Lecture 4: Tue Aug 25, 2020

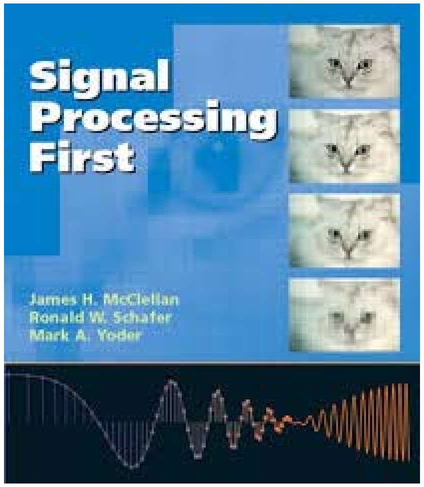
Reminder: HW1 due midnight tonight.

Lecture: systems and their properties

- Memoryless
- Causal
- Invertible
- (BIBO) stable
- Linear
- Time-invariant
- LTI

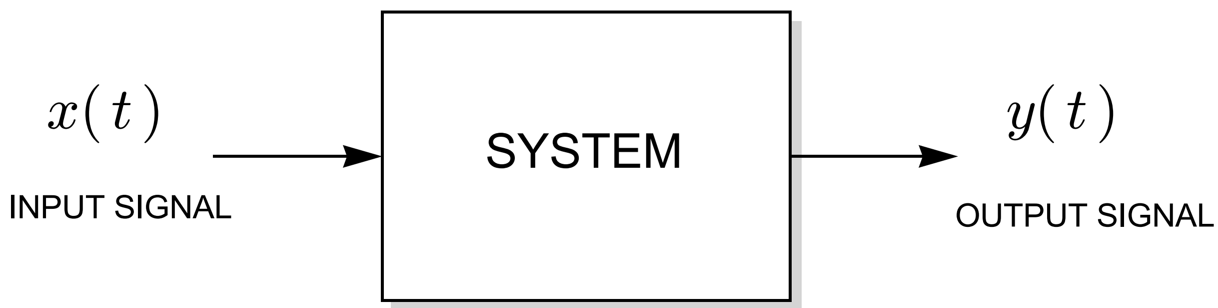
Reminder: Reading Assignment

9 Continuous-Time Signals and LTI Systems 245



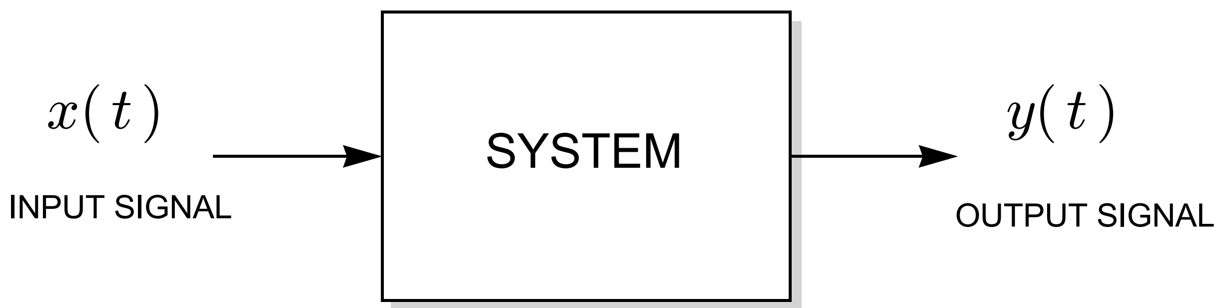
System

A deterministic transformation of an “input” signal to an “output” signal.



System

A deterministic transformation of an “input” signal to an “output” signal.
Examples: FM radio receiver, microphone, photodetector.



Examples:

- the “delay-by-0.5” system: $y(t) = x(t - 0.5)$
- the “squaring” system: $y(t) = x^2(t)$

System Categories

- *Memoryless* — output at a given instant t_0 depends only on input at same t_0. No dependence on past and future.
- *Causal* — output at a given instant t_0 depends only on input at or before t_0. No dependence on future.
- *Invertible* — input can always be recovered from output
- *(BIBO) Stable* — A bounded input always results in a bounded output.
- *Linear* — Additive and Scalable:

  The response to $\alpha x_1(t) + \beta x_2(t)$

  is always $\alpha y_1(t) + \beta y_2(t)$,

  where $y_1(t)$ is the response to $x_1(t)$, and $y_2(t)$ is ... to $x_2(t)$.

- *Time-Invariant* —

  The response to $x_1(t - t_0)$

  is always $y_1(t - t_0)$, where $y_1(t)$ is the response to $x_1(t)$.

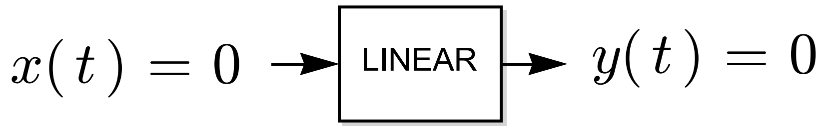
Yes or No?

	memoryless	causal	invertible
$y(t) = x(t - 0.5)$			
$y(t) = x^2(t)$			

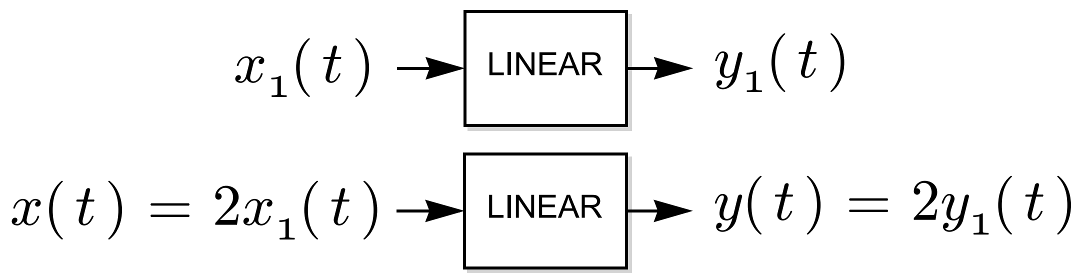
Comments

- Memoryless implies causal.
- Before performing full-blown α/β test for linearity, try *litmus* tests:

  ▷ *Zero-in Zero Out* — the response of linear system to zero must be zero.



  ▷ *Double-In Double-Out* — doubling the input must result in the same output, only doubled



Failing either test means that the system is NOT linear.
Passing both tests means that it could be linear. Must use α/β to be sure.

About the Litmus Tests

They are special cases of the full-blown α/β tests:

▷ *Zero-in Zero Out:* $\alpha = \beta = 0$

▷ *Double-In Double Out:* $\alpha = 2, \beta = 0$

Time-Invariant

Intuition: The response to an input does not depend on *when* it is applied.

Today, tomorrow, morning, afternoon, next year ... doesn't matter.

Mathematically: A system is **time-invariant** (TI) when

The response to $x_1(t - t_0)$

is always $y_1(t - t_0)$,

where $y_1(t)$ is the response to $x_1(t)$.

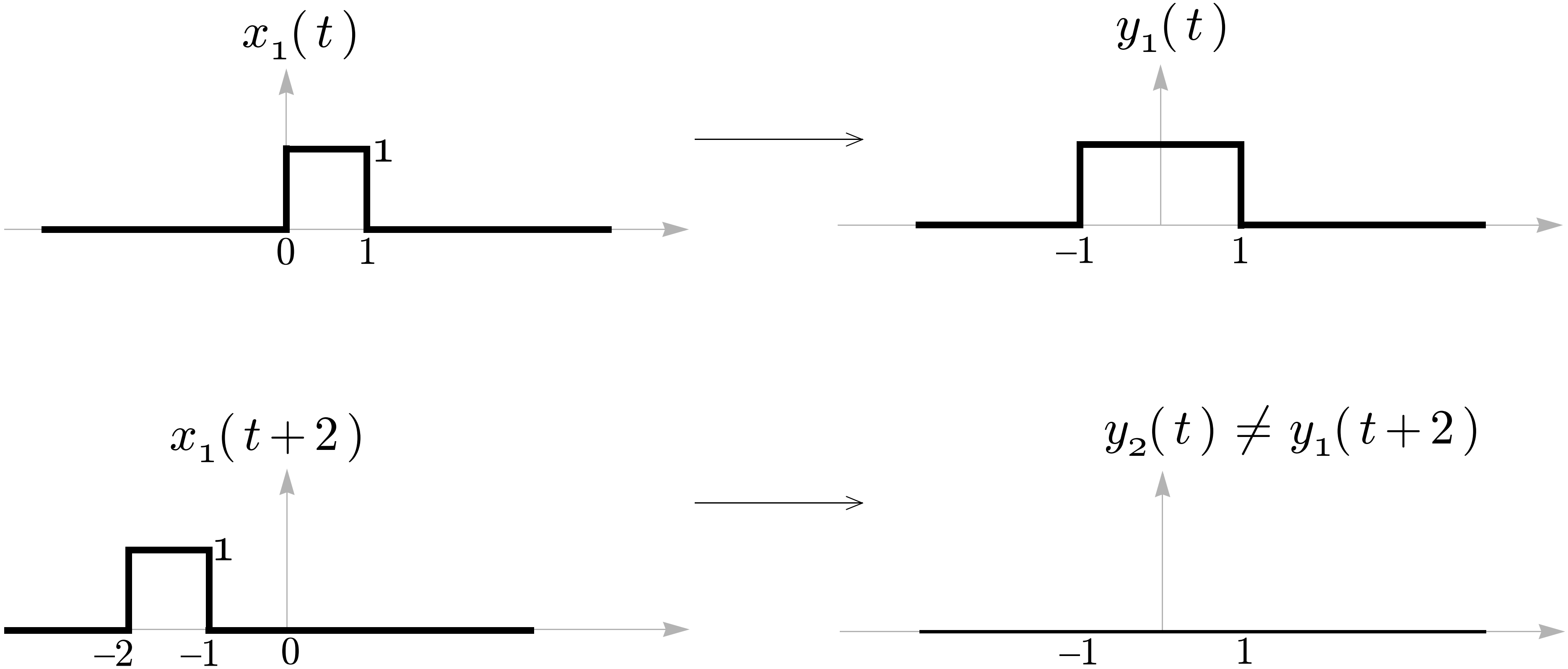
Like linearity, it's easier to show that something is *not* TI.

Just need one counterexample.
Try something simple at first, like inputs of $\delta(t)$ and $\delta(t-1)$.

	memoryless	causal	invertible	stable	linear	time-invariant
$y(t) = 1 + x(t)$						
$y(t) = x(1-t)$						
$y(t) = x^2(t)$						
$y(t) = x(3t)$						
$y(t) = x(t^2)$						
$y(t) = (2 + \cos(t))x(t)$						
$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$						

	memoryless	causal	invertible	stable	linear	time-invariant
$y(t) = 1 + x(t)$	Y	Y	Y	Y	N	Y
$y(t) = x(1-t)$	N	N	Y	Y	Y	N
$y(t) = x^2(t)$	Y	Y	N	Y	N	Y
$y(t) = x(3t)$	N	N	Y	Y	Y	N
$y(t) = x(t^2)$	N	N	N	Y	Y	N
$y(t) = (2 + \cos(t))x(t)$	Y	Y	Y	Y	Y	N
$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$	N	Y	Y	N	Y	Y

Is $y(t) = x(t^2)$ Time-Invariant?



No. Delaying the input $x_1(t)$ results in a completely different output.