Lecture 5: Tue Sep 1, 2020

Reminder:

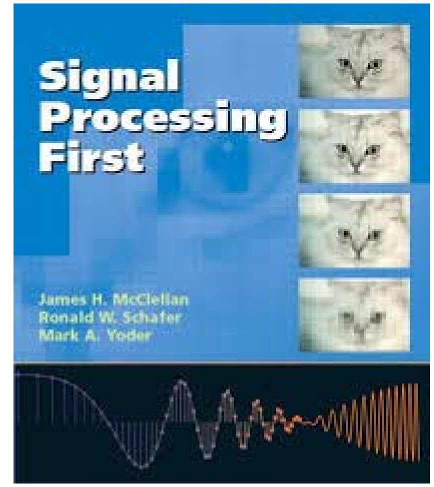
- HW1 solutions posted
- HW2 due Thursday.

Lecture

- LTI systems
- impulse response
- convolution integral

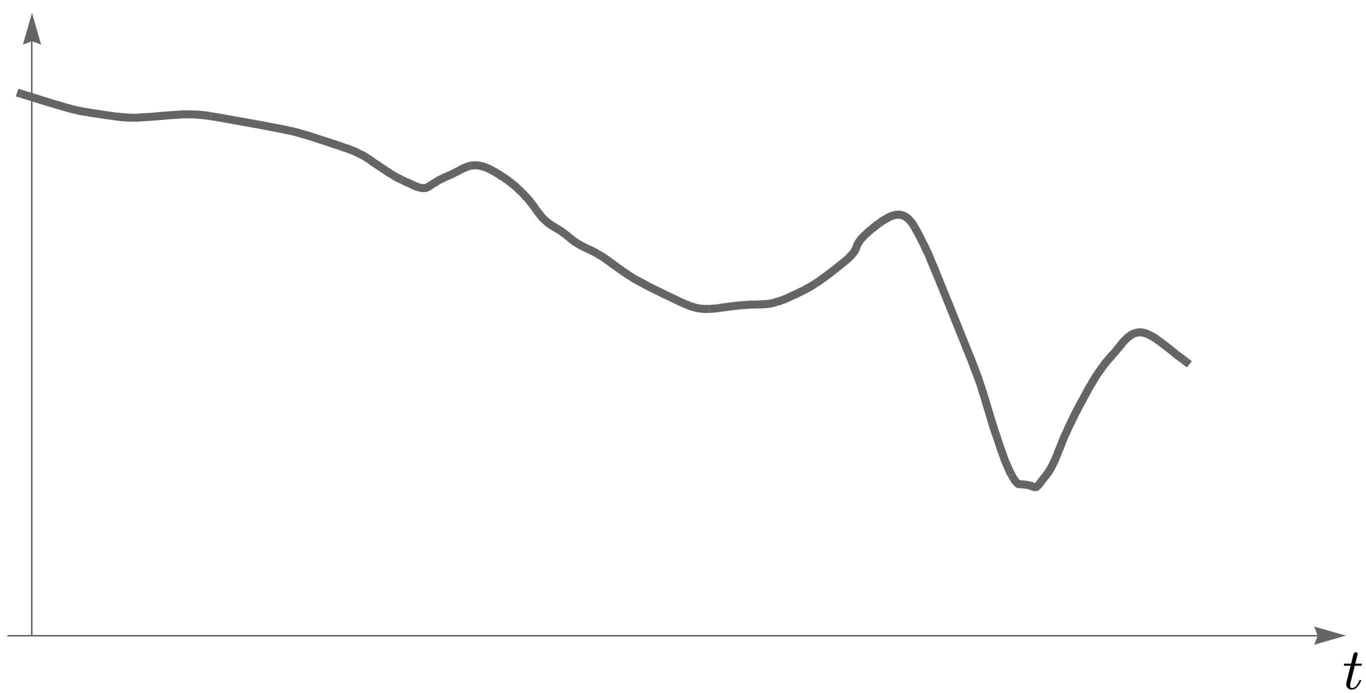
Reminder: Reading Assignment

9 Continuous-Time Signals and LTI Systems 245



Representation Property

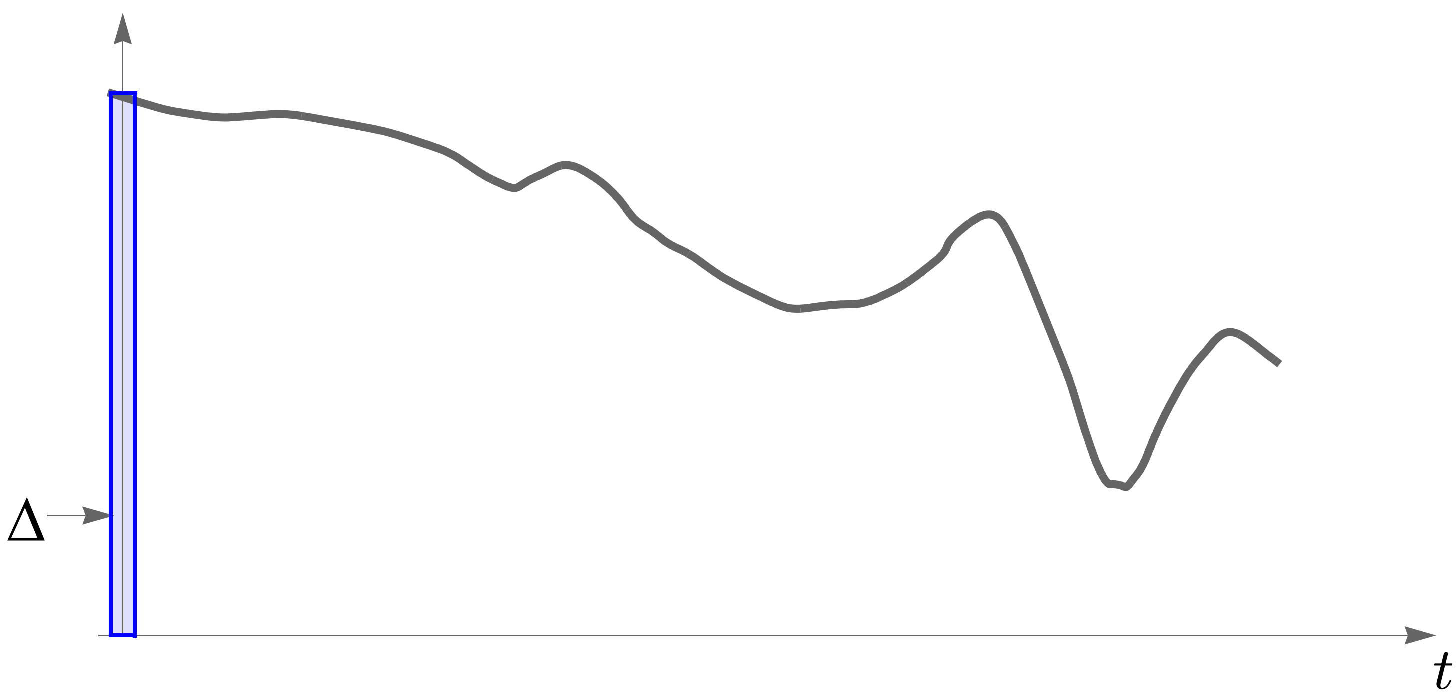
Any signal can be written as “sum” of Diracs:



Representation Property

Any signal can be written as “sum” of Diracs:

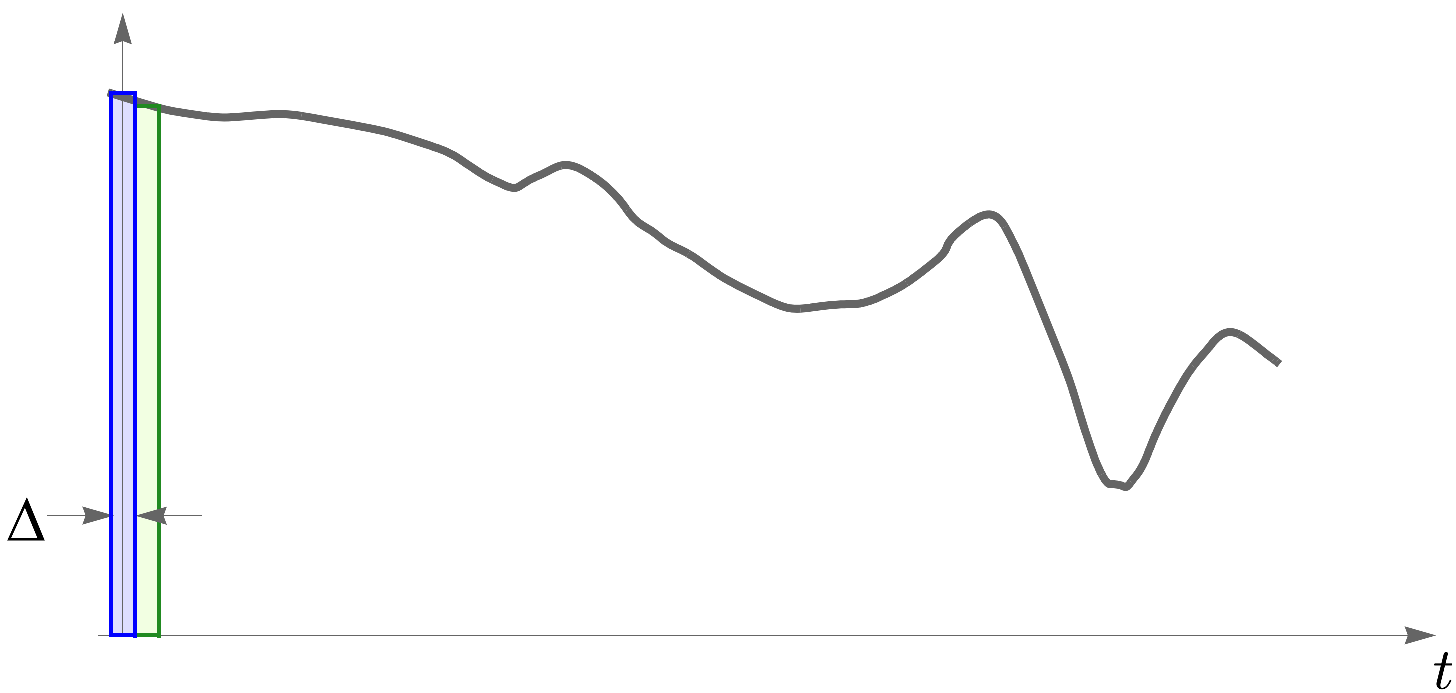
$$x(t) \approx \Delta x(0) g(t) + \ldots$$



Representation Property

Any signal can be written as “sum” of Diracs:

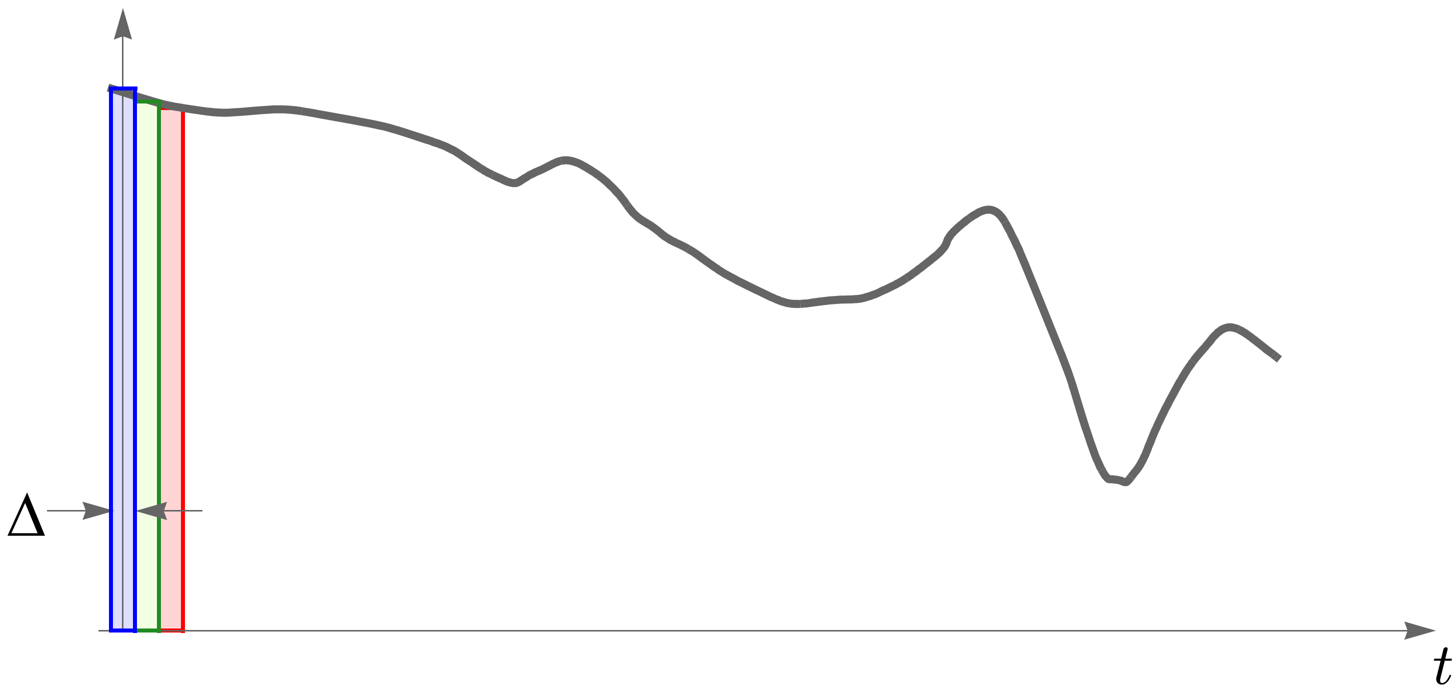
$$x(t) \approx \Delta x(0) g(t) + \Delta x(\Delta) g(t - \Delta) + \ldots$$



Representation Property

Any signal can be written as “sum” of Diracs:

$$x(t) \approx \Delta x(0)g(t) + \Delta x(\Delta)g(t-\Delta) + \Delta x(2\Delta)g(t-2\Delta) + \ldots$$



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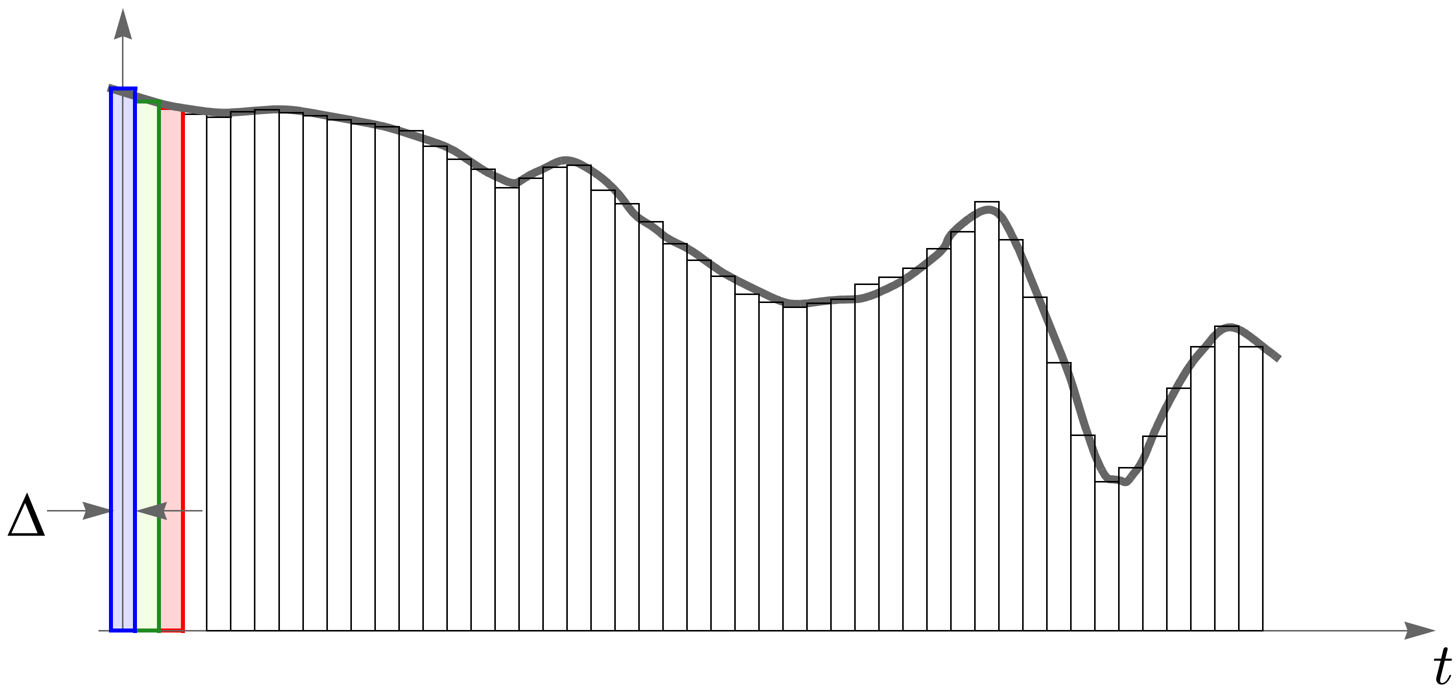
$$\approx \sum_k x(k\Delta)g(t-k\Delta)\Delta$$



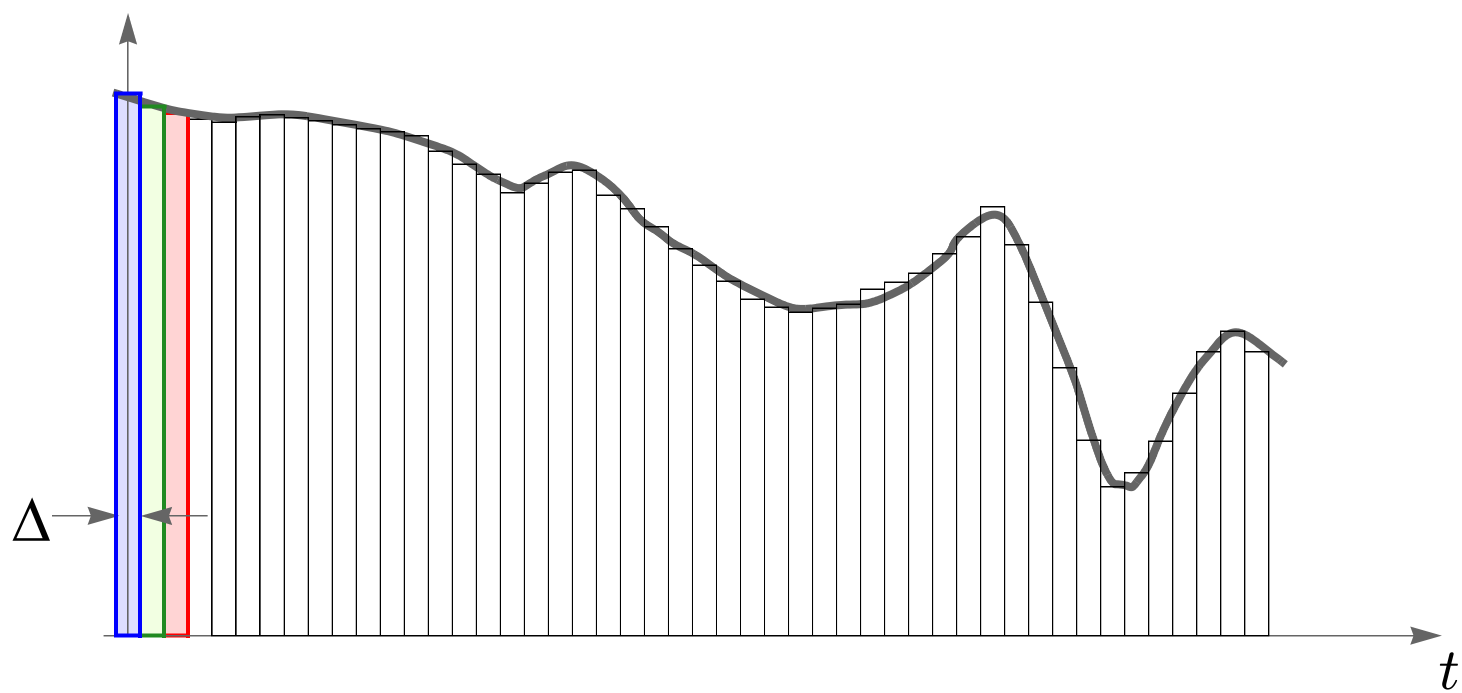
Representation Property

Any signal can be written as "sum" of Diracs:

$$x(t) \approx \Delta x(0)g(t) + \Delta x(\Delta)g(t-\Delta) + \Delta x(2\Delta)g(t-2\Delta) + \ldots$$

$$\approx \sum_k x(k\Delta)\delta(t-k\Delta)\Delta$$

$$\Rightarrow \boxed{x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau}$$



Impulse Response

Let $h(t)$ denote response of system to $\delta(t)$

Incredibly important for LTI systems. Why?

TI	$\Rightarrow$	response to	$\delta(t-\tau)$	is	$h(t-\tau)$,
L	$\Rightarrow$	response to	$x(\tau)\delta(t-\tau)$	is	$x(\tau)h(t-\tau)$.
L	$\Rightarrow$	response to integral (limiting case of a sum):			

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\mathrm{d}\tau$$

is

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\mathrm{d}\tau$$

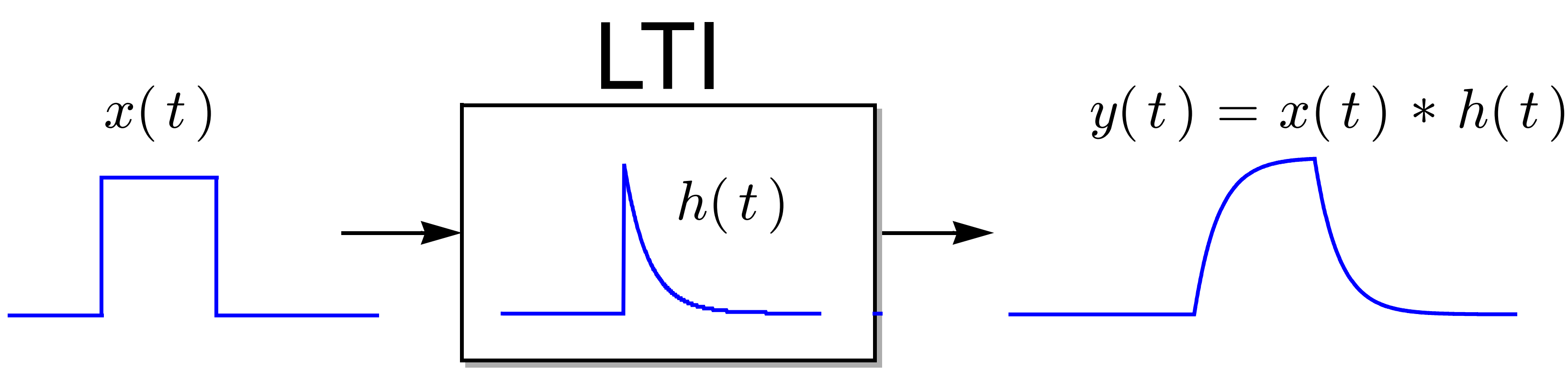
$$= x(t) * h(t)$$

CONVOLUTION

Why Impulse Response is Important

An LTI system is *completely* characterized by $h(t)$.

Response to ***any*** input can be found by convolving it with $h(t)$:



Convolution Properties

- commutative: $x(t) * h(t) = h(t) * x(t)$ (change of variables $t' = t - \tau$)

  $\Rightarrow$ doesn't matter which is input, which is impulse response!

- associative $(x(t) * h(t)) * z(t) = x(t) * (h(t) * z(t))$

- shift property: $x(t) * h(t - t_0) = x(t - t_0) * h(t)$

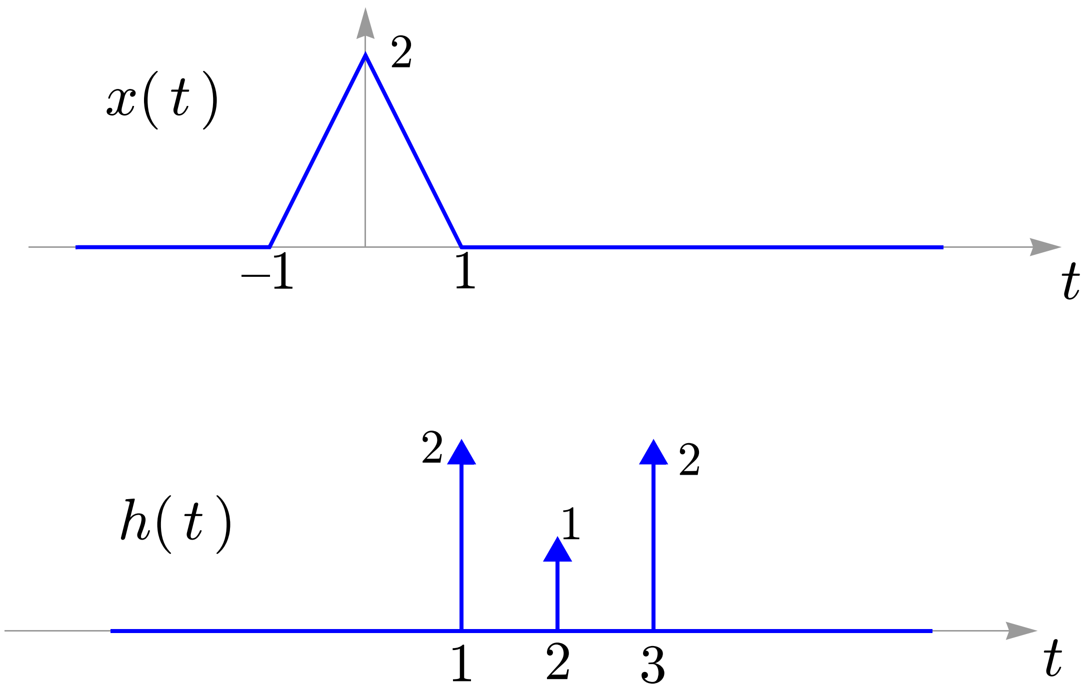
- Derivative: $\frac{d}{dt}(x(t) * h(t)) = (\frac{d}{dt}x(t)) * h(t) = x(t) * \frac{d}{dt}h(t)$

- Convolving with an impulse:

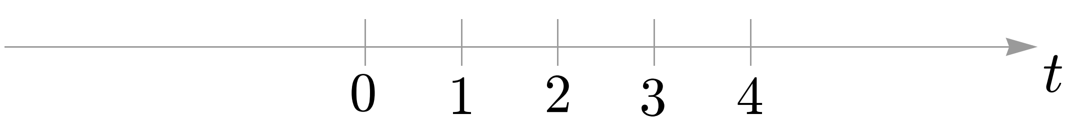
  ▷ $x(t) * \delta(t) = x(t)$

  ▷ $x(t) * \delta(t - t_0) = x(t - t_0)$

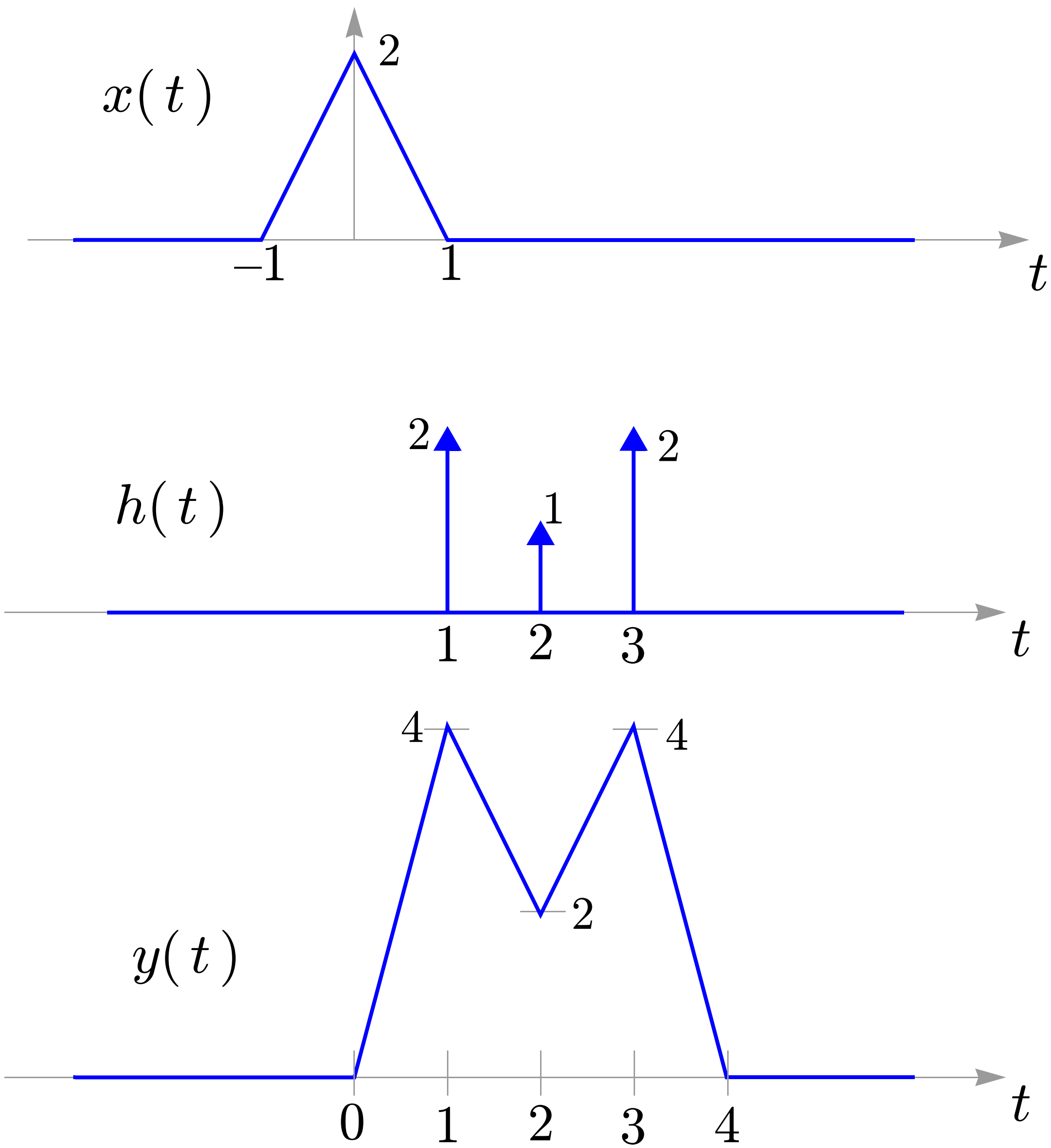
Example: Convolving w Impulses



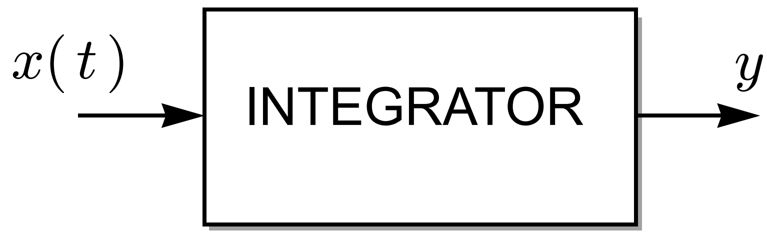
$y(t) = ?$



Example: Convolving w Impulses



Pop Quiz: The Integrator

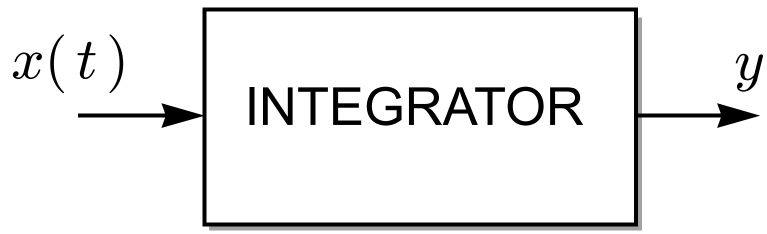


$$y(t) = \int_{-\infty}^{t} x(\tau)\,\mathrm{d}\tau$$

(a): Find its *impulse* response

(b): Find its *step* response

Pop Quiz: The Integrator



$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

<u>(a):</u> Impulse response is $h(t) = u(t) =$ step.

<u>(b):</u> Convolve step input with $h(t)$ to get step response:

$$s(t) = u(t) * u(t)$$

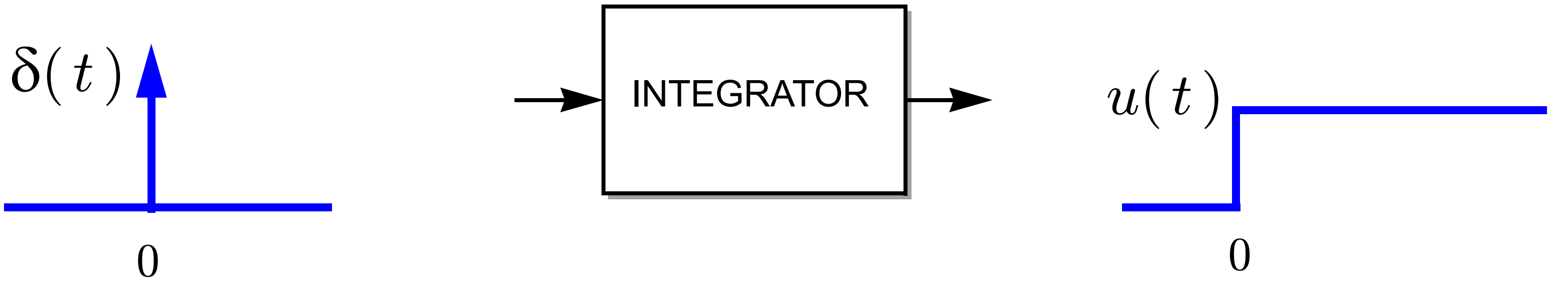
$$= \int_{-\infty}^{\infty} u(\tau)u(t - \tau)\,d\tau$$

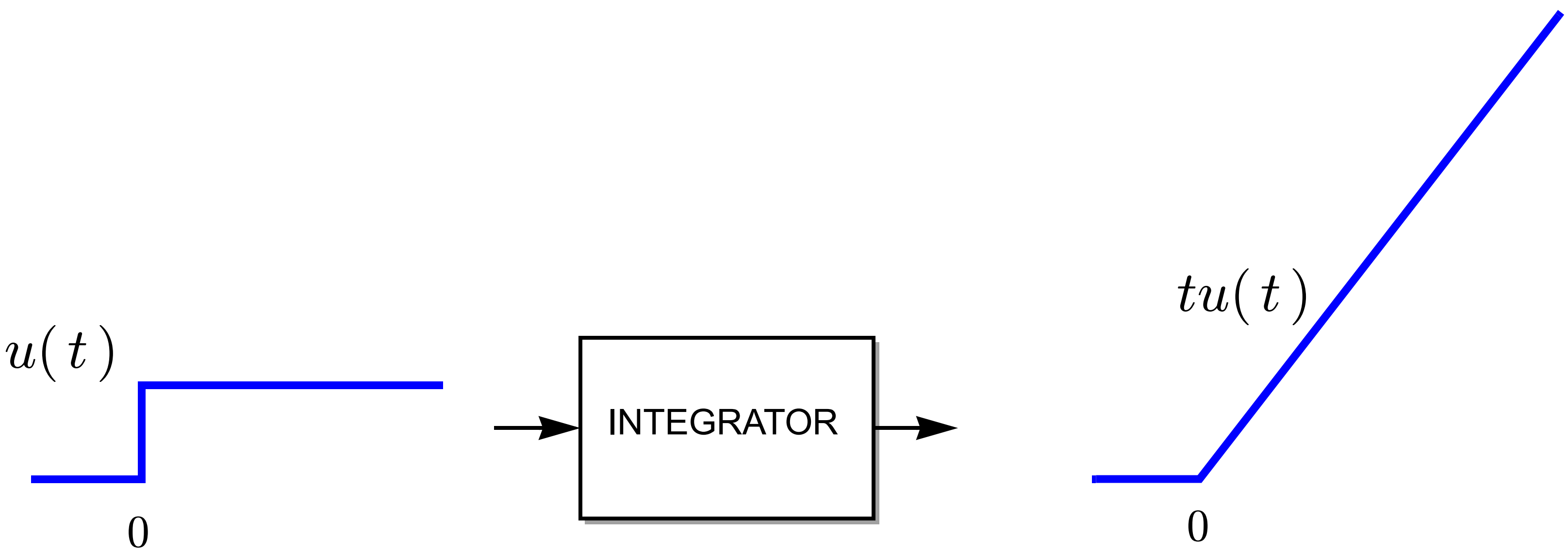
$$= \int_{0}^{\infty} 1u(t - \tau)\,d\tau$$

$$= \int_{0}^{t} 1\,d\tau = tu(t) = \text{ramp.}$$

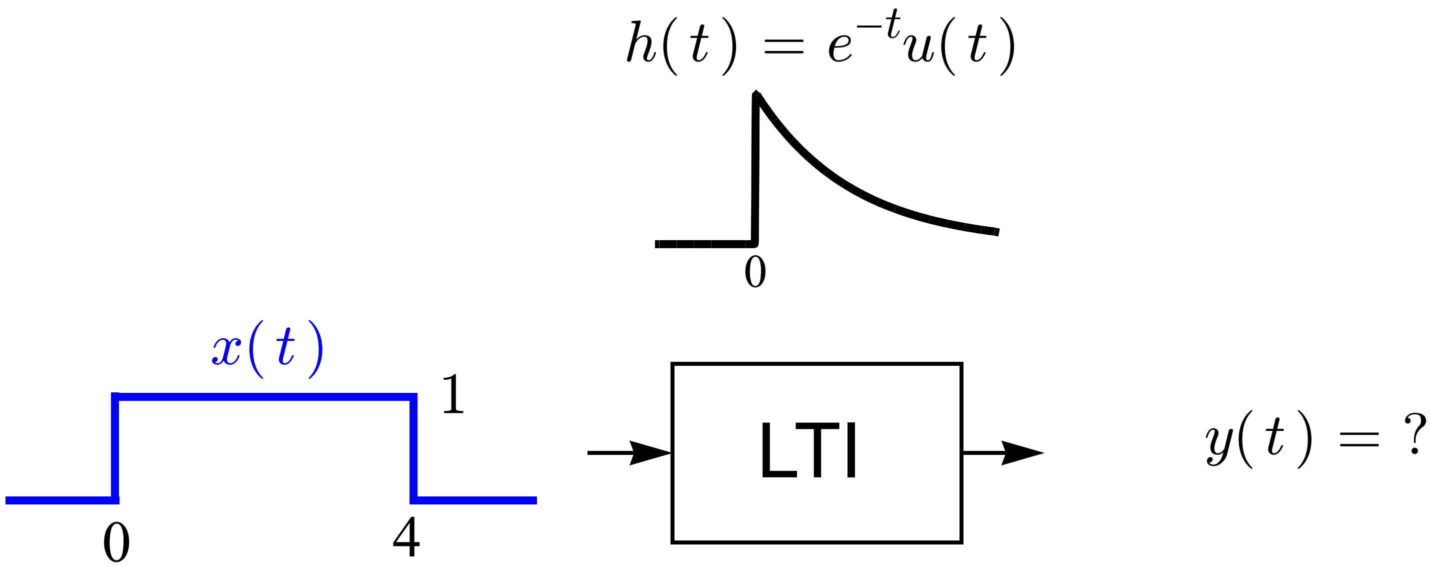
Alternatively: Check answer by applying step input to integrator!

Integrator



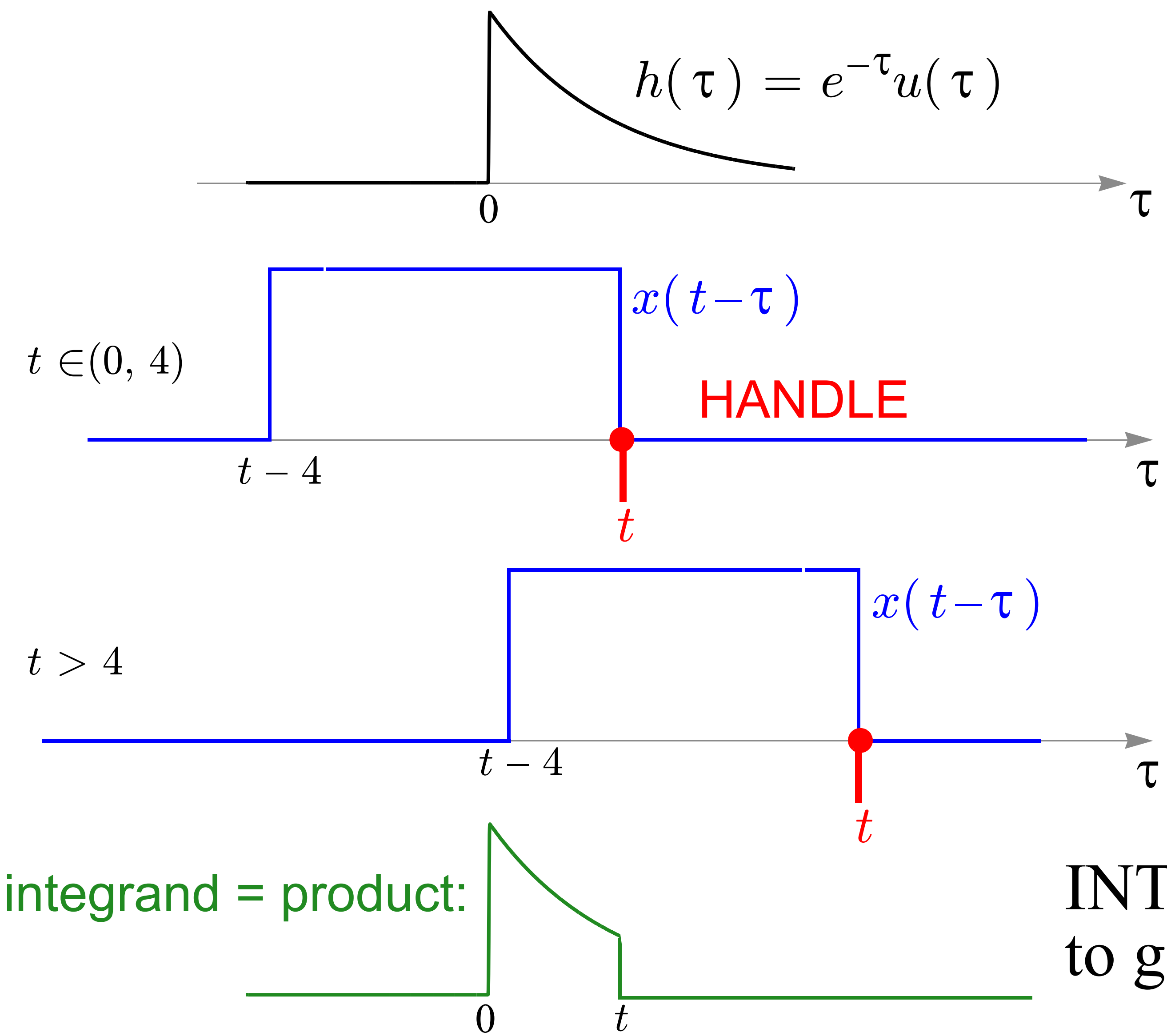


Example: Find Output



Graphical Convolution

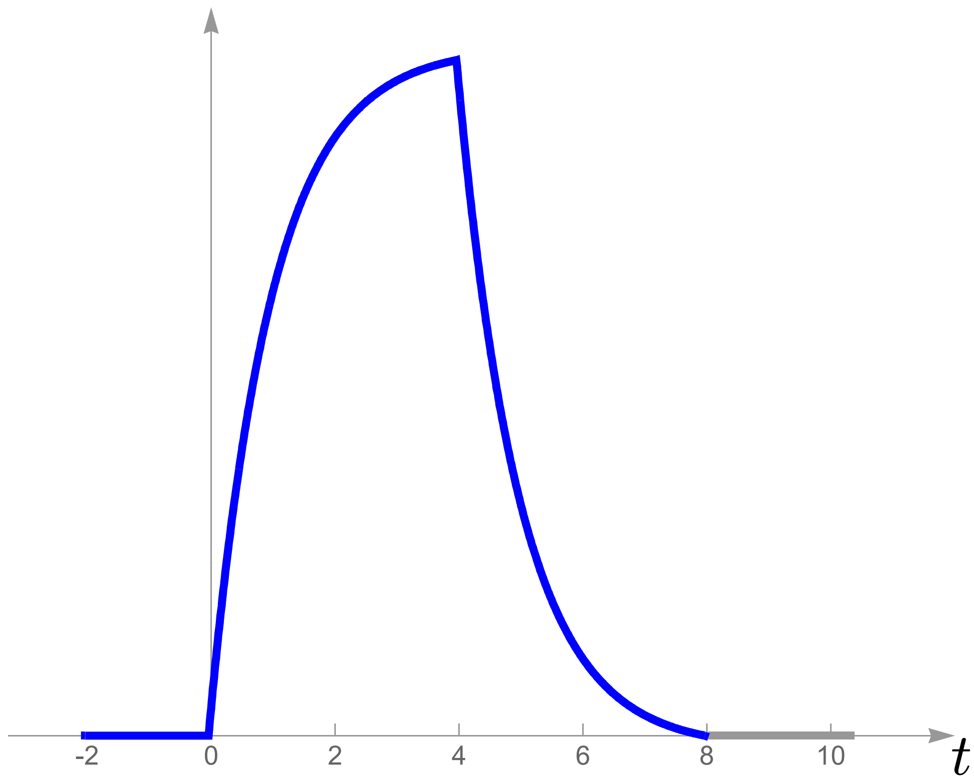
$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$



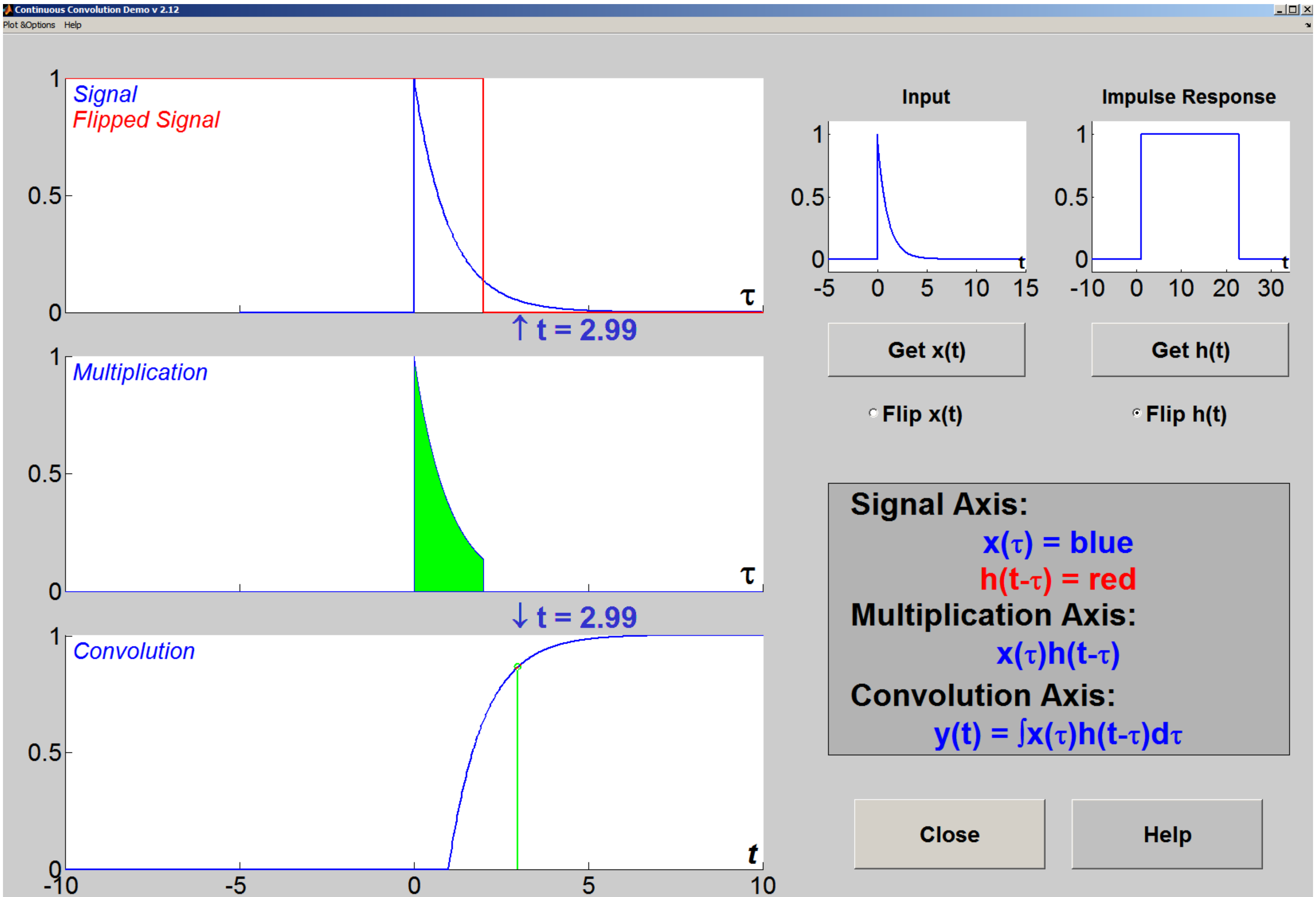
INTEGRATE
to get $y(t)$

Final Answer

$$y(t) = (1 - e^{-t})u(t) - (1 - e^{-(t-4)})u(t-4)$$



cconvdemo


Continuous Convolution Demo v 2.12
Plot &Options Help
Signal
Flipped Signal
t = 2.99
Multiplication
t = 2.99
Convolution
Input
Impulse Response
Get x(t)
Get h(t)
Flip x(t)
Flip h(t)
Signal Axis:
x(τ) = blue
h(t-τ) = red
Multiplication Axis:
x(τ)h(t-τ)
Convolution Axis:
y(t) = ∫x(τ)h(t-τ)dτ
Close
Help