

Lecture 24: Thu Nov 5, 2020

Reminder:

- Homework 10 due tonight
- Quiz 2: One week from today

Lecture

- Solving differential equations using Laplace
- initial and final value theorem

Quiz 2

Coverage:

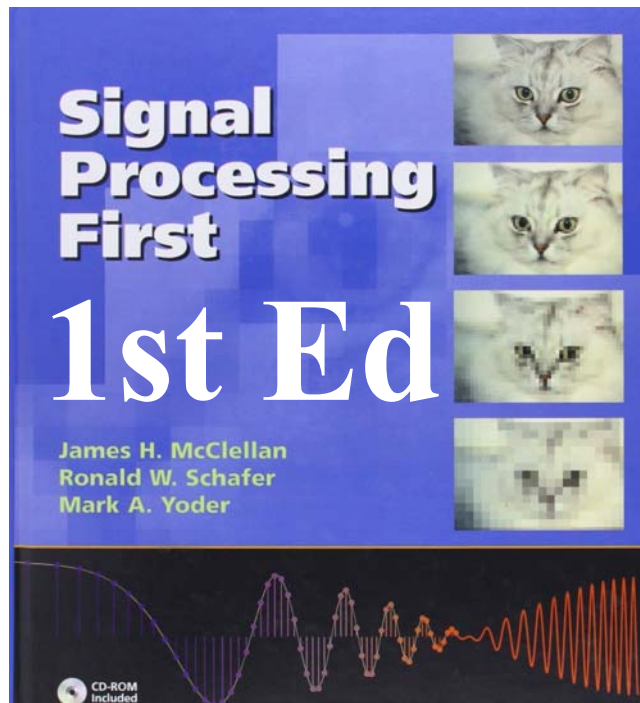
- lectures 10 – 23
- homeworks 6 – 10
- Chapters 7 – 10
of 3084 PDF book
- Chapters 4, 11 & 12
of 2026 SP1st (1st Ed)
- Youtube module 18 (lec 15)
through module 33 (lec 31)

lec10	Thu Sep 17	More Fourier series, intro to Fourier transform
lec11	Tue Sep 22	The Fourier Transform: famous pairs
lec12	Thu Sep 24	Fourier Transform properties
lec13	Tue Sep 29	Fourier transform: using the tables
lec14	Thu Oct 1	QUIZ 1
lec15	Tue Oct 6	MF in the frequency domain; Parseval's relation
lec16	Thu Oct 8	Time-bandwidth product, the Gaussian pulse, AM modulation and demodulation
lec17	Tue Oct 13	AM demodulation, downconversion, I&Q
lec18	Thu Oct 15	up and downconversion, complex envelopes, I & Q, envelope & phase
lec19	Tue Oct 20	Review I & Q; downconversion in the frequency domain
lec20	Thu Oct 22	The sampling theorem revisited; intro to Laplace
lec21	Tue Oct 27	The Laplace transform
lec22	Thu Oct 29	Laplace: Tables 1 and 2
lec23	Tue Nov 3	Translating between pole/zero, $H(s)$, $h(t)$; inverse Laplace; PFE

Coverage from 3084 PDF

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Coverage from SP First



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Coverage from Youtube

Modules 18 - 33:

Handouts for ECE 3084

- [syllabus](#)
- [3084 book](#)

Supplemental Material

- [Step and Delta functions](https://math.mit.edu/~stoon/18.031/) (from <https://math.mit.edu/~stoon/18.031/>)
- [Summer 2026 ECE3084 video lecture playlist \(youtube\)](#)
- [ECE3084 Resource Page](#): videos and lecture notes.
- [MIT Class](#): Videos, lecture notes, and problems.

ECE3084 Signals and Systems
53 videos • 2,060 views • Last updated on Aug 15, 2020

These are video lectures from the Summer 2020 offering of ECE3084: Signals and Systems at Georgia Tech, brought to you as a result of Covid-19 forcing Tech to operate in "distance learning" mode.

Lantertronics [SUBSCRIBE](#)

Module	Video Title	Duration	Channel
18	ECE3084 Lecture 15: Intro to Fourier Transforms (Ex: Decaying Exponential) (Signals & Systems, 2020)	12:34	Lantertronics
19	ECE3084 Lecture 16: Revisiting Frequency Response (Signals & Systems, Summer 2020, Georgia Tech)	9:09	Lantertronics
20	ECE3084 Lecture 17: Fourier Transforms of Impulses & Sinusoids (Signals & Systems, 2020, GA Tech)	13:18	Lantertronics
21	ECE3084 Lecture 18: The Fourier Time-Delay Property (Signals & Systems, Summer 2020, Georgia Tech)	7:00	Lantertronics
22	ECE3084 Lecture 19: The Fourier Time-Derivative Property (Signals & Systems, Summer 2020, GA Tech)	7:25	Lantertronics
23	ECE3084 Lecture 20: The Fourier Frequency-Shift Property (Signals & Systems, Summer 2020, GA Tech)	12:00	Lantertronics
24	ECE3084 Lecture 21: Fourier Transforms of Boxcars and Sinc Functions (Signals & Systems, 2020, GT)	18:36	Lantertronics
25	ECE3084 Lecture 22: Fourier Transforms of Periodic Signals (Signals & Systems, Summer 2020, GA Tech)	5:31	Lantertronics
26	ECE3084 Lecture 23: Amplitude Modulation for Communication, UPDATED VERSION (Signals & Systems)	21:02	Lantertronics
27	ECE3084 Lecture 24: Sampling in Time -- Replication in Frequency (Signals & Systems, 2020, GA Tech)	21:25	Lantertronics
28	ECE3084 Lecture 25: Replication in Time -- Sampling in Frequency (Signals & Systems, 2020, GA Tech)	16:23	Lantertronics
29	ECE3084 Lecture 27: Intro to Laplace Transforms (Exponentials & Sinusoids) (Signals & Systems, 2020)	20:10	Lantertronics
30	ECE3084 Lecture 28: The Laplace Time-Delay Property & Transforms of Impulses (Summer 2020, GA Tech)	16:23	Lantertronics
31	ECE3084 Lecture 29: The Laplace Time-Integral Property (Signals & Systems, Summer 2020, GA Tech)	6:22	Lantertronics
32	ECE3084 Lecture 30: Laplace Transforms of Polynomials (Ramps, etc.) (Signals & Systems, Summer 2020)	6:45	Lantertronics
33	ECE3084 Lecture 31: The Laplace "Frequency-Shift" Property (Signals & Systems, Summer 2020, GA Tech)	4:44	Lantertronics

Table 1: Pairs

Impulse	$\delta(t)$	1	✓
Delayed Impulse	$\delta(t - t_0), \quad t_0 \geq 0$	e^{-st_0}	✓
Step	$u(t)$	$\frac{1}{s}$	✓
Rectangular Pulse	$u(t) - u(t - T), \quad T > 0$	$\frac{1 - e^{-sT}}{s}$	✓
Ramp	$tu(t)$	$\frac{1}{s^2}$	✓
Polynomial	$t^k u(t), \quad k \geq 0$	$\frac{k!}{s^{k+1}}$	✓
Exponential	$e^{-at}u(t)$	$\frac{1}{s + a}$	✓
Polynomial $\times$ Exponential	$t^k e^{-at}u(t)$	$\frac{k!}{(s + a)^{k+1}}$	✓
Cosine	$\cos(\omega_0 t)u(t)$	$\frac{s}{s^2 + \omega_0^2}$	✓
Sine	$\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	✓
Exponential $\times$ Cosine	$e^{-at} \cos(\omega_0 t)u(t)$	$\frac{s + a}{(s + a)^2 + \omega_0^2}$	✓
Exponential $\times$ Sine	$e^{-at} \sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(s + a)^2 + \omega_0^2}$	✓

Table 2: Properties

Linearity	$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(s) + \beta X_2(s)$	✓
Right-Shift	$x(t - t_0), \quad t_0 \geq 0$	$e^{-st_0} X(s)$	✓
Time Scaling	$x(at), \quad a > 0$	$\frac{1}{a} X\left(\frac{s}{a}\right)$	
First Derivative	$\dot{x}(t) = \frac{d}{dt}x(t)$	$sX(s) - x(0)$	
Second Derivative	$\ddot{x}(t) = \frac{d^2}{dt^2}x(t)$	$s^2X(s) - sx(0) - \dot{x}(0)$	
Integration	$\int_0^t x(\tau) d\tau$	$\frac{X(s)}{s}$	✓
Modulation	$x(t)e^{at}$	$X(s - a)$	✓
Convolution	$x(t) * h(t)$	$X(s)H(s)$	✓
Final Value Theorem	$\lim_{t \rightarrow \infty} x(t), \quad (\text{if limit exists})$	$\lim_{s \rightarrow 0} sX(s)$	
Initial Value Theorem	$\lim_{t \rightarrow 0} x(t), \quad (\text{if limit exists})$	$\lim_{s \rightarrow \infty} sX(s)$	

s-domain differentiation $-tx(t)$ $\frac{d}{ds}X(s)$ ✓

Differentiation

$$\mathcal{L}\left\{\frac{d}{dt}x(t)\right\} =$$

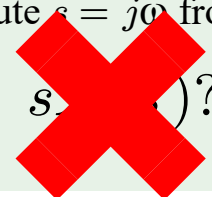
Differentiation

Substitute $s = j\omega$ from FT:

$$\mathcal{L}\left\{\frac{d}{dt}x(t)\right\} = sX(s)?$$

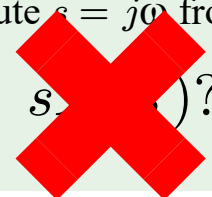
Differentiation

Substitute $s = j\omega$ from FT:

$$\mathcal{L}\left\{\frac{d}{dt}x(t)\right\} = sX(s) \quad ?$$


Differentiation

Substitute $s = j\omega$ from FT:

$$\mathcal{L}\left\{\frac{d}{dt}x(t)\right\} = sX(s) \quad \text{?}$$


$$\mathcal{L}\left\{\frac{d}{dt}x(t)\right\} = \int_{0^-}^{\infty} \underbrace{\frac{d}{dt}x(t)}_{dv} \underbrace{e^{-st}}_u dt \quad \text{(INTEGRATE BY PARTS)}$$

$$= uv - \int v du$$

$$= x(t)e^{-st} \Big|_0^{\infty} + s \int_{0^-}^{\infty} x(t)e^{-st} dt$$

$$= \overset{0}{x(t)e^{-st}} \Big|_{t=\infty} - x(0) + sX(s)$$

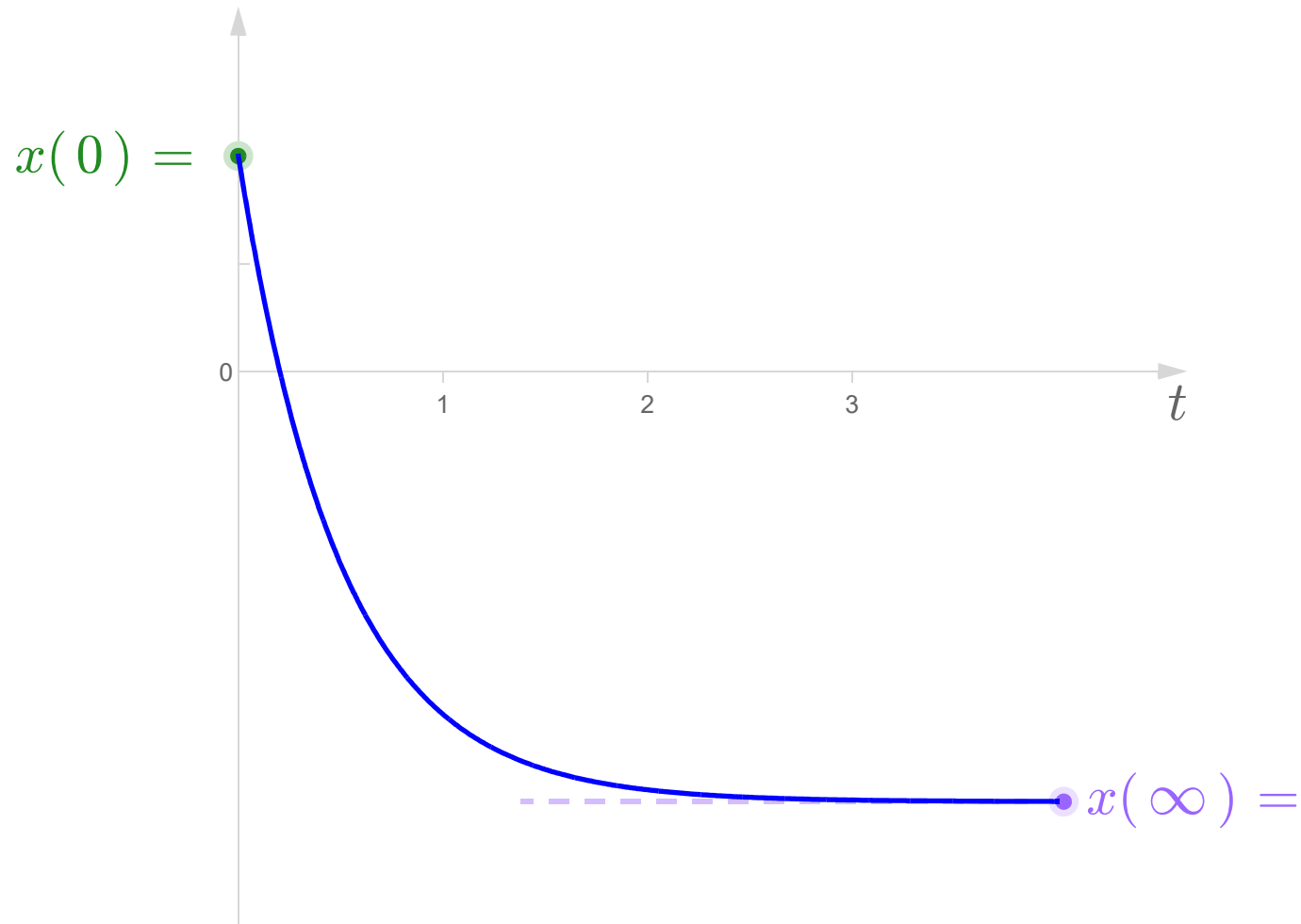
Differentiation

$$\mathcal{L}\left\{\frac{d}{dt}x(t)\right\} = sX(s) - x(0)$$

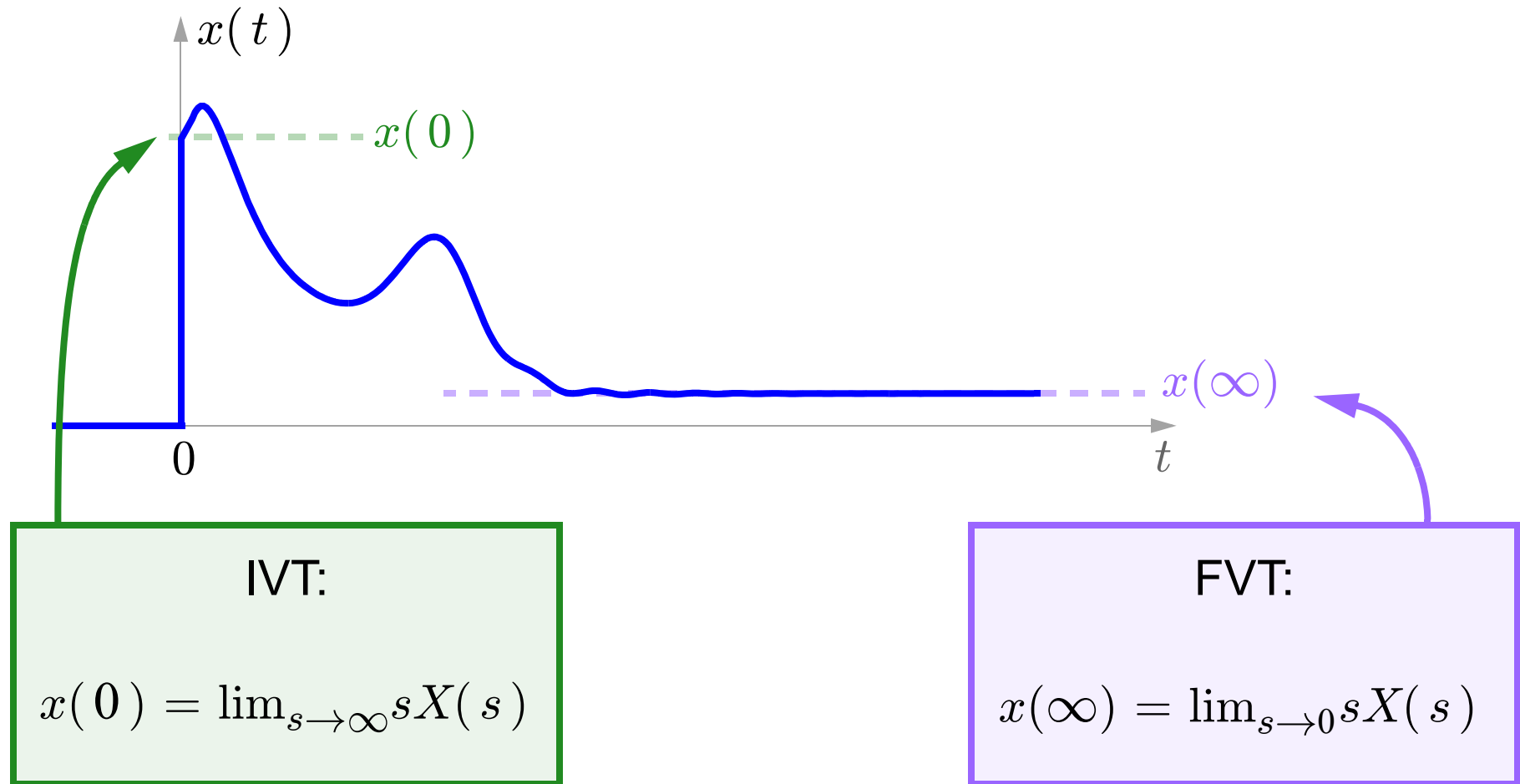
$$\begin{aligned}\mathcal{L}\left\{\frac{d}{dt}x(t)\right\} &= \int_{0^-}^{\infty} \underbrace{\frac{d}{dt}x(t)}_{dv} \underbrace{e^{-st}}_u dt && \text{(INTEGRATE BY PARTS)} \\ &= uv - \int v du \\ &= x(t)e^{-st} \Big|_0^{\infty} + s \int_{0^-}^{\infty} x(t)e^{-st} dt \\ &= \overset{0}{x(t)e^{-st}} \Big|_{t=\infty} - x(0) + sX(s)\end{aligned}$$

Pop Quiz

If $X(s) = \frac{3s - 12}{s^2 + 2s}$, find *initial value* $x(0)$ and *final value* $x(\infty)$:



Initial & Final-Value Theorems*



Why? Because $sX(s) = x(0) + \int_0^{\infty} \frac{d}{dt}x(t)e^{-st}dt \dots$

Pop Quiz

What is $x(\infty)$ when $X(s) = \frac{1}{s^2 + 1}$?

Solving Differential Equations

Consider a system defined by:

$$\frac{d^2}{dt^2} y(t) + \frac{d}{dt} y(t) + 1.25y(t) = x(t) + \frac{d}{dt} x(t),$$

with initial conditions $y(0) = 6$, $\dot{y}(0) = -3$.

- (a) Is the system LTI?
- (b) Is the system stable?
- (c) Find the “zero-input response:” i.e., find $y(t)$ when $x(t) = 0$.
- (c) Find the “step response:” i.e., find $y(t)$ when $x(t) = u(t)$.

Solution

Take Laplace transform of both sides of diff-eq:

$$\Rightarrow Q(s)Y(s) - P_{\text{init}}(s) = P(s)X(s)$$

$$\Rightarrow Y(s) = \underbrace{\frac{P_{\text{init}}(s)}{Q(s)}}_{\text{zero-input response}} + \underbrace{H(s)X(s)}_{\text{zero-state response}}$$

where $H(s) = \frac{P(s)}{Q(s)} =$

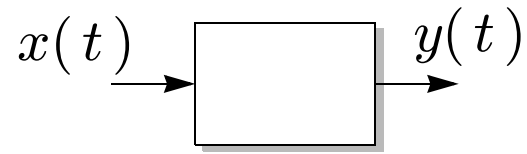
Zero-Input Response?

Step Response?

Solution (c)

Write $Y(s) =$

5 Ways to Specify an LTI System

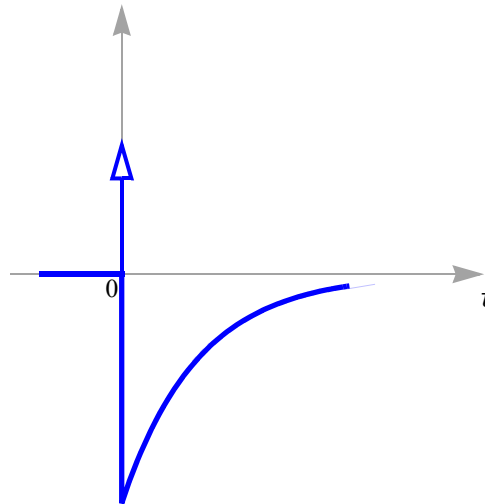


1. impulse response $h(t)$
2. differential equation, e.g. $y(t) = \frac{d}{dt}x(t) + 3x(t - 1)$
3. frequency response $H(j\omega)$
4. system transfer function $H(s)$
5. pole-zero plot

Key skill (e.g. homeworks and exams): *translate from any one to any other*

Pop Quiz

Let $h(t) = \delta(t) - 2\pi e^{-\pi t}$:



- Find $H(s)$
- Sketch pole-zero plot
- Sketch $|H(j\omega)|$ using geometry of pole-zero plot

Pop Quiz

Find $x(t)$ when $X(s) = \frac{3s + 6}{s^2 + 4s + 29}$

Pop Quiz

Find $x(t)$ when $X(s) = \frac{3s + 6}{s^2 + 4s + 29}$

```
>> roots([1 4 29])  
ans =  
    -2 + 5i  
    -2 - 5i
```


Pop Quiz

Find $x(t)$ when: $X(s) = \frac{3s + 6}{s^2 + 4s + 29}$

1. Brute force: $X(s) = \frac{A}{s + 2 - 5j} + \frac{B}{s + 2 + 5j} \dots$

```
>> roots([1 4 29])
```

```
ans =
```

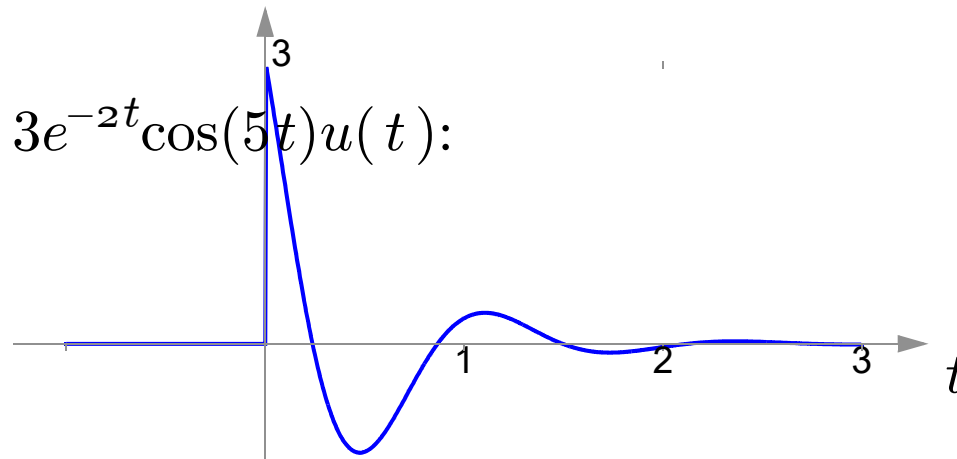
```
-2 + 5i
```

```
-2 - 5i
```

2. Easier to complete square:

$$X(s) = \frac{3s + 6}{s^2 + 4s + 29} = 3 \frac{(s + 2)}{(s + 2)^2 + 5^2}$$

$$\Rightarrow x(t) = 3e^{-2t} \cos(5t) u(t):$$



Coming Next:

2nd-Order Systems