Lecture 3: Tue Aug 25, 2020

Reminder: HW1 due Thursday midnight.

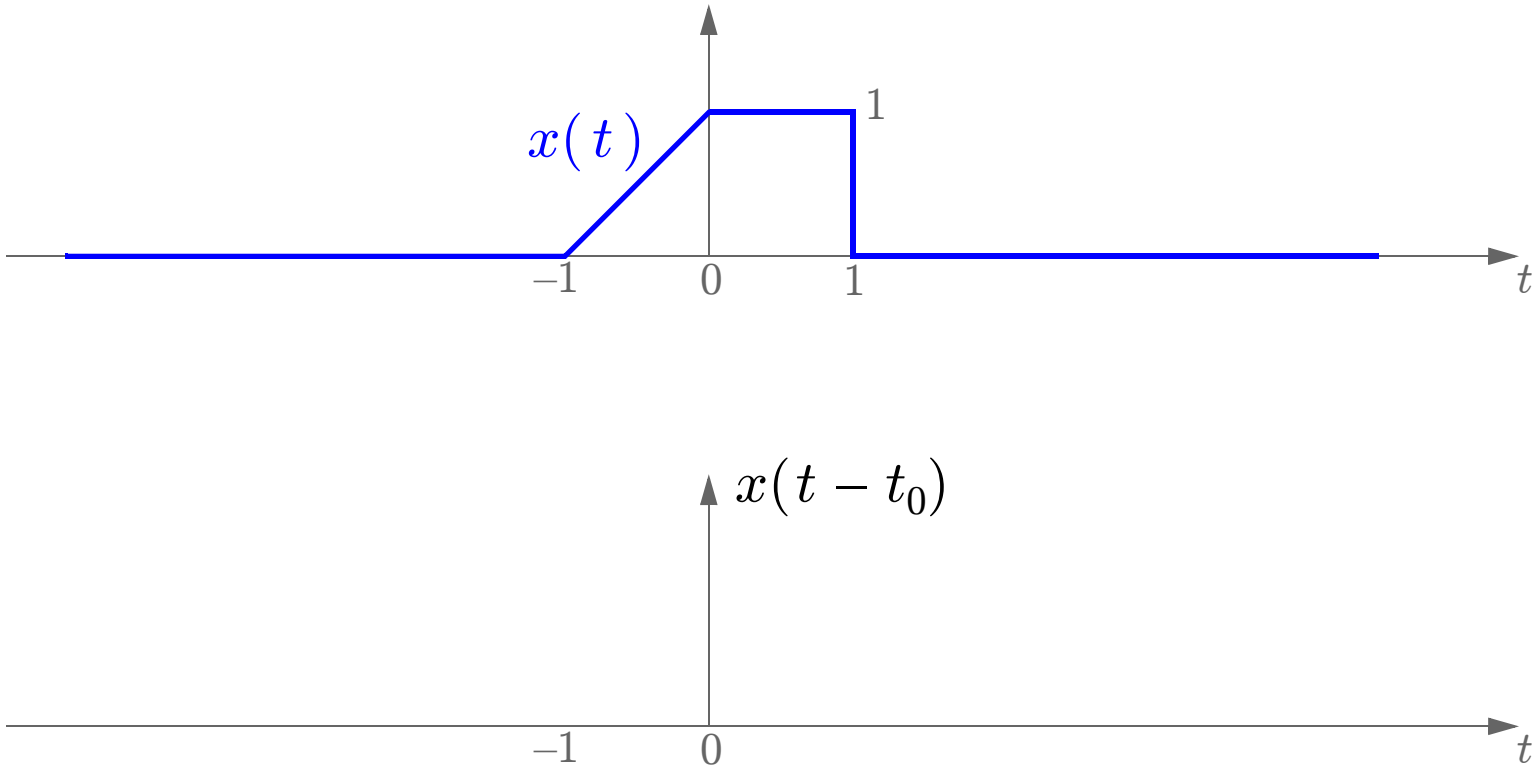
Signal Manipulation

- time shift (delay)
- time reversal
- time scale

Signal Characterization

- [even][odd][neither]
- [periodic][nonperiodic]
- [causal][anticausal][noncausal]
- [energy][power]

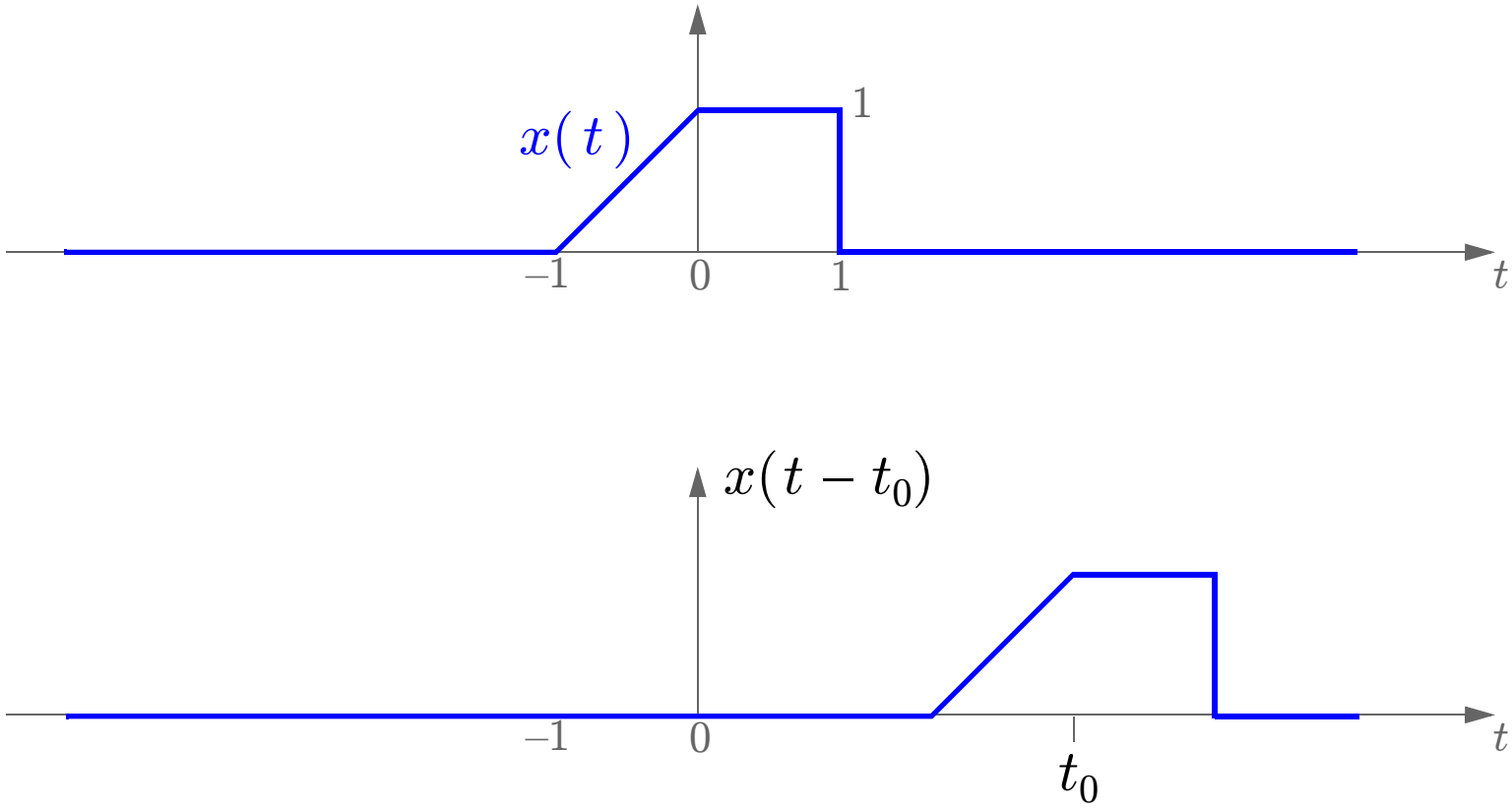
Time Shift



▷ shifts to right (delay) when $t_0 > 0$

▷ shifts to left (advance) when $t_0 < 0$

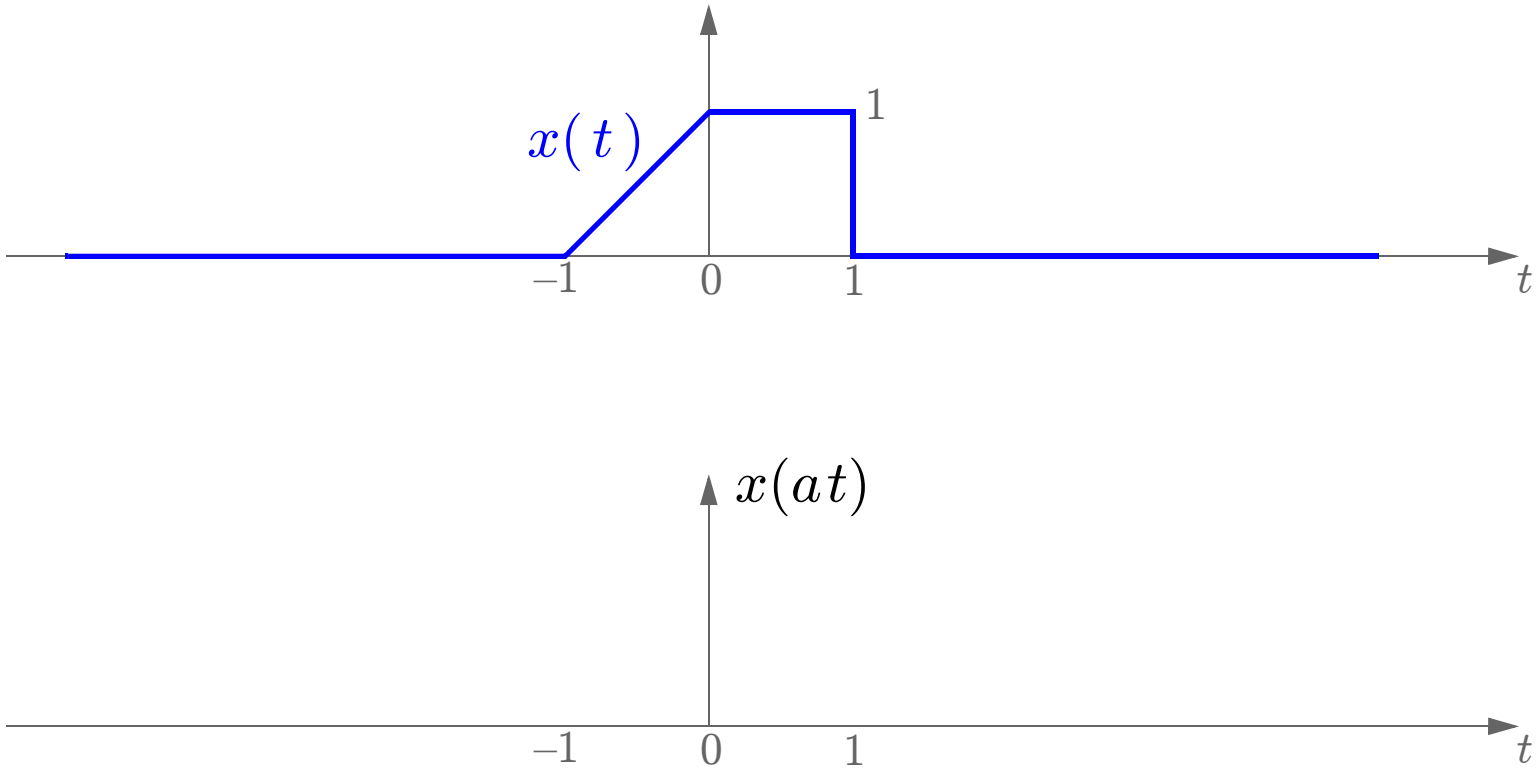
Time Shift



▷ shifts to right (delay) when $t_0 > 0$

▷ shifts to left (advance) when $t_0 < 0$

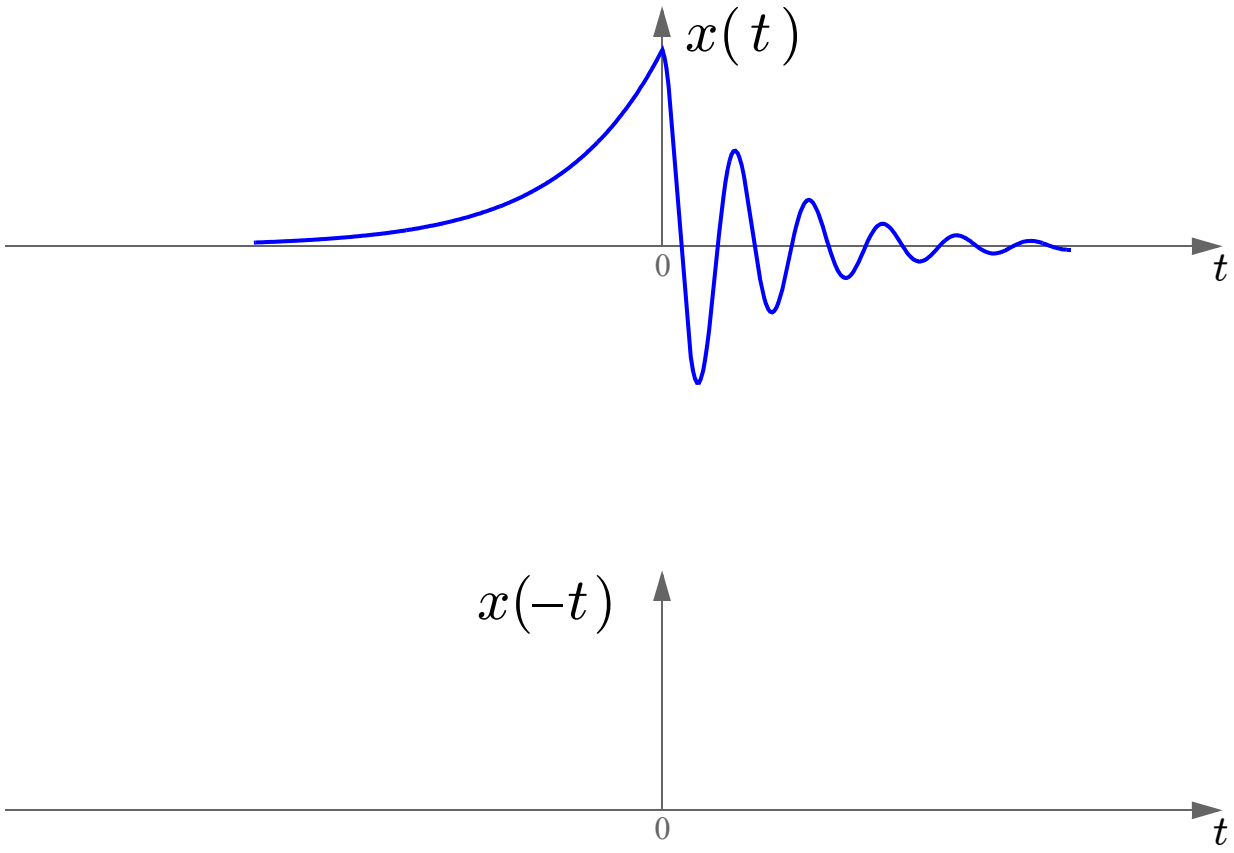
Time Scale



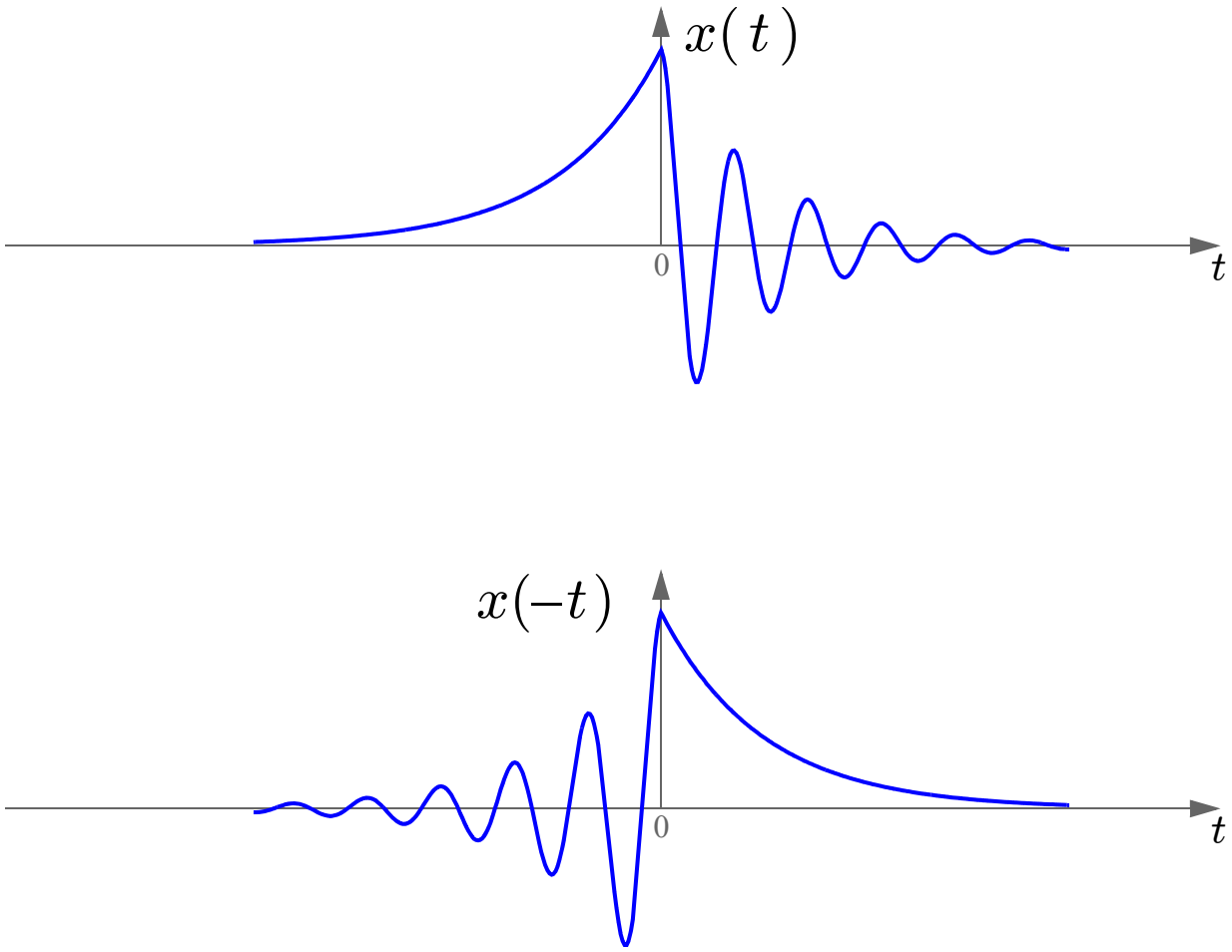
$x(at)$

- ▷ compresses when $a > 1$
- ▷ expands when $0 < a < 1$

Time Reversal

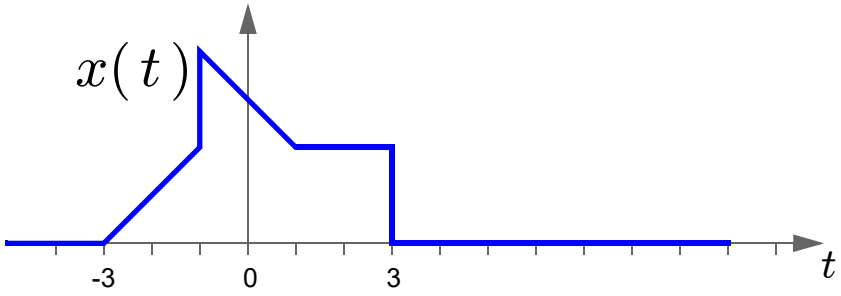


Time Reversal



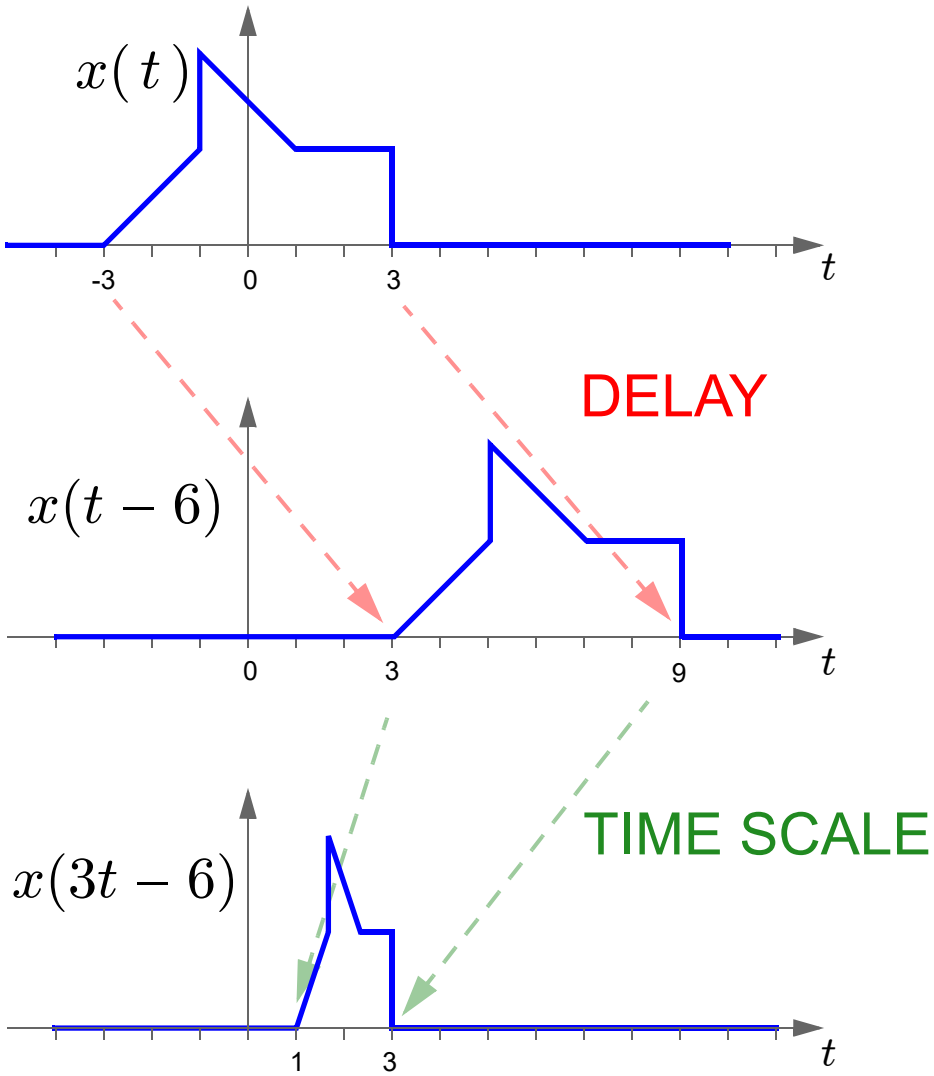
Example:

Sketch $z(t) = x(3t - 6)$, when $x(t)$ is defined below:

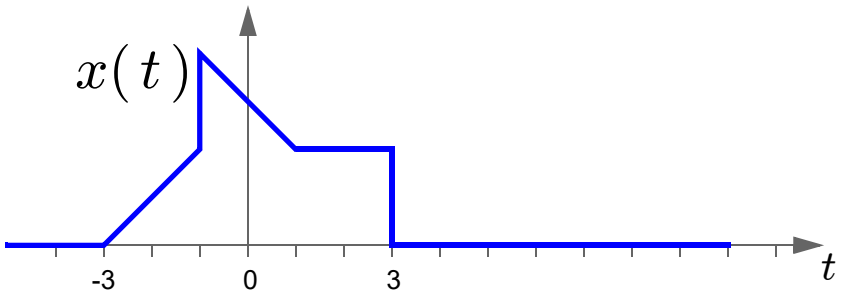


Example: Delay, then Time Scale

Sketch $z(t) = x(3t - 6)$, when $x(t)$ is defined below:



First Time Scale, Then Delay?



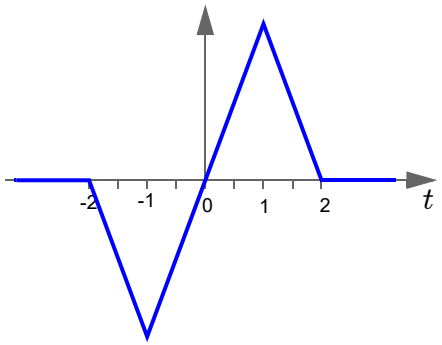
$$y(t) = x(3t)$$

$$z(t) = y(t-2)$$

Another Time Scale and Shift Example

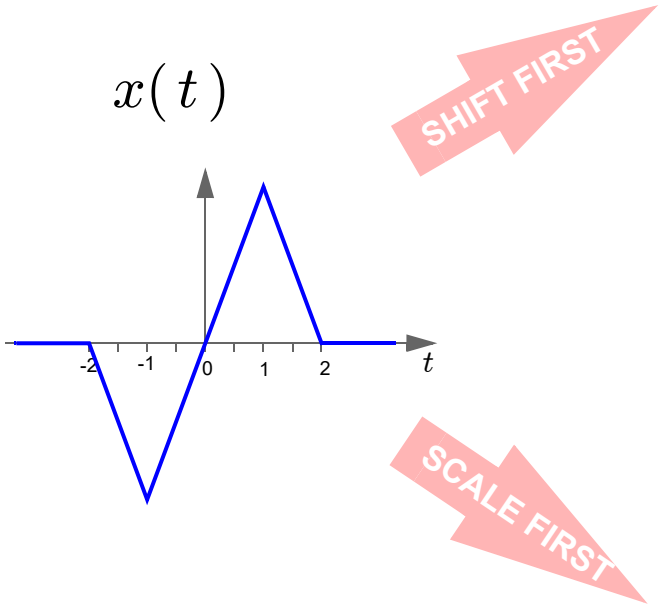
Sketch $x(2t-1)$, when $x(t)$ is defined on the left-hand side below:

$x(t)$



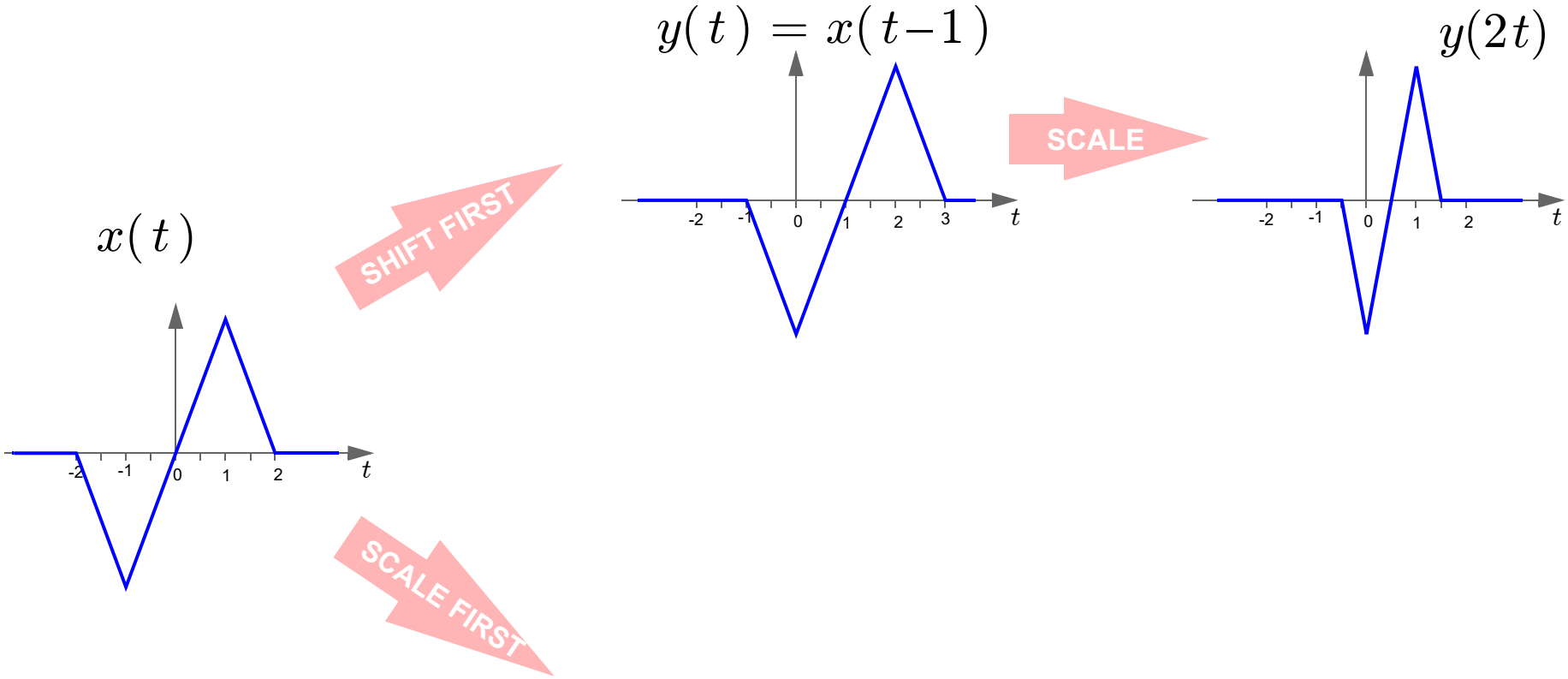
Another Time Scale and Shift Example

Sketch $x(2t-1)$, when $x(t)$ is defined on the left-hand side below:



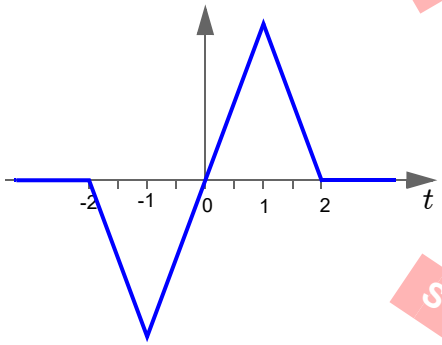
Another Time Scale and Shift Example

Sketch $x(2t-1)$, when $x(t)$ is defined on the left-hand side below:

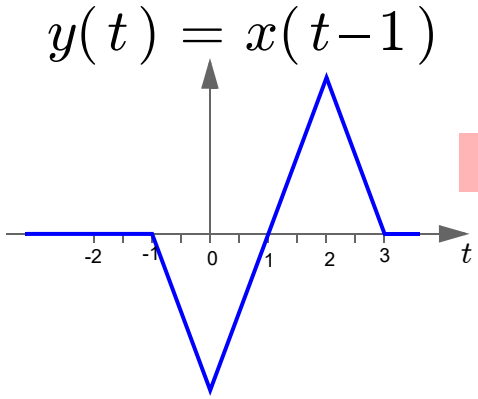


Another Time Scale and Shift Example

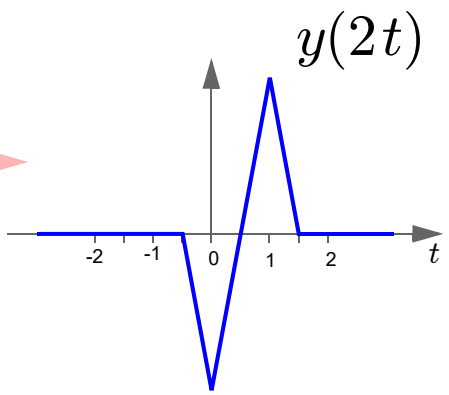
Sketch $x(2t-1)$, when $x(t)$ is defined on the left-hand side below:



SHIFT FIRST

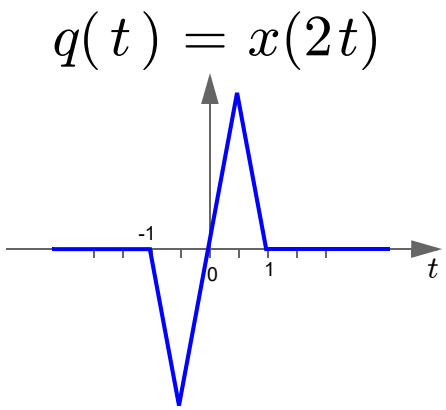


SCALE

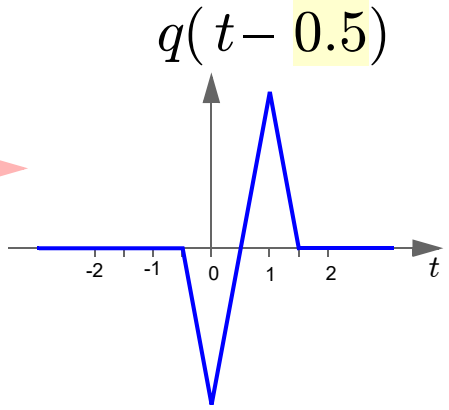


same answer either way!

SCALE FIRST



SHIFT



Time Scaling an Impuse

Pop Quiz 1: What is $\delta(-t)$?

Time Scaling an Impuse

Pop Quiz 1: What is $\delta(-t)$? $\delta(-t) = \delta(t)$

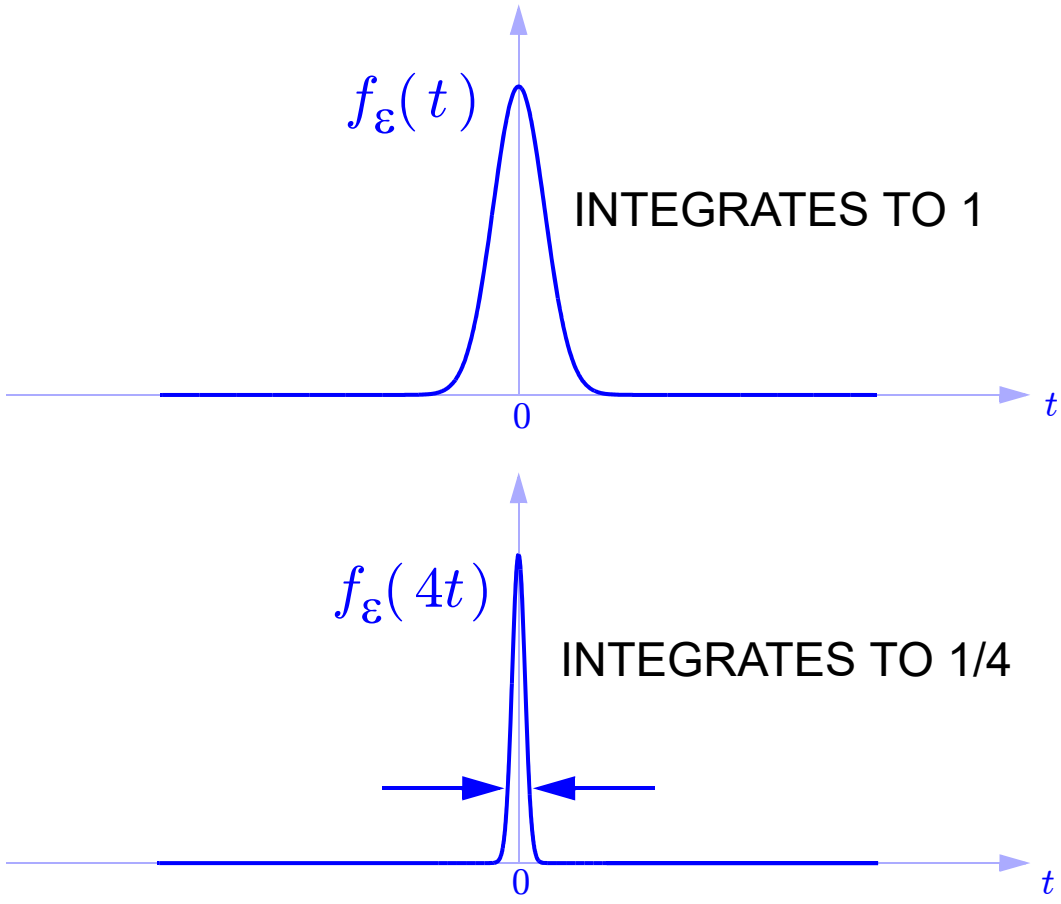
Pop Quiz 2: What is $\delta(4t)$?

Time Scaling an Impuse

Pop Quiz 1: What is $\delta(-t)$? $\delta(-t) = \delta(t)$

Pop Quiz 2: What is $\delta(4t)$?

Answer: $\delta(4t) = \frac{1}{4}\delta(t)$:

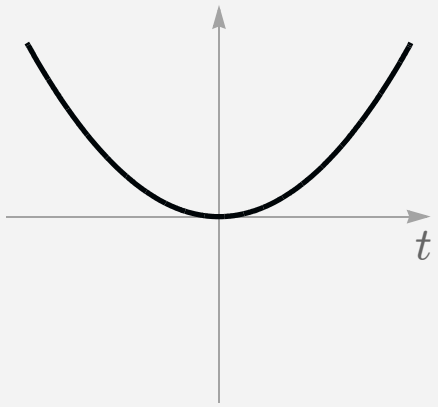


Categorizing Signals

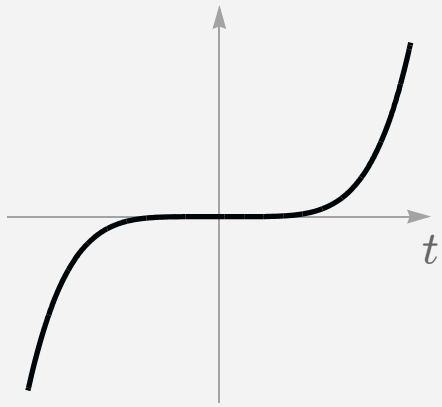
- symmetry: odd, even, neither
- periodic, nonperiodic
- causal, noncausal, neither
- “energy” (finite energy, zero power)
- “power” (infinite energy, finite power)

Types of Symmetry

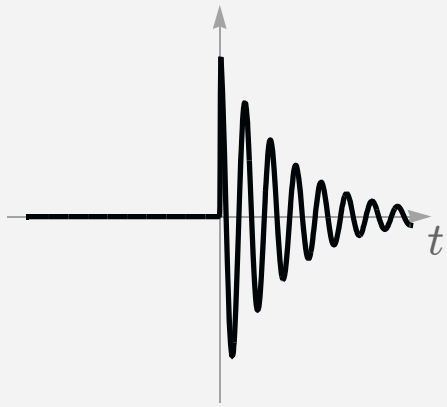
EVEN



ODD



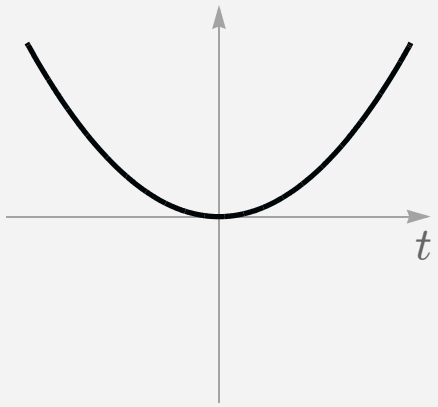
NEITHER



Types of Symmetry

EVEN

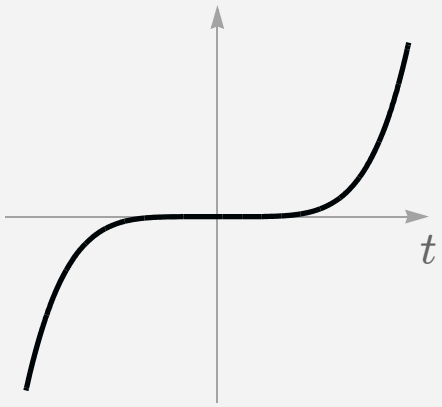
$$x(-t) = x(t)$$



E.g.: t^4, $e^{-|t|}$, $\cos(t)$

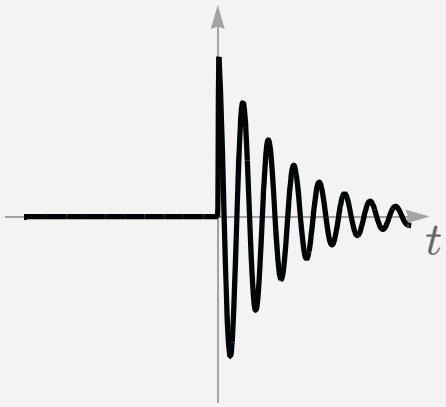
ODD

$$x(-t) = -x(t).$$



E.g.: t^3, $\sin(t)$

NEITHER



E.g.: $u(t)$

Even-Odd Decomposition

Theorem: Any signal can be decomposed into even and odd parts:

$$x(t) = x_e(t) + x_o(t)$$

where $x_e(t)$ is even

and $x_o(t)$ is odd.

Even-Odd Decomposition

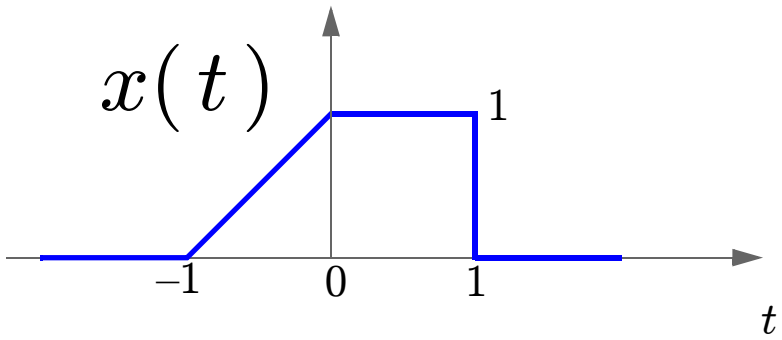
Theorem: Any signal can be decomposed into even and odd parts:

$$x(t) = x_e(t) + x_o(t)$$

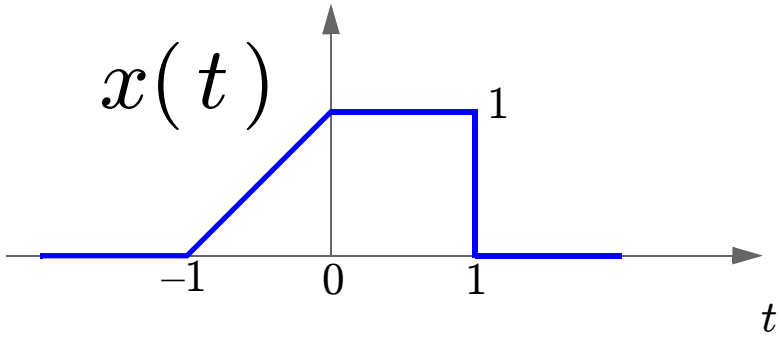
where $x_e(t) = \dfrac{x(t) + x(-t)}{2}$ is even

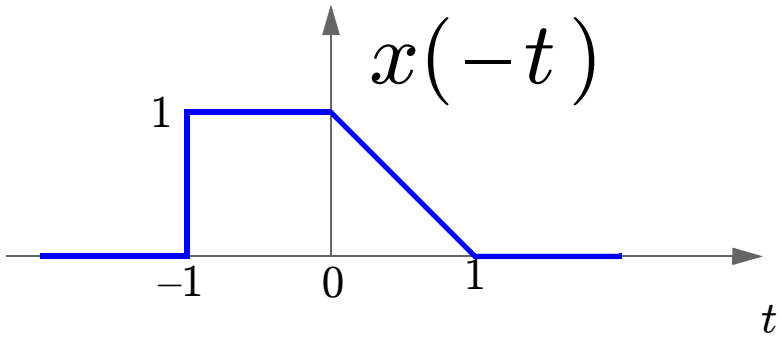
and $x_o(t) = \dfrac{x(t) - x(-t)}{2}$ is odd.

Example: Write as Even + Odd Signal

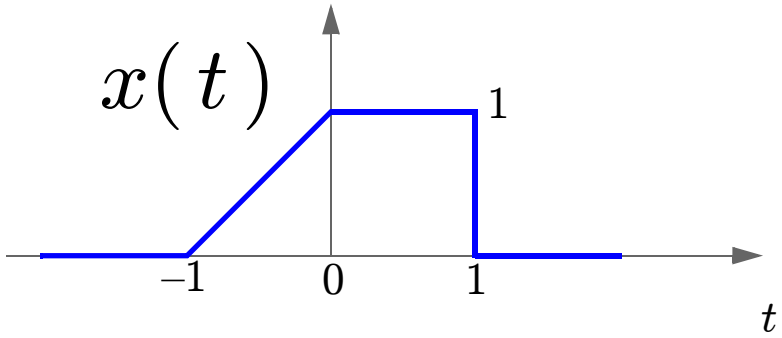


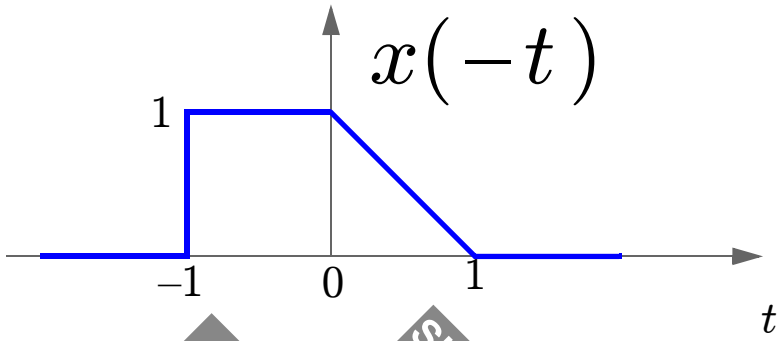
Example: Write as Even + Odd Signal





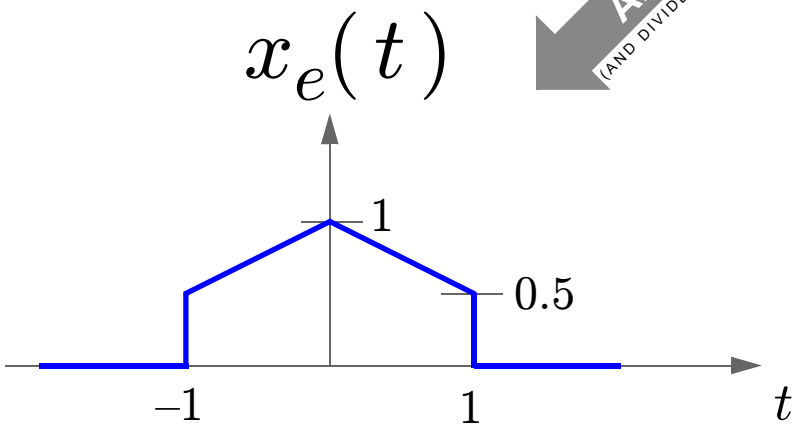
Example: Write as Even + Odd Signal

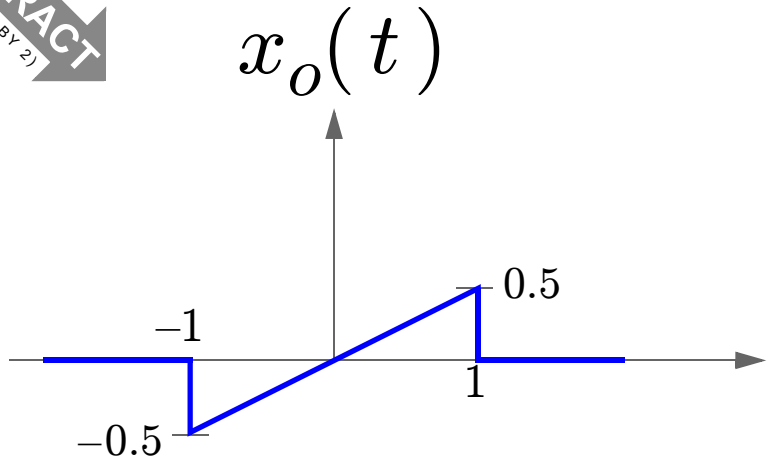




ADD (AND DIVIDE BY 2)

SUBTRACT (AND DIVIDE BY 2)





Pop Quiz

Let:

- $x_e(t)$ and $y_e(t)$ be even
- $x_o(t)$ and $y_o(t)$ be odd

(a) $x_e(t)y_e(t)$ is [even][odd][neither]

(b) $x_o(t)y_o(t)$ is [even][odd][neither]

(c) $x_e(t)x_o(t)$ is [even][odd][neither]

Energy and Power

Energy:

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

Power:

$$P = \left\langle x^2(t) \right\rangle = \lim_{\tau \to \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} x^2(t)\,dt$$

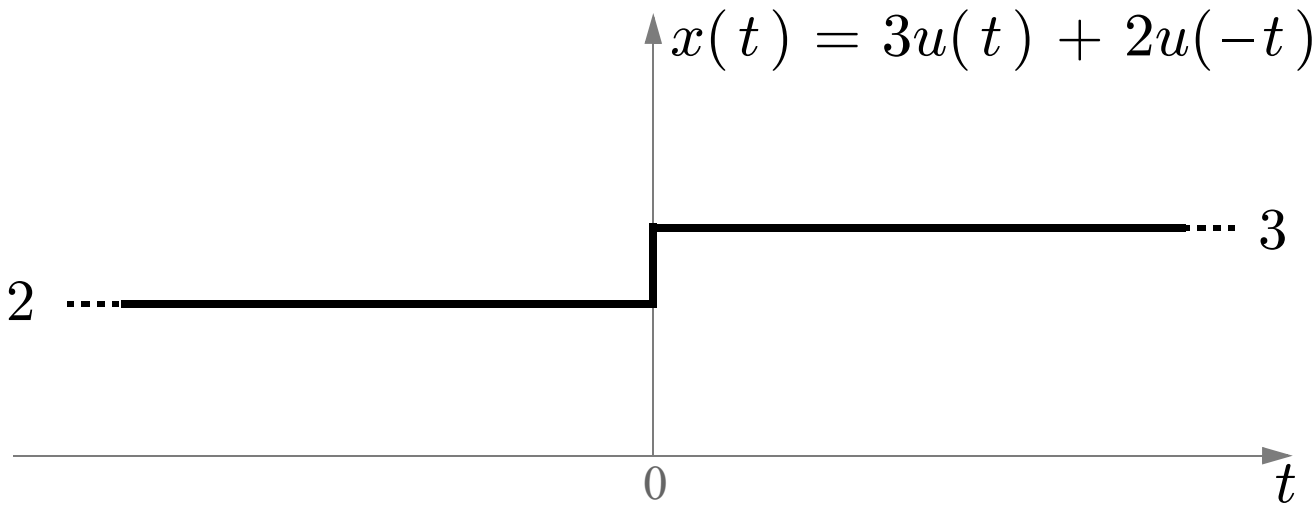
Three Examples

(a) Energy and power of $s(t) = 5\cos(200\pi t)$?

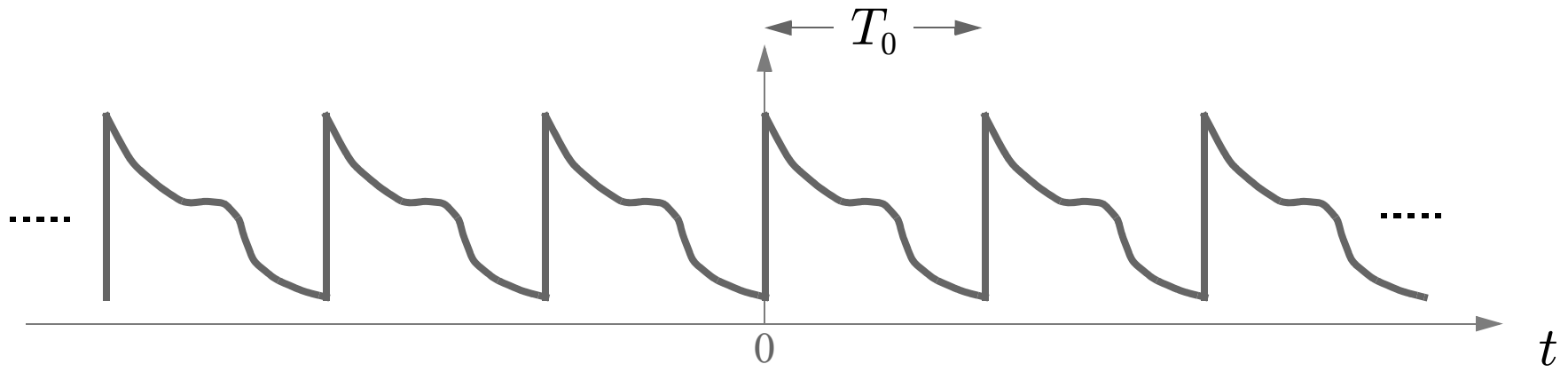
(b) Energy and power of $x(t) = e^{-2t}u(t)$?

(c) Energy and power of $x(t) = u(t)$?

Energy and Power?



Power of a Periodic Signal



Find the power P of a periodic signal satisfying $x(t) = x(t + T_0)$, expressed in terms of

- the energy E of one period,
- the fundamental period T_0.