

Lecture 4: Tue Aug 25, 2020

Reminder: HW1 due midnight tonight.

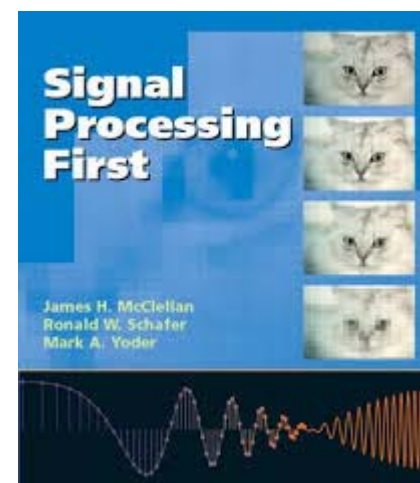
Lecture: systems and their properties

- Memoryless
- Causal
- Invertible
- (BIBO) stable
- Linear
- Time-invariant
- LTI

Reminder: Reading Assignment

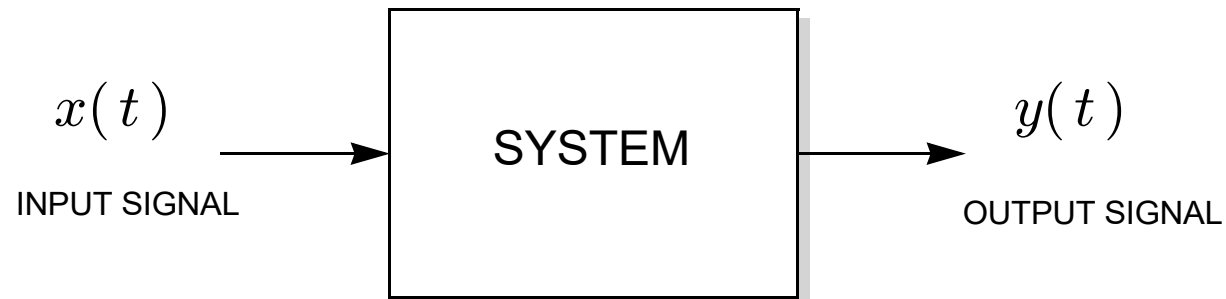
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System

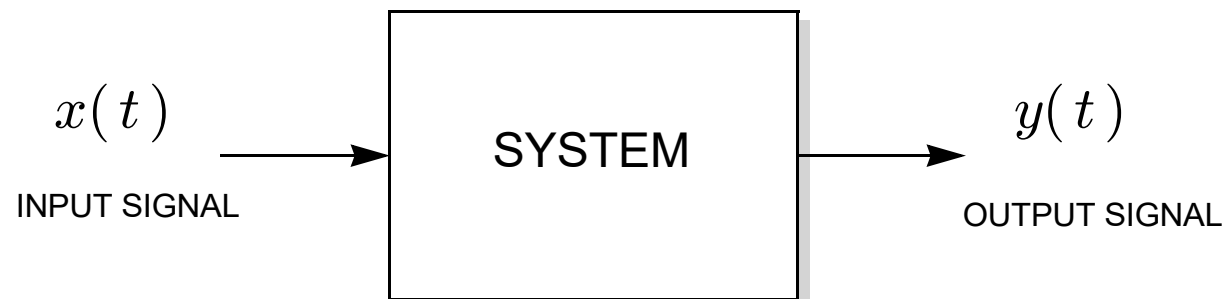
A deterministic transformation of an “input” signal to an “output” signal.



System

A deterministic transformation of an “input” signal to an “output” signal.

Examples: FM radio receiver, microphone, photodetector.



Examples:

- the “delay-by-0.5” system: $y(t) = x(t - 0.5)$
- the “squaring” system: $y(t) = x^2(t)$

System Categories

- *Memoryless* — output at a given instant t_0 depends only on input at same t_0 .
No dependence on past and future.
- *Causal* — output at a given instant t_0 depends only on input at or before t_0 .
No dependence on future.
- *Invertible* — input can always be recovered from output
- *(BIBO) Stable* — A bounded input always results in a bounded output.
- *Linear* — Additive and Scalable:

The response to $\alpha x_1(t) + \beta x_2(t)$

is always $\alpha y_1(t) + \beta y_2(t)$,

where $y_1(t)$ is the response to $x_1(t)$, and $y_2(t)$ is ... to $x_2(t)$.

- *Time-Invariant* —

The response to $x_1(t - t_0)$

is always $y_1(t - t_0)$, where $y_1(t)$ is the response to $x_1(t)$.

Yes or No?

	memoryless	causal	invertible
$y(t) = x(t - 0.5)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$y(t) = x^2(t)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Comments

- Memoryless implies causal.
- Before performing full-blown α/β test for linearity, try *litmus* tests:
 - ▷ *Zero-in Zero Out* — the response of linear system to zero must be zero.

$$x(t) = 0 \rightarrow \boxed{\text{LINEAR}} \rightarrow y(t) = 0$$

- ▷ *Double-In Double-Out* — doubling the input must result in the same output, only doubled

$$\begin{aligned} x_1(t) &\rightarrow \boxed{\text{LINEAR}} \rightarrow y_1(t) \\ x(t) = 2x_1(t) &\rightarrow \boxed{\text{LINEAR}} \rightarrow y(t) = 2y_1(t) \end{aligned}$$

Failing either test means that the system is NOT linear.

Passing both tests means that it could be linear. Must use α/β to be sure.

About the Litmus Tests

They are special cases of the full-blown α/β tests:

▷ *Zero-in Zero Out:* $\alpha = \beta = 0$

▷ *Double-In Double Out:* $\alpha = 2, \beta = 0$

Time-Invariant

Intuition: The response to an input does not depend on *when* it is applied.
Today, tomorrow, morning, afternoon, next year ... doesn't matter.

Mathematically: A system is **time-invariant** (TI) when

The response to $x_1(t - t_0)$

is always $y_1(t - t_0)$,

where $y_1(t)$ is the response to $x_1(t)$.

Like linearity, it's easier to show that something is *not* TI.

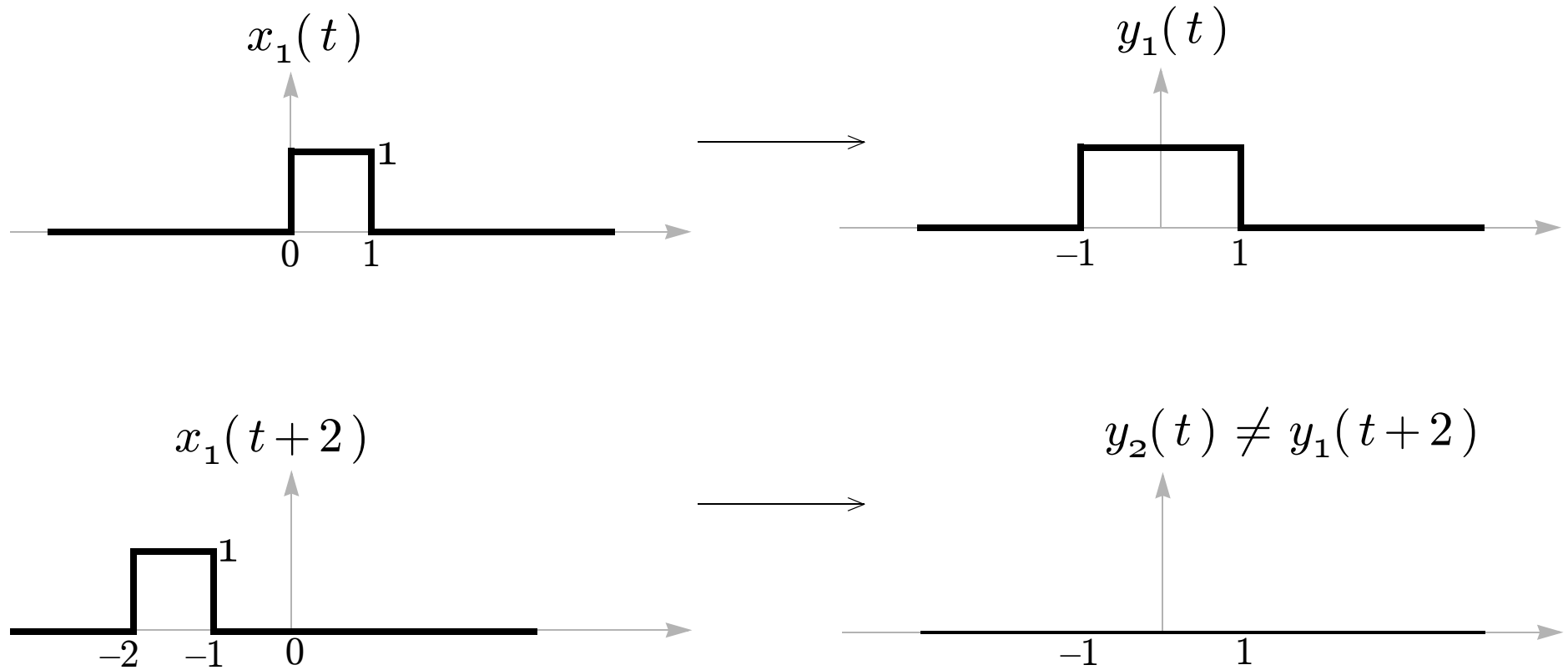
Just need one counterexample.

Try something simple at first, like inputs of $\delta(t)$ and $\delta(t - 1)$.

	memoryless	causal	invertible	stable	linear	time-invariant
$y(t) = 1 + x(t)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$y(t) = x(1 - t)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$y(t) = x^2(t)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$y(t) = x(3t)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$y(t) = x(t^2)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$y(t) = (2 + \cos(t))x(t)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$y(t) = \int_{-\infty}^t x(\tau) d\tau$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

	memoryless	causal	invertible	stable	linear	time-invariant
$y(t) = 1 + x(t)$	Y	Y	Y	Y	N	Y
$y(t) = x(1 - t)$	N	N	Y	Y	Y	N
$y(t) = x^2(t)$	Y	Y	N	Y	N	Y
$y(t) = x(3t)$	N	N	Y	Y	Y	N
$y(t) = x(t^2)$	N	N	N	Y	Y	N
$y(t) = (2 + \cos(t))x(t)$	Y	Y	Y	Y	Y	N
$y(t) = \int_{-\infty}^t x(\tau) d\tau$	N	Y	Y	N	Y	Y

Is $y(t) = x(t^2)$ Time-Invariant?



No. Delaying the input $x_1(t)$ results in a completely different output.