with $h(t)$:



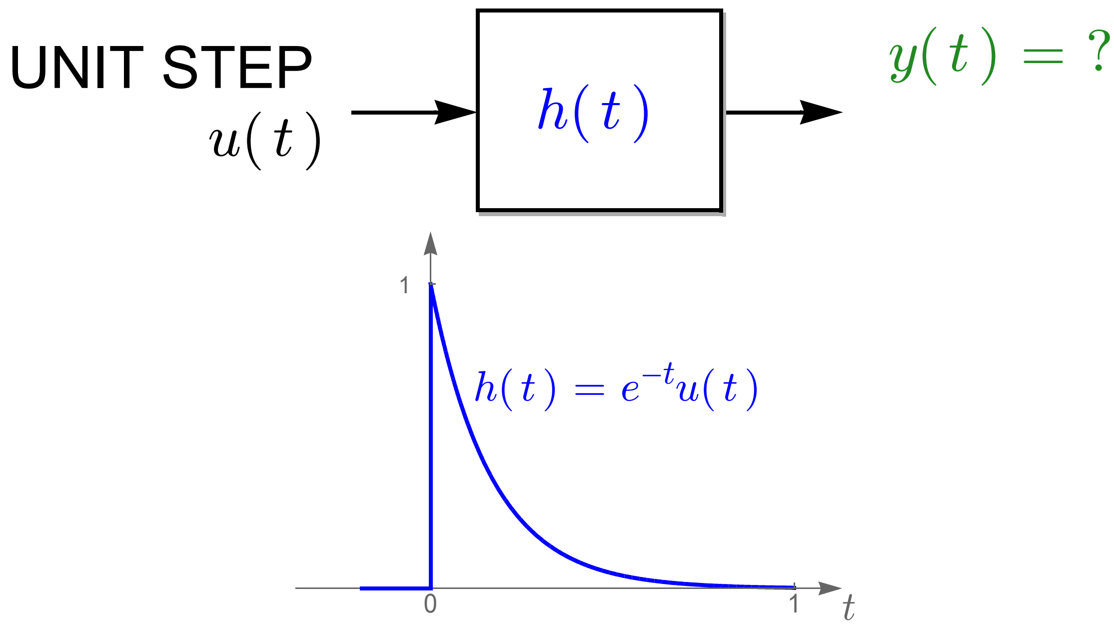
Convolution Properties

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

- commutative: $x(t) * h(t) = h(t) * x(t)$ ✓
- associative $(x(t) * h(t)) * z(t) = x(t) * (h(t) * z(t))$ ✓
- Convolving with an impulse:
  - ▷ $x(t) * \delta(t) = x(t)$ ✓
  - ▷ $x(t) * \delta(t-t_0) = x(t-t_0)$ ✓
- Delay property: $x(t) * h(t-t_0) = x(t-t_0) * h(t)$
- Derivative: $\frac{d}{dt}(x(t) * h(t)) = (\frac{d}{dt}x(t)) * h(t) = x(t) * (\frac{d}{dt}h(t))$

Pop Quiz

Find the "step response" of a filter with an exponential impulse response:



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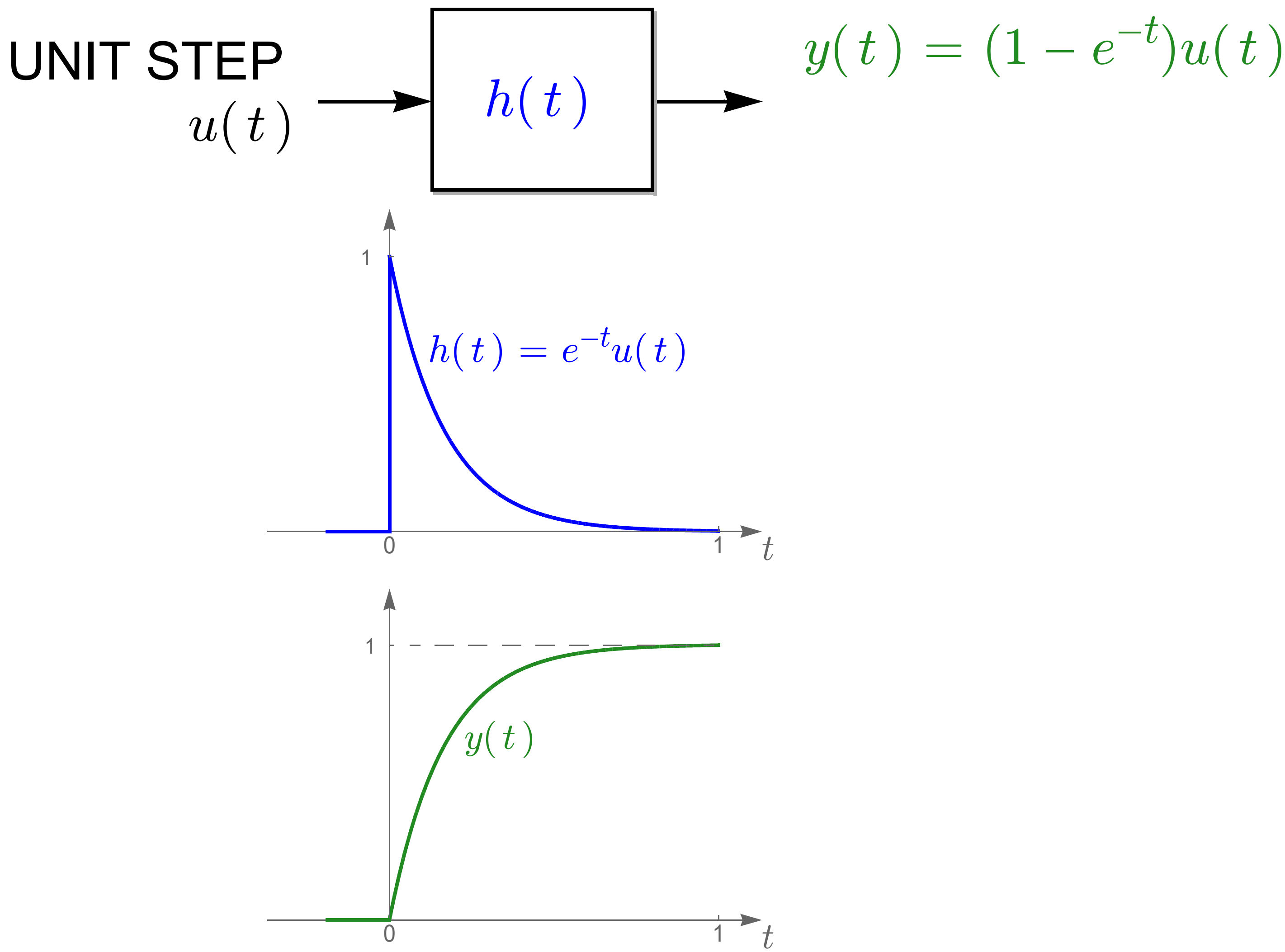
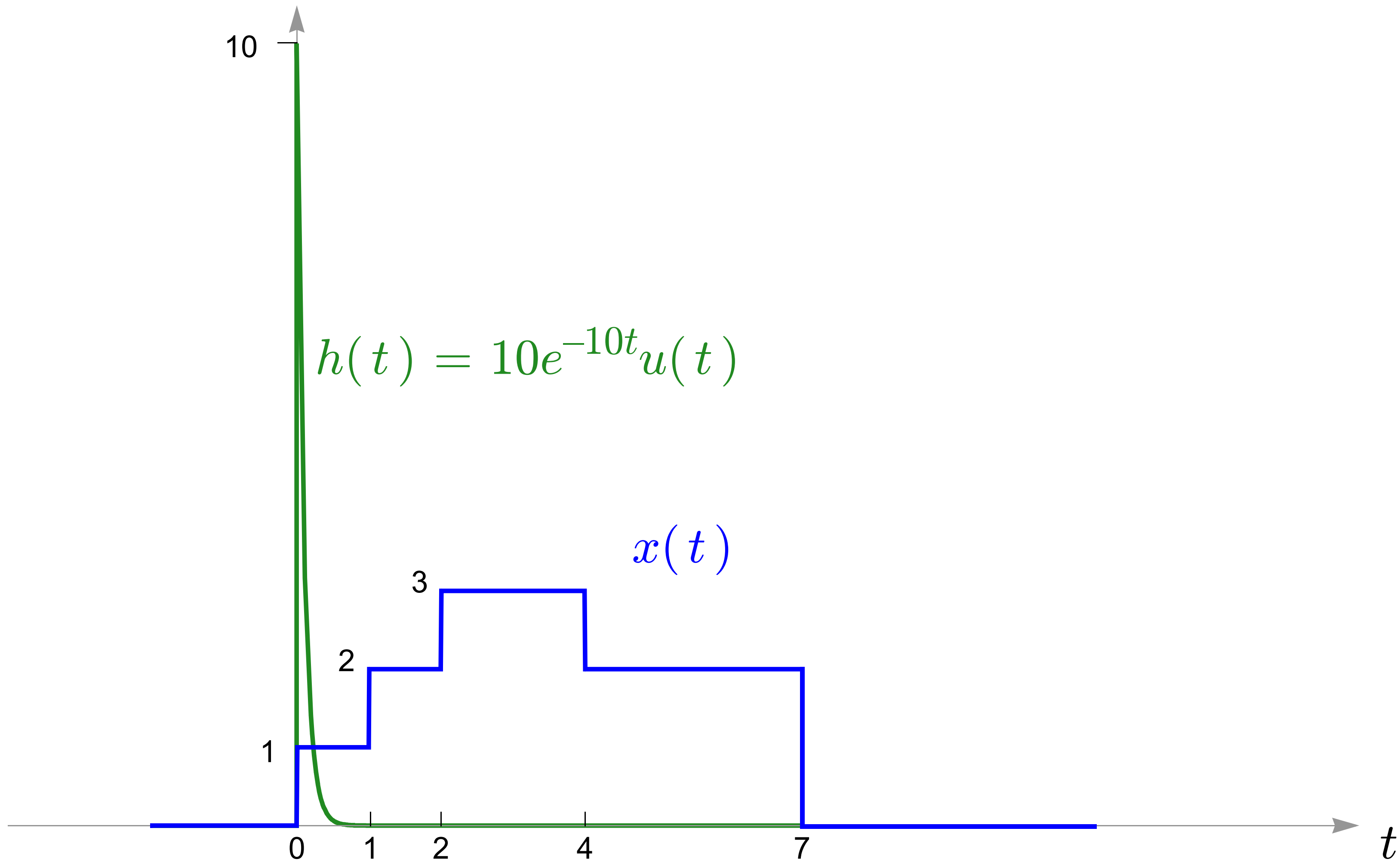


Sketch Convolution of x(t) with h(t):



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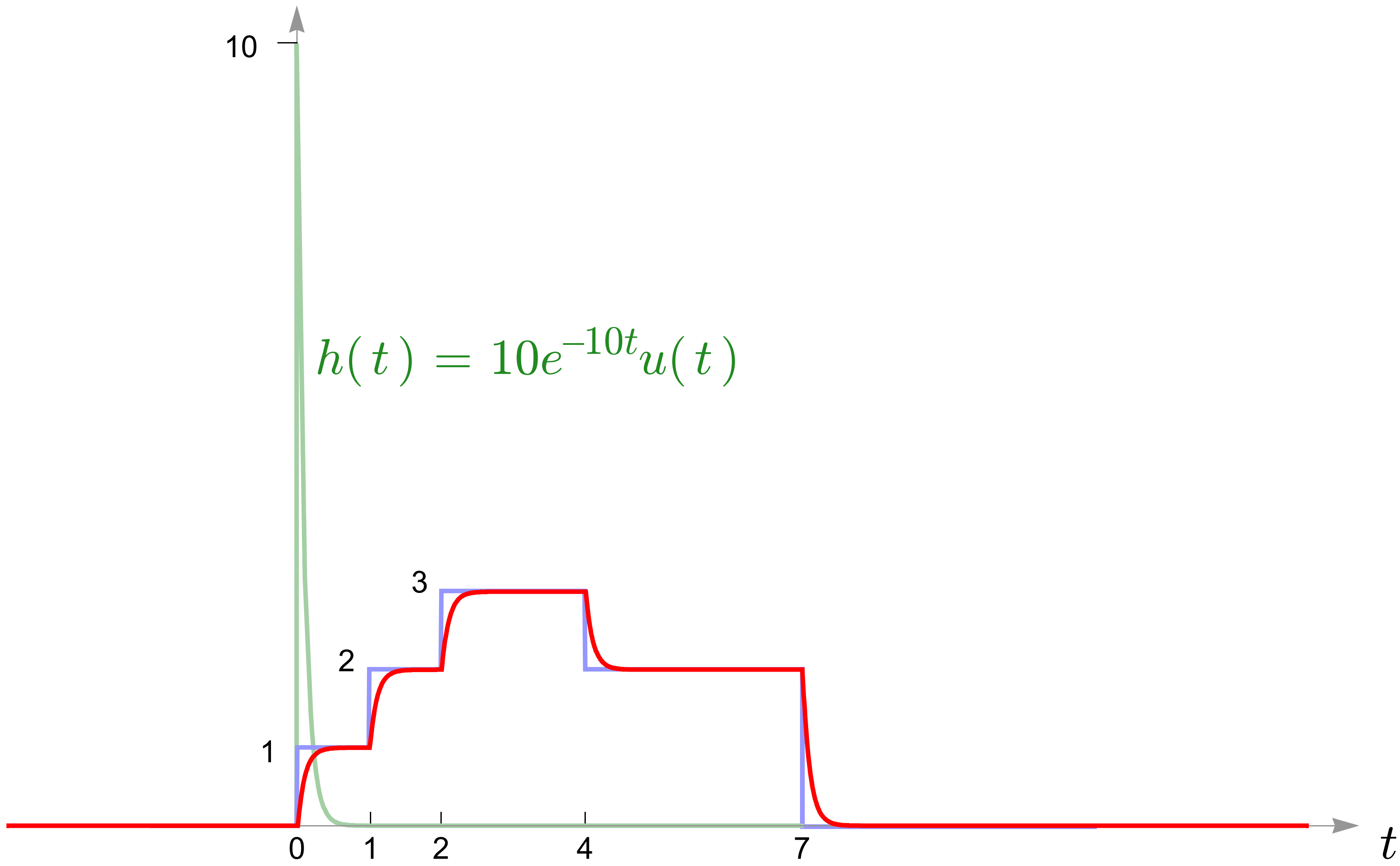


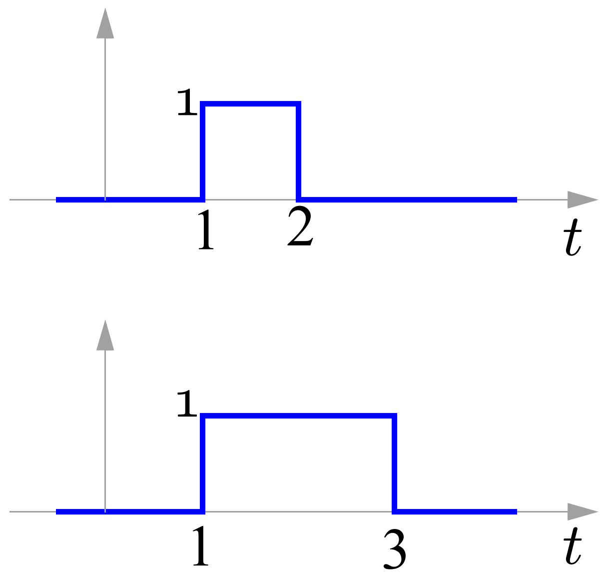
Convolution Tips

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\mathrm{d}\tau$$

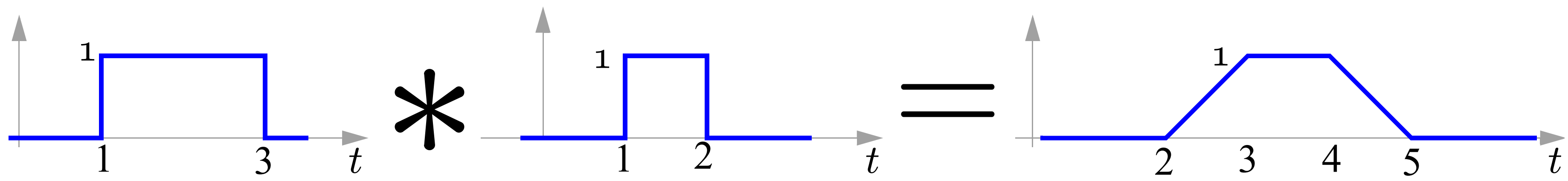
- graphical approach mimics convolution integral
- graphs are plotted versus dummy variable τ
- convolution is commutative $\Rightarrow$ pick "simpler" signal to flip!
- Getting the right limits of integration is 90% of job, integration itself is easy
- Use **`cconvdemo`** to learn the basics

Example of Convolution

Convolve these two rectangles:



Answer is a Trapezoid



- ▷ One rectangle floats past another
- ▷ Once they begin to overlap,
  amount of overlap increases *linearly* with time $\Rightarrow$ ramp up
- ▷ While they overlap fully, the signal plateaus
- ▷ When the amount of overlap decreases it decreases linearly
  $\Rightarrow$ ramp back down

Fun Convolution Facts

- convolving two steps yields a ramp
- convolving a rect with itself yields a triangle
- convolving rects of different widths yields a trapezoid
- duration of convolution is sum of durations of each
- convolving a signal that starts at time t_1 with another starting at t_2 yields a signal that starts at $t_1 + t_2$

Example 1: