

Lecture 5: Tue Sep 1, 2020

Reminder:

- HW1 solutions posted
- HW2 due Thursday.

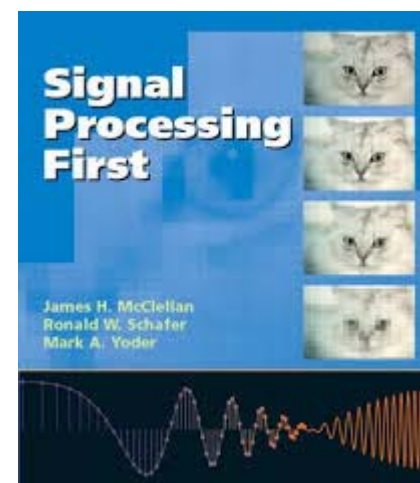
Lecture

- LTI systems
- impulse response
- convolution integral

Reminder: Reading Assignment

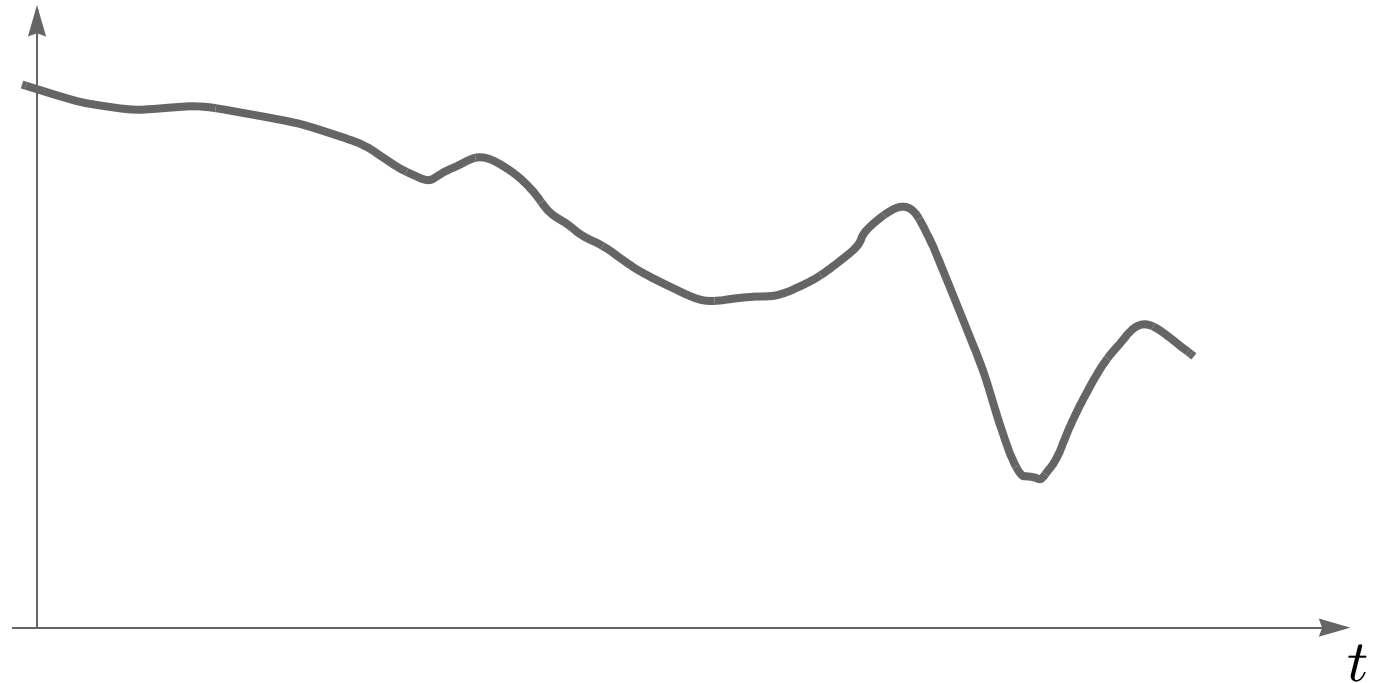
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Representation Property

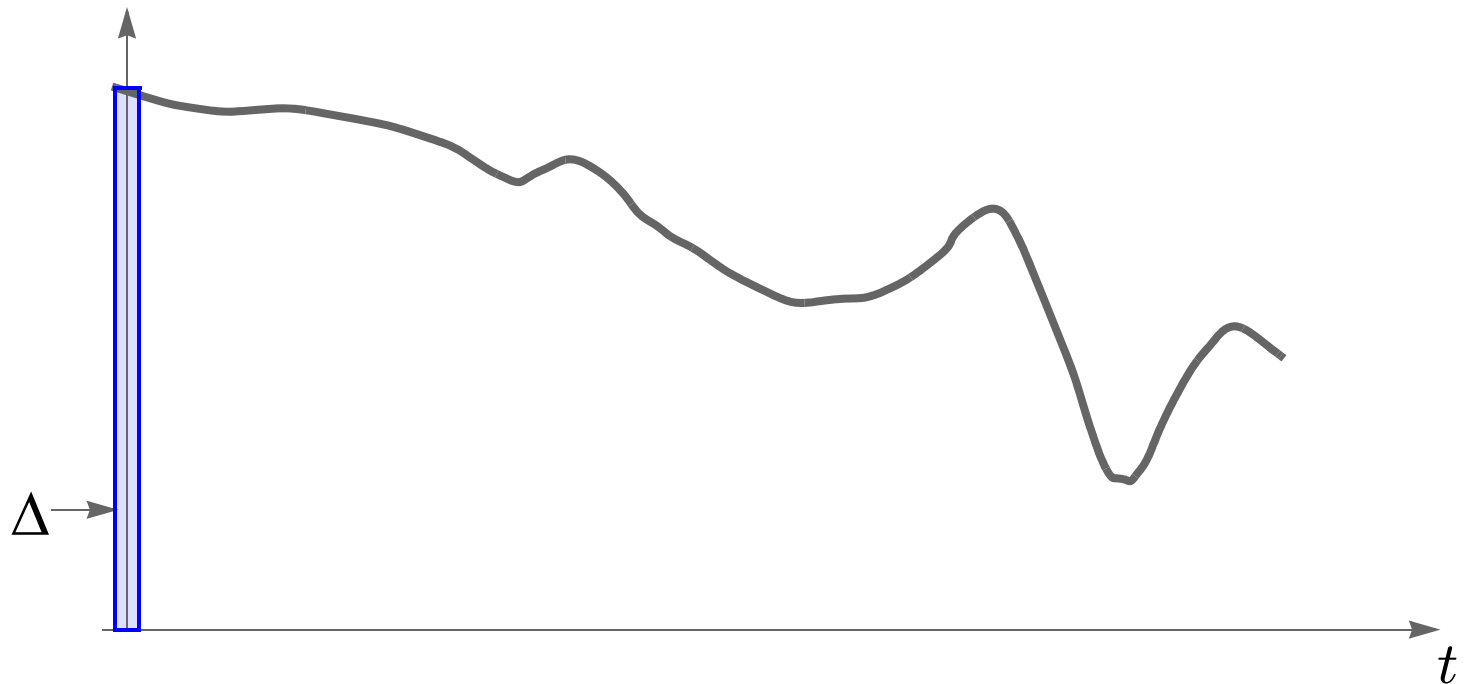
Any signal can be written as “sum” of Diracs:



Representation Property

Any signal can be written as “sum” of Diracs:

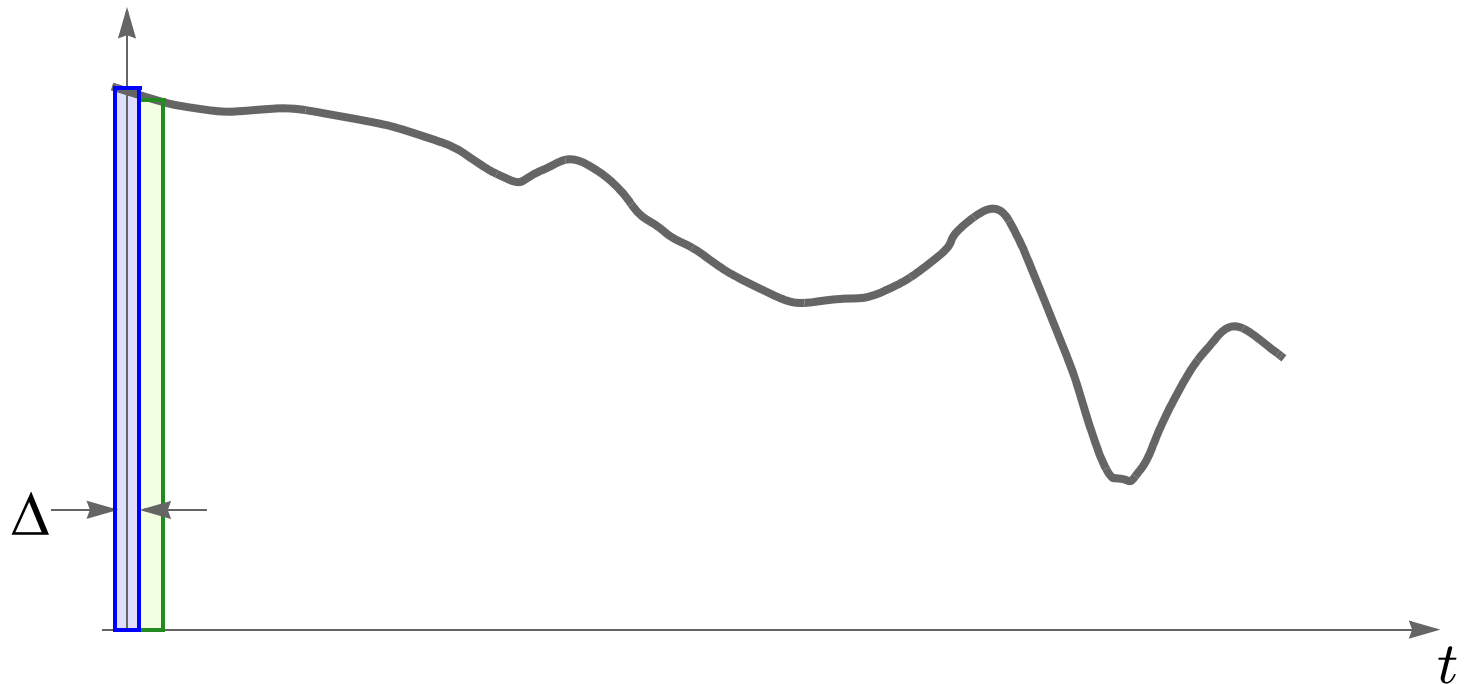
$$x(t) \approx \Delta x(0)g(t) + \dots$$



Representation Property

Any signal can be written as “sum” of Diracs:

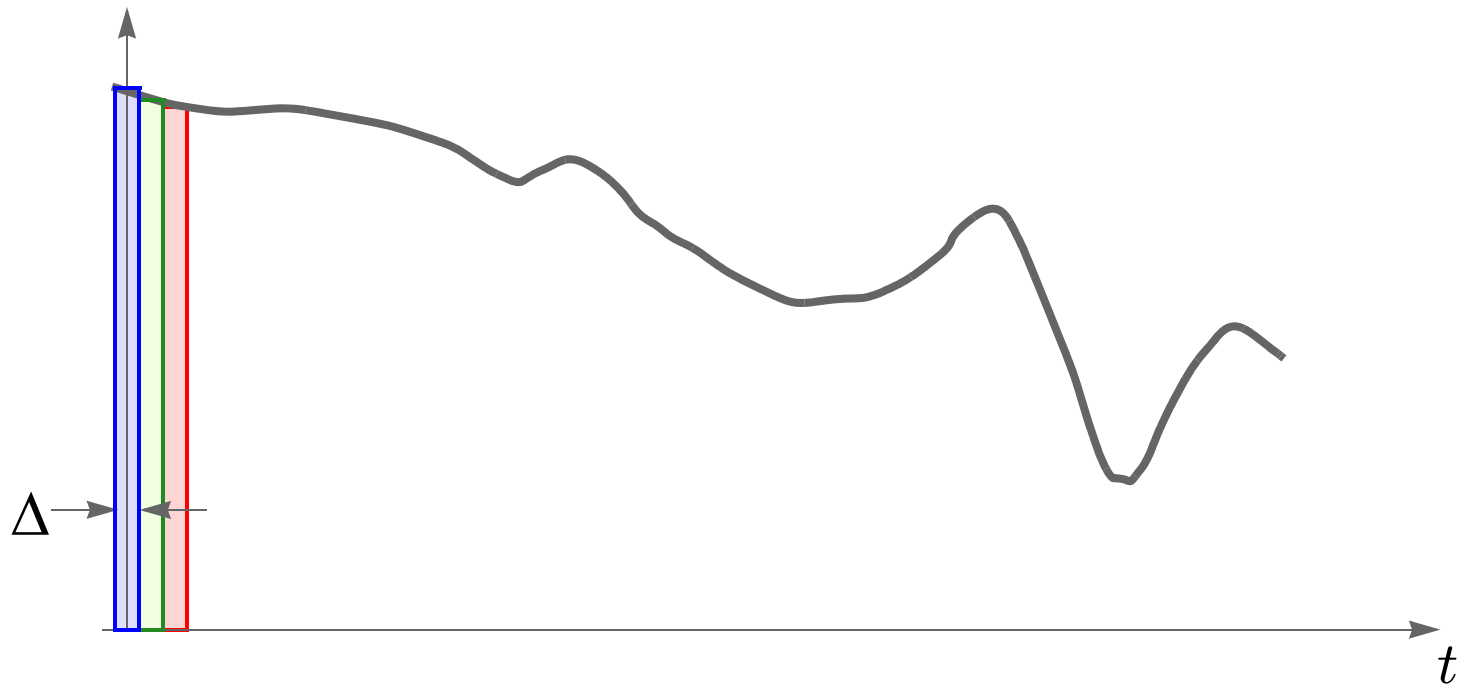
$$x(t) \approx \Delta x(0)g(t) + \Delta x(\Delta)g(t - \Delta) + \dots$$



Representation Property

Any signal can be written as “sum” of Diracs:

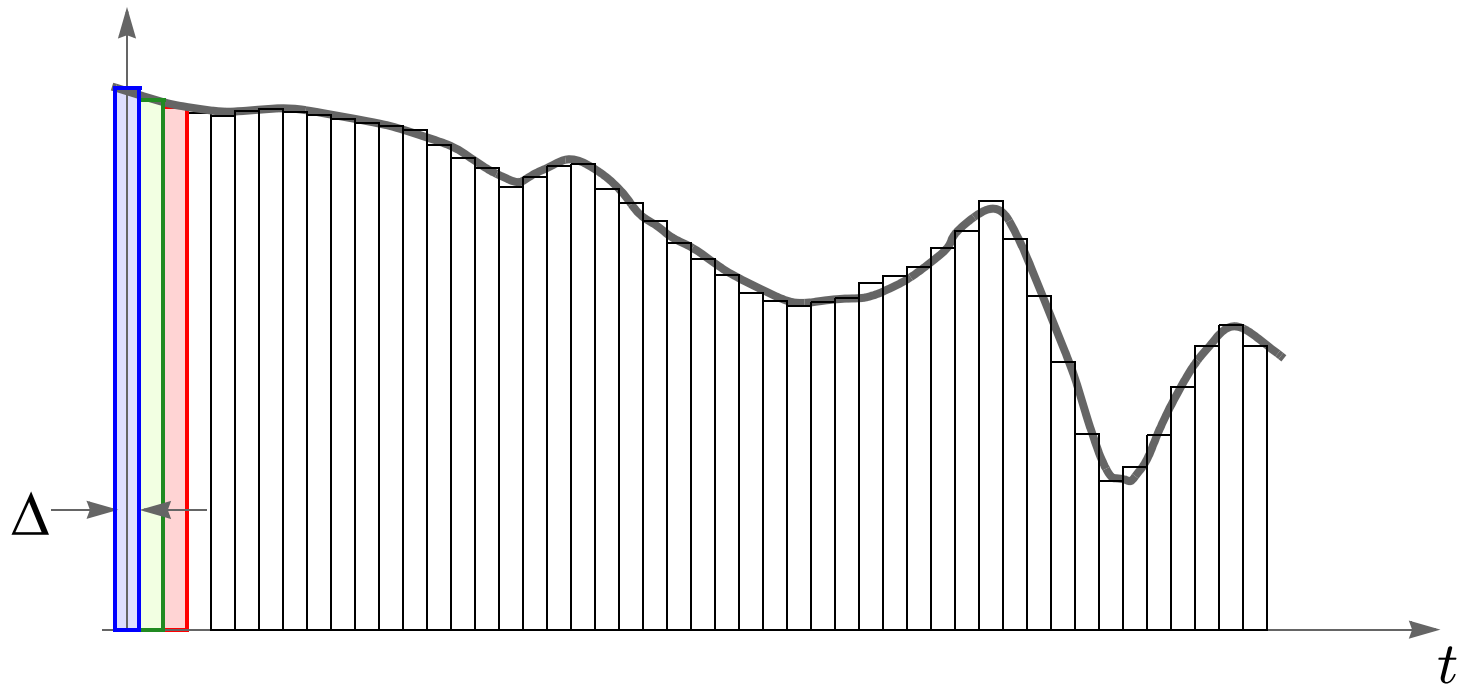
$$x(t) \approx \Delta x(0)g(t) + \Delta x(\Delta)g(t - \Delta) + \Delta x(2\Delta)g(t - 2\Delta) + \dots$$



Representation Property

Any signal can be written as “sum” of Diracs:

$$\begin{aligned}x(t) &\approx \Delta x(0)g(t) + \Delta x(\Delta)g(t - \Delta) + \Delta x(2\Delta)g(t - 2\Delta) + \dots \\&\approx \sum_k x(k\Delta)g(t - k\Delta)\Delta\end{aligned}$$



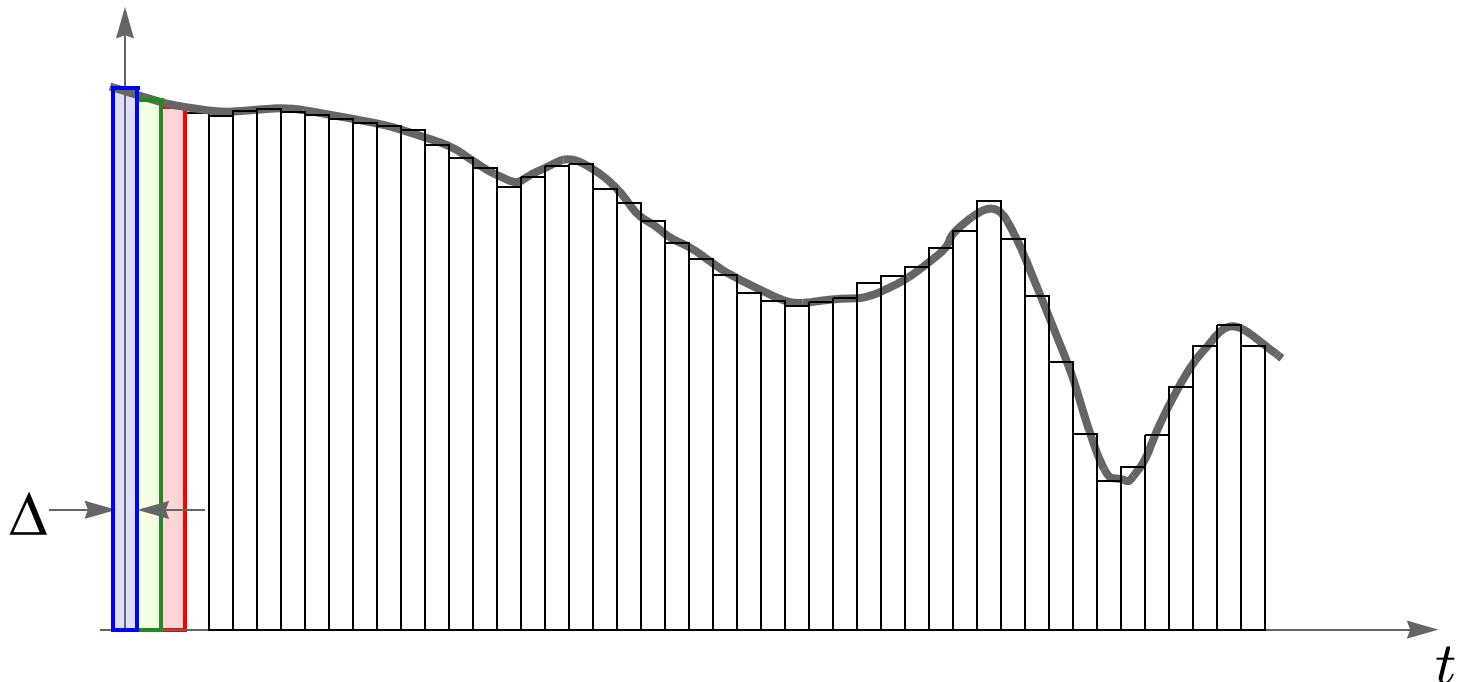
Representation Property

Any signal can be written as “sum” of Diracs:

$$x(t) \approx \Delta x(0)g(t) + \Delta x(\Delta)g(t - \Delta) + \Delta x(2\Delta)g(t - 2\Delta) + \dots$$

$$\approx \sum_k x(k\Delta)\delta(t - k\Delta)\Delta$$

$$\Rightarrow x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau$$



Impulse Response

Let $h(t)$ denote response of system to $\delta(t)$

Incredibly important for LTI systems. Why?

TI \Rightarrow response to $\delta(t - \tau)$ is $h(t - \tau)$,

L \Rightarrow response to $x(\tau)\delta(t - \tau)$ is $x(\tau)h(t - \tau)$.

L \Rightarrow response to integral (limiting case of a sum):

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau$$

is
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

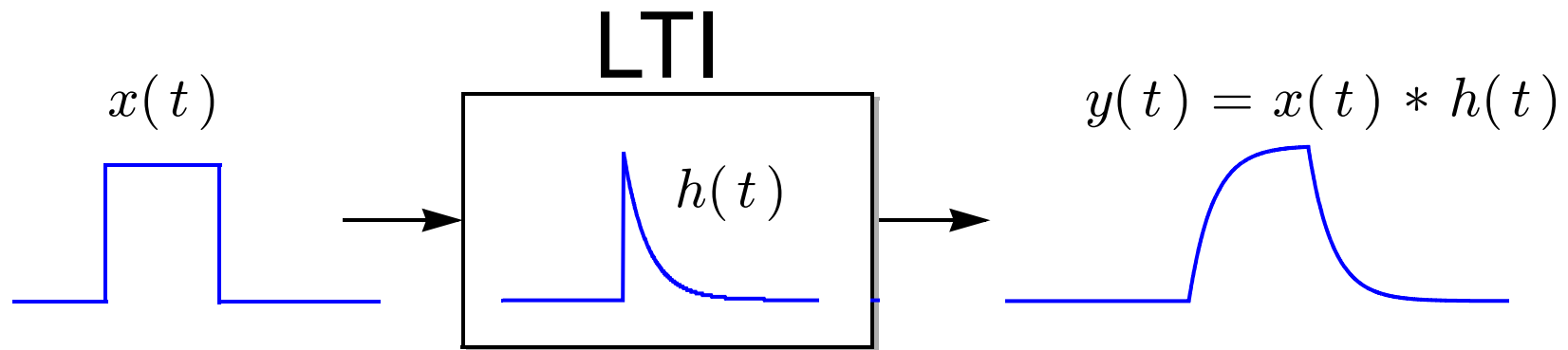
$$= x(t) * h(t)$$

CONVOLUTION

Why Impulse Response is Important

An LTI system is *completely* characterized by $h(t)$.

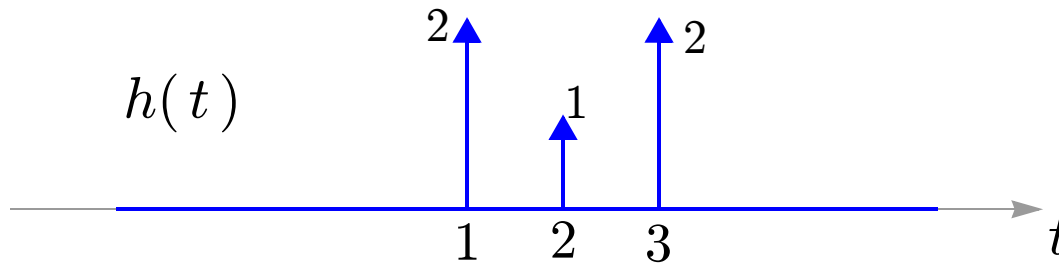
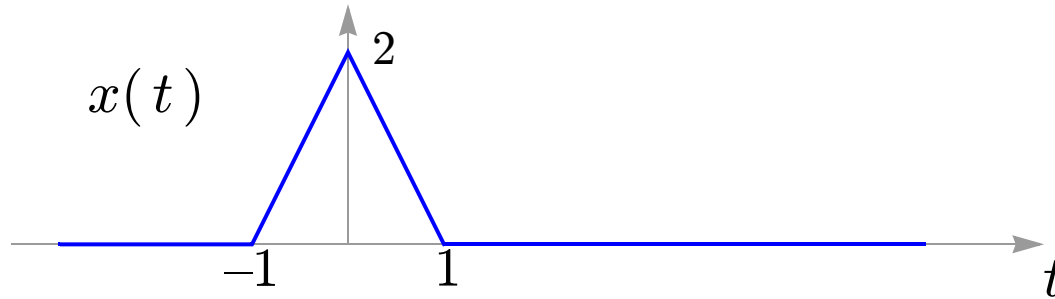
Response to *any* input can be found by convolving it with $h(t)$:



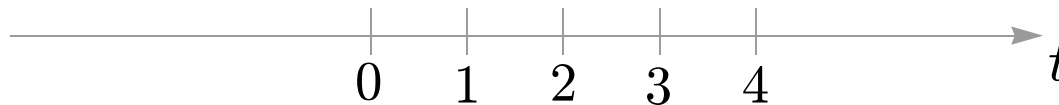
Convolution Properties

- commutative: $x(t) * h(t) = h(t) * x(t)$ (change of variables $t' = t - \tau$)
 \Rightarrow doesn't matter which is input, which is impulse response!
- associative $(x(t) * h(t)) * z(t) = x(t) * (h(t) * z(t))$
- shift property: $x(t) * h(t - t_0) = x(t - t_0) * h(t)$
- Derivative: $\frac{d}{dt}(x(t) * h(t)) = (\frac{d}{dt}x(t)) * h(t) = x(t) * \frac{d}{dt}h(t)$
- Convoluting with an impulse:
 - ▷ $x(t) * \delta(t) = x(t)$
 - ▷ $x(t) * \delta(t - t_0) = x(t - t_0)$

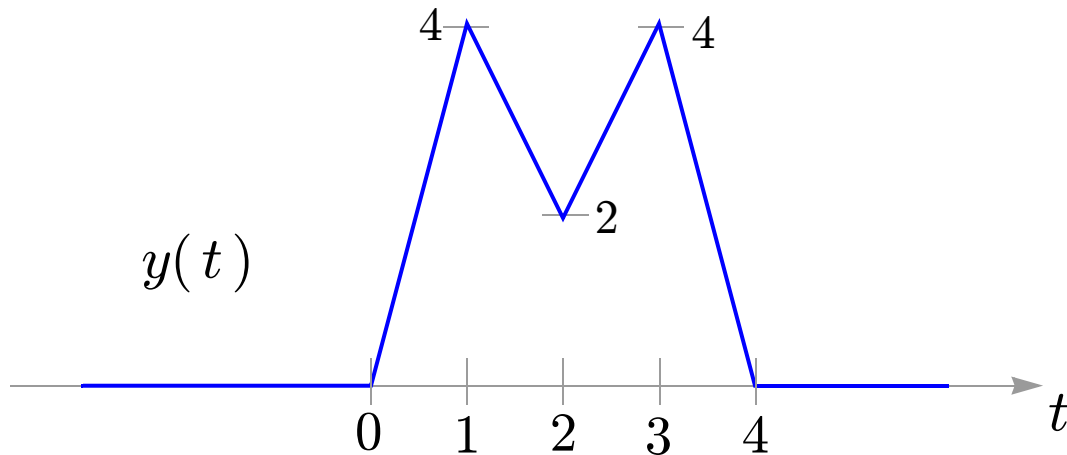
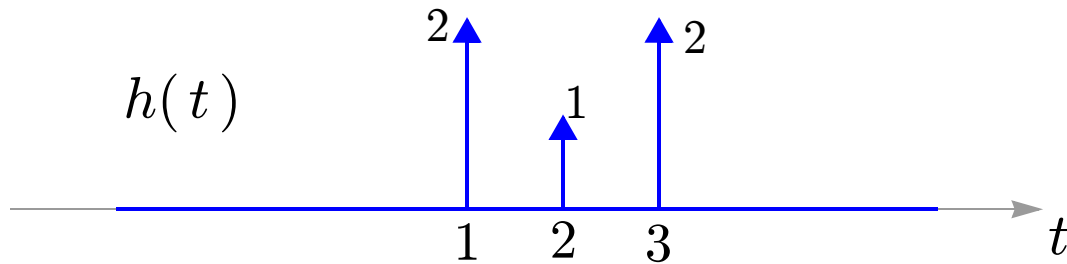
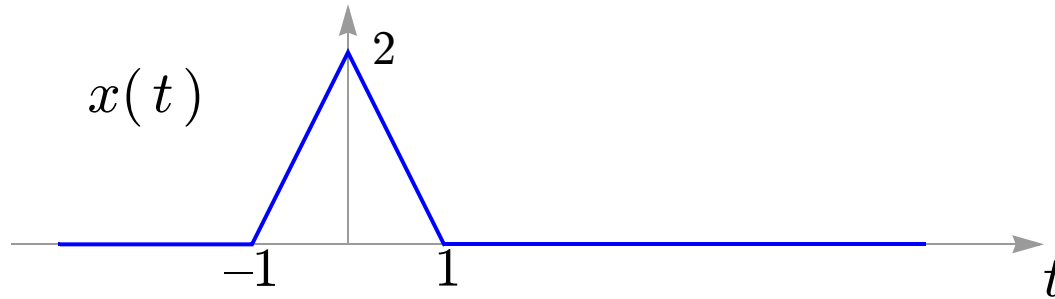
Example: Convolving w Impulses



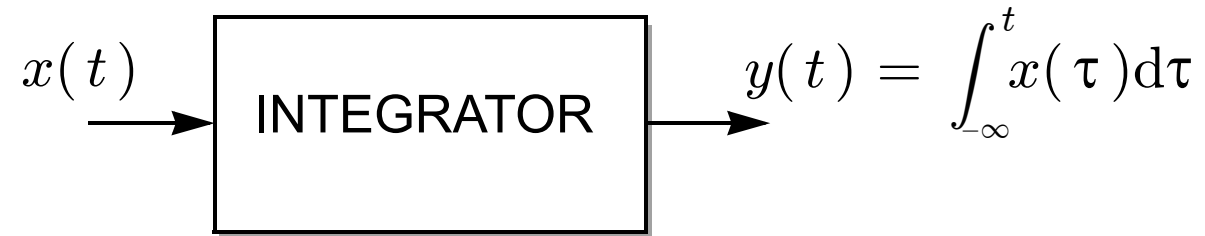
$$y(t) = ?$$



Example: Convolving w Impulses



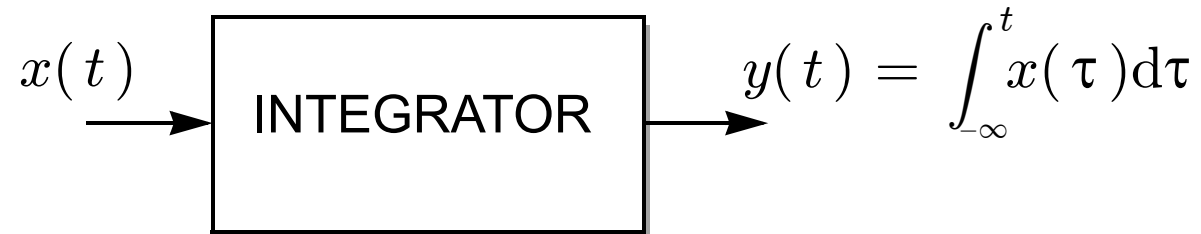
Pop Quiz: The Integrator



(a): Find its *impulse* response

(b): Find its *step* response

Pop Quiz: The Integrator



(a): Impulse response is $h(t) = u(t) = \text{step}$.

(b): Convolve step input with $h(t)$ to get step response:

$$s(t) = u(t) * u(t)$$

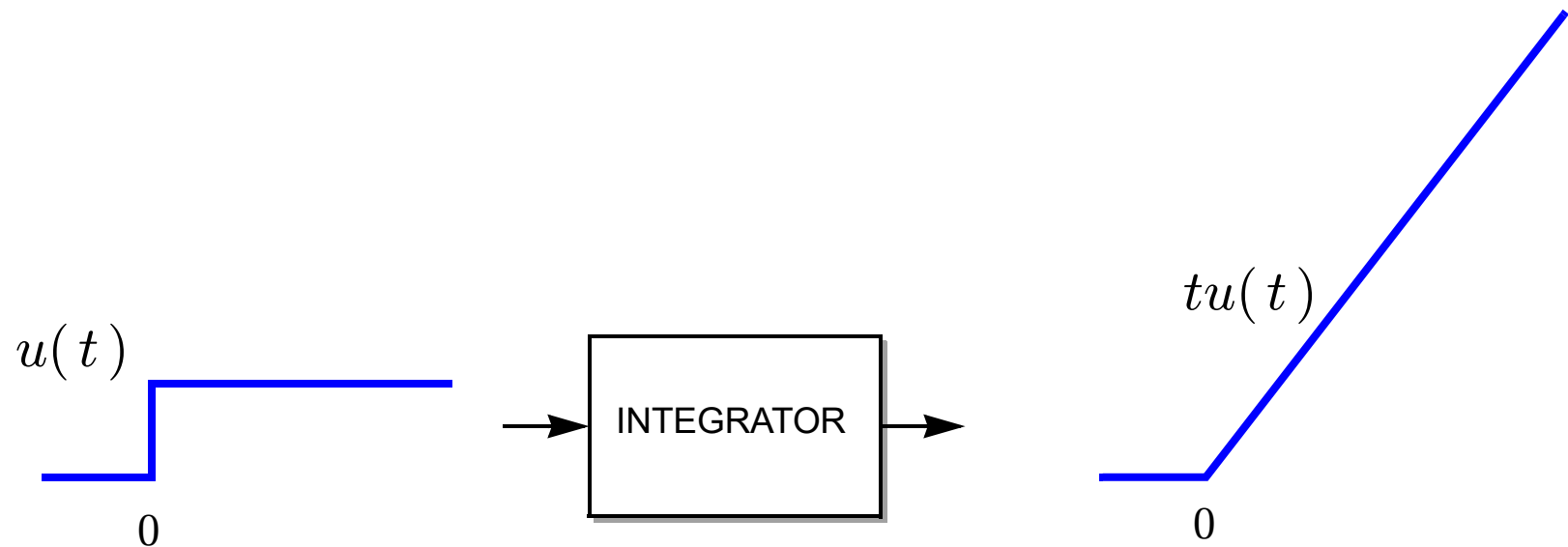
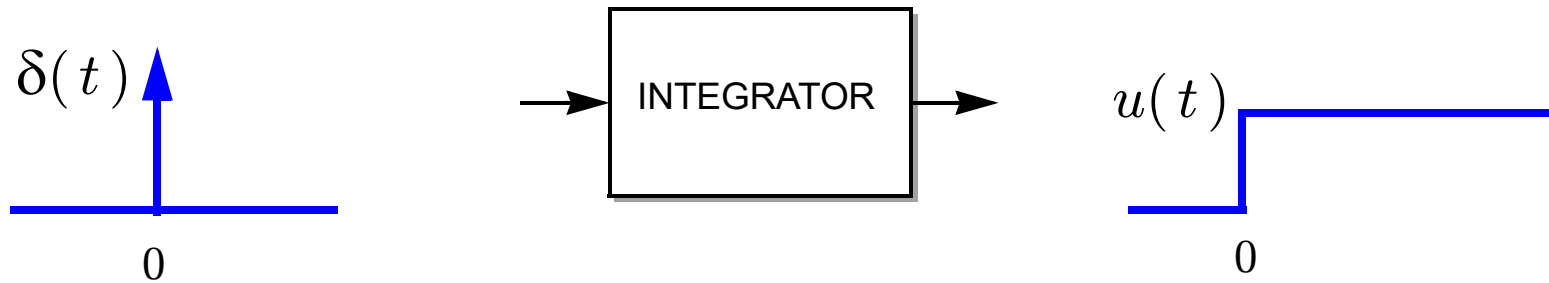
$$= \int_{-\infty}^{\infty} u(\tau) u(t - \tau) d\tau$$

$$= \int_0^{\infty} 1 u(t - \tau) d\tau$$

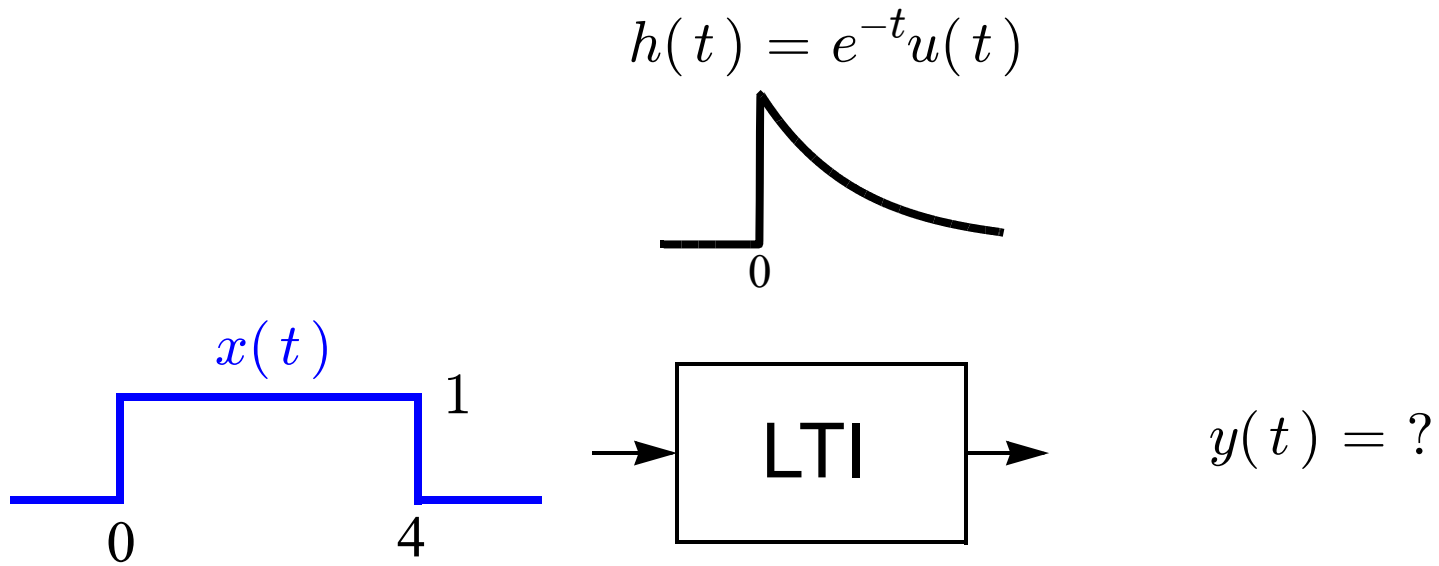
$$= \int_0^t 1 d\tau = t u(t) = \text{ramp}.$$

Alternatively: Check answer by applying step input to integrator!

Integrator

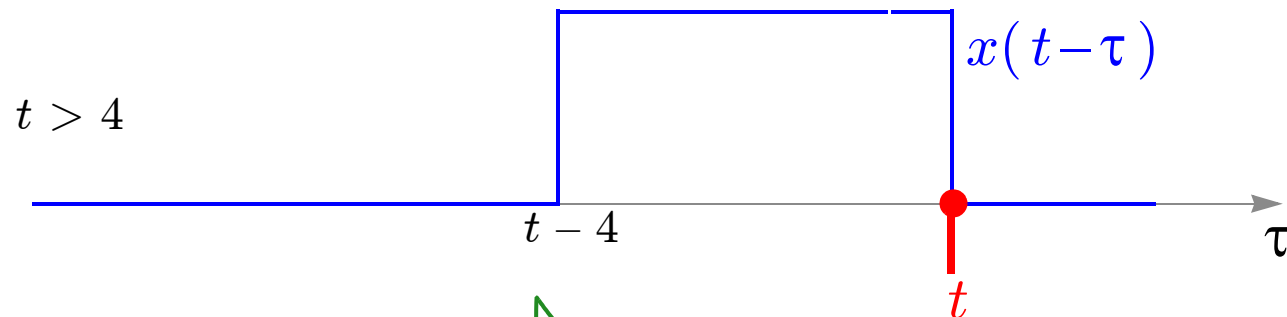
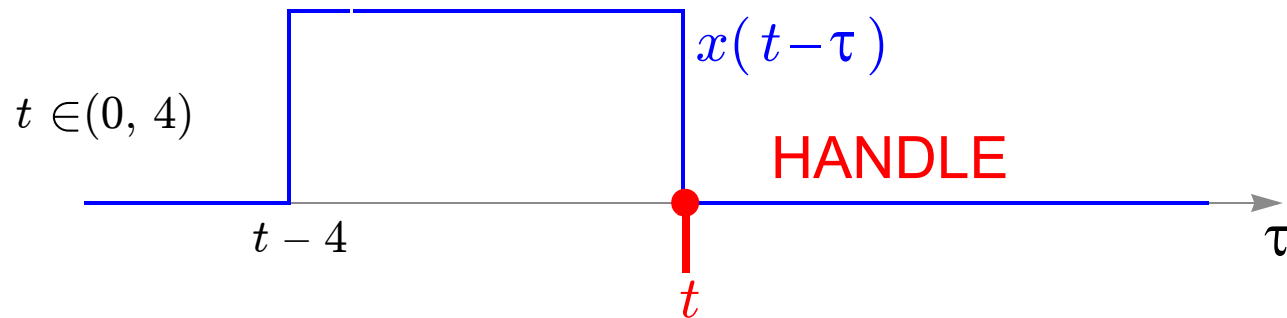
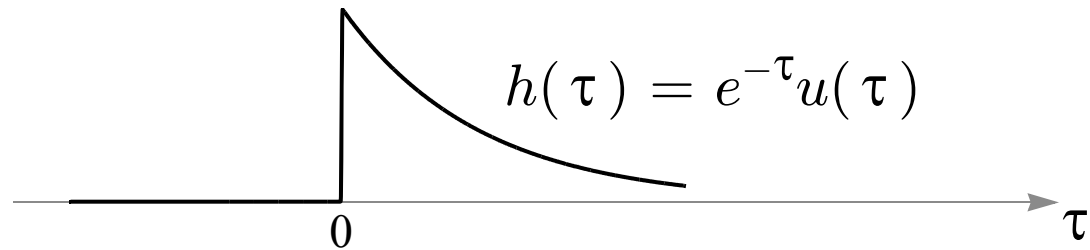


Example: Find Output

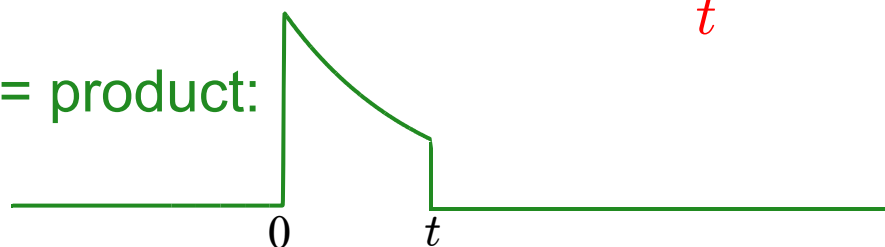


Graphical Convolution

$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau$$



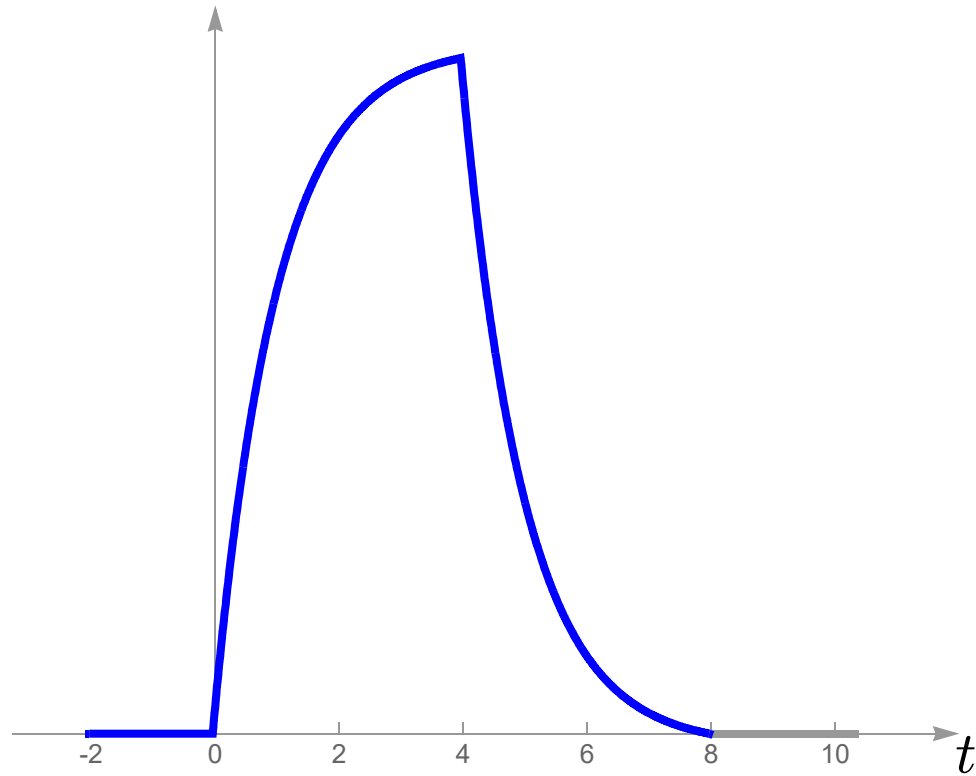
integrand = product:



INTEGRATE
to get $y(t)$

Final Answer

$$y(t) = (1 - e^{-t})u(t) - (1 - e^{-(t-4)})u(t-4)$$



cconvdemo

