Lecture 24: Thu Nov 5, 2020

Reminder:

- Homework 10 due tonight
- Quiz 2: One week from today

Lecture

- Solving differential equations using Laplace
- initial and final value theorem

Quiz 2

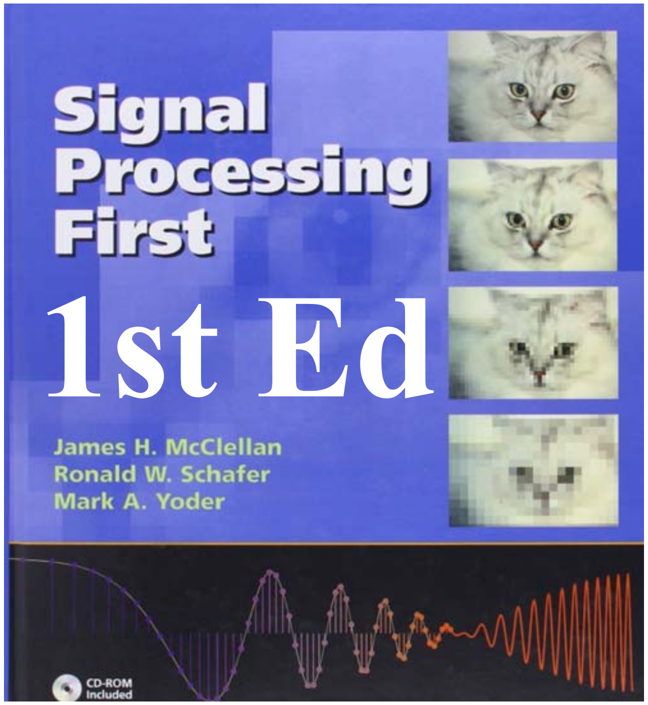
Coverage:

- lectures 10 – 23
- homeworks 6 – 10
- Chapters 7 – 10
  of 3084 PDF book
- Chapters 4, 11 & 12
  of 2026 SP1st (1st Ed)
- Youtube module 18 (lec 15)
  through module 33 (lec 31)

lec10	Thu Sep 17	More Fourier series, intro to Fourier transform
lec11	Tue Sep 22	The Fourier Transform: famous pairs
lec12	Thu Sep 24	Fourier Transform properties
lec13	Tue Sep 29	Fourier transform: using the tables
lec14	Thu Oct 1	QUIZ 1
lec15	Tue Oct 6	MF in the frequency domain; Parseval's relation
lec16	Thu Oct 8	Time-bandwidth product, the Gaussian pulse, AM modulation and demod
lec17	Tue Oct 13	AM demodulation, downconversion, I&Q
lec18	Thu Oct 15	up and downconversion, complex envelopes, I & Q, envelope & phase
lec19	Tue Oct 20	Review I & Q; downconversion in the frequency domain
lec20	Thu Oct 22	The sampling theorem revisited; intro to Laplace
lec21	Tue Oct 27	The Laplace transform
lec22	Thu Oct 29	Laplace: Tables 1 and 2
lec23	Tue Nov 3	Translating between pole/zero, H(s), h(t); inverse Laplace; PFE

Coverage from 3084 PDF

Coverage from SP First



Coverage from Youtube

Modules 18 - 33:

Handouts for ECE 3084

- syllabus
- 3084 book

Supplemental Material

- Step and Delta functions (from https://math.mit.edu/~stoopn/18.031/)
- Summer 2026 ECE3084 video lecture playlist (youtube)
- ECE3084 Resource Page: Videos and lecture notes.
- MIT Class: Videos, lecture notes, and problems.

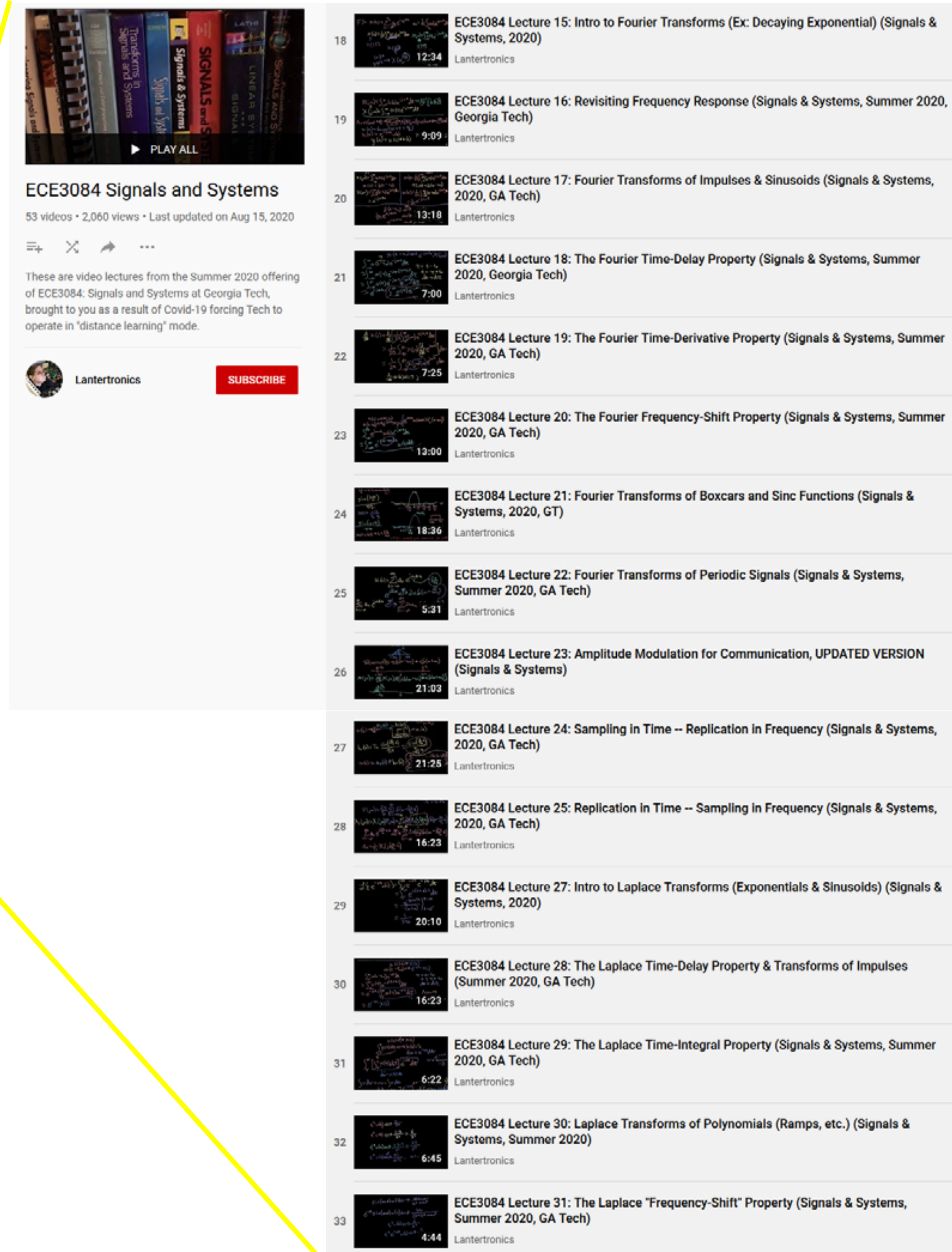


Table 1: Pairs

Impulse	$\delta(t)$	1
Delayed Impulse	$\delta(t-t_0), \quad t_0 \geq 0$	e^{-st_0}
Step	$u(t)$	$\dfrac{1}{s}$
Rectangular Pulse	$u(t)-u(t-T), \quad T>0$	$\dfrac{1-e^{-sT}}{s}$
Ramp	$tu(t)$	$\dfrac{1}{s^2}$
Polynomial	$t^k u(t), \quad k \geq 0$	$\dfrac{k!}{s^{k+1}}$
Exponential	$e^{-at}u(t)$	$\dfrac{1}{s+a}$
Polynomial $\times$ Exponential	$t^k e^{-at} u(t)$	$\dfrac{k!}{(s+a)^{k+1}}$
Cosine	$\cos(\omega_0 t)u(t)$	$\dfrac{s}{s^2+\omega_0^2}$
Sine	$\sin(\omega_0 t)u(t)$	$\dfrac{\omega_0}{s^2+\omega_0^2}$
Exponential $\times$ Cosine	$e^{-at}\cos(\omega_0 t)u(t)$	$\dfrac{s+a}{(s+a)^2+\omega_0^2}$
Exponential $\times$ Sine	$e^{-at}\sin(\omega_0 t)u(t)$	$\dfrac{\omega_0}{(s+a)^2+\omega_0^2}$

Table 2: Properties

Linearity	$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(s) + \beta X_2(s)$ ✓
Right-Shift	$x(t - t_0), \quad t_0 \geq 0$	$e^{-st_0} X(s)$ ✓
Time Scaling	$x(at), \quad a > 0$	$\frac{1}{a} X\left(\frac{s}{a}\right)$
First Derivative	$\dot{x}(t) = \frac{d}{dt} x(t)$	$sX(s) - x(0)$
Second Derivative	$\ddot{x}(t) = \frac{d^2}{dt^2} x(t)$	$s^2 X(s) - sx(0) - \dot{x}(0)$
Integration	$\int_0^t x(\tau) d\tau$	$\frac{X(s)}{s}$ ✓
Modulation	$x(t) e^{at}$	$X(s - a)$ ✓
Convolution	$x(t) * h(t)$	$X(s) H(s)$ ✓
Final Value Theorem	$\lim_{t \to \infty} x(t)$, (if limit exists)	$\lim_{s \to 0} sX(s)$
Initial Value Theorem	$\lim_{t \to 0} x(t)$, (if limit exists)	$\lim_{s \to \infty} sX(s)$

s-domain differentiation $\quad -tx(t) \quad \frac{d}{ds} X(s)$ ✓

Differentiation

$$\mathcal{L}\{\frac{d}{dt}x(t)\} =$$

Differentiation

Substitute $s = j\omega$ from FT:

$$\mathcal{L}\{\frac{d}{dt}x(t)\} = \qquad sX(s)?$$

Differentiation

Substitute $s = j\omega$ from FT:

$$\mathcal{L}\{\frac{d}{dt}x(t)\} = \cancel{sX(s)}?$$

Differentiation

Substitute $s = j\omega$ from FT:

$$\mathcal{L}\{\frac{d}{dt}x(t)\} = \cancel{sX(s)}?$$

$$\mathcal{L}\{\frac{d}{dt}x(t)\} = \int_{0^-}^{\infty} \underbrace{\frac{d}{dt}x(t)}_{dv}\underbrace{e^{-st}}_{u}dt$$

(INTEGRATE BY PARTS)

$$= uv - \int v\,du$$

$$= x(t)e^{-st}\Big|_{0}^{\infty} + s\int_{0^-}^{\infty} x(t)e^{-st}dt$$

$$= \overbrace{x(t)e^{-st}\Big|_{t=\infty}}^{0} - x(0) + sX(s)$$

Differentiation

$$\mathcal{L}\{\frac{d}{dt}x(t)\} = \quad sX(s) - x(0)$$

$$\mathcal{L}\{\frac{d}{dt}x(t)\} = \int_{0^-}^{\infty} \underbrace{\frac{d}{dt}x(t)}_{dv}\underbrace{e^{-st}}_{u}dt$$

(INTEGRATE BY PARTS)

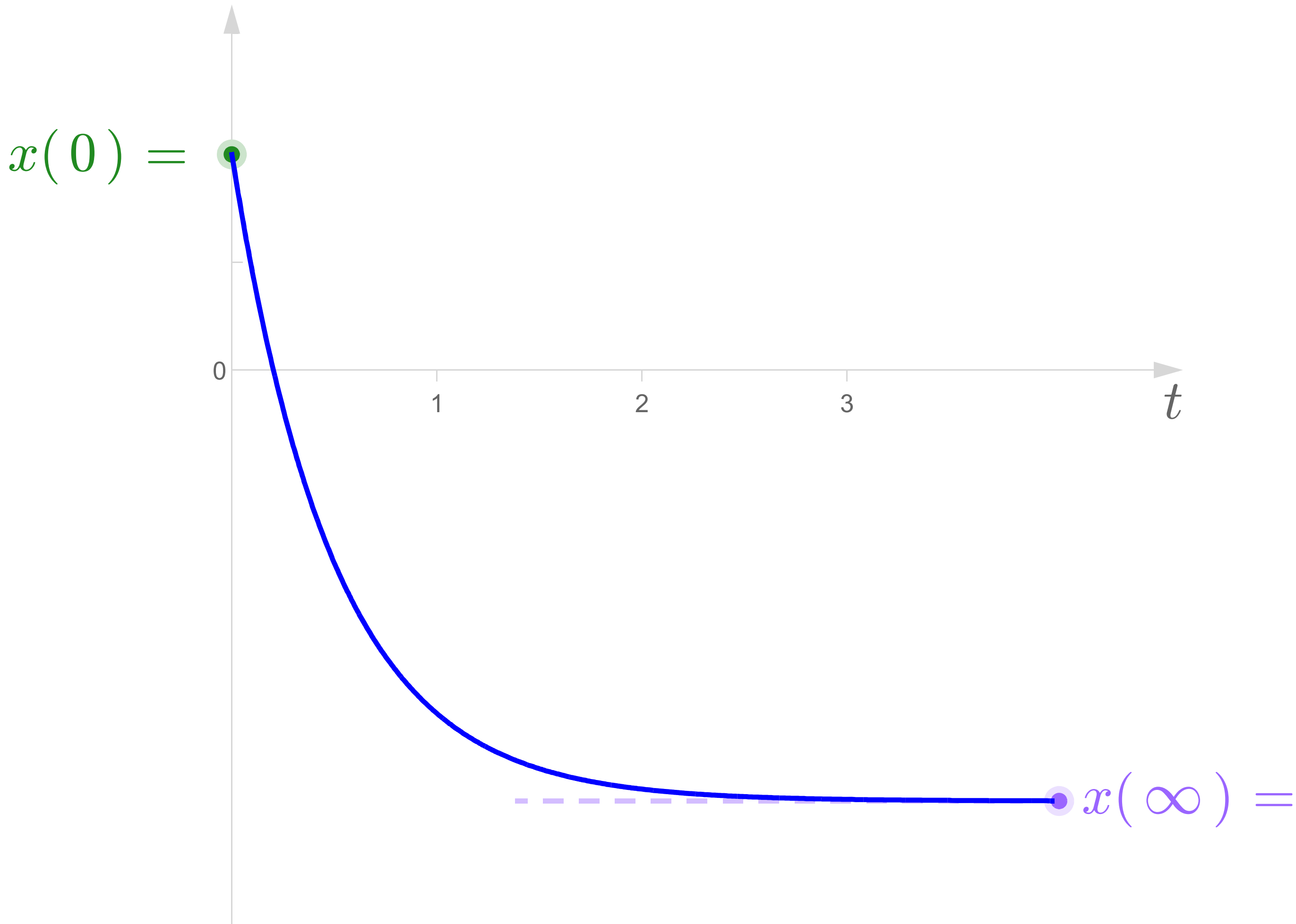
$$= \quad uv \;-\; \int v\,du$$

$$= x(t)e^{-st}\Big|_{0}^{\infty} + s\int_{0^-}^{\infty} x(t)e^{-st}dt$$

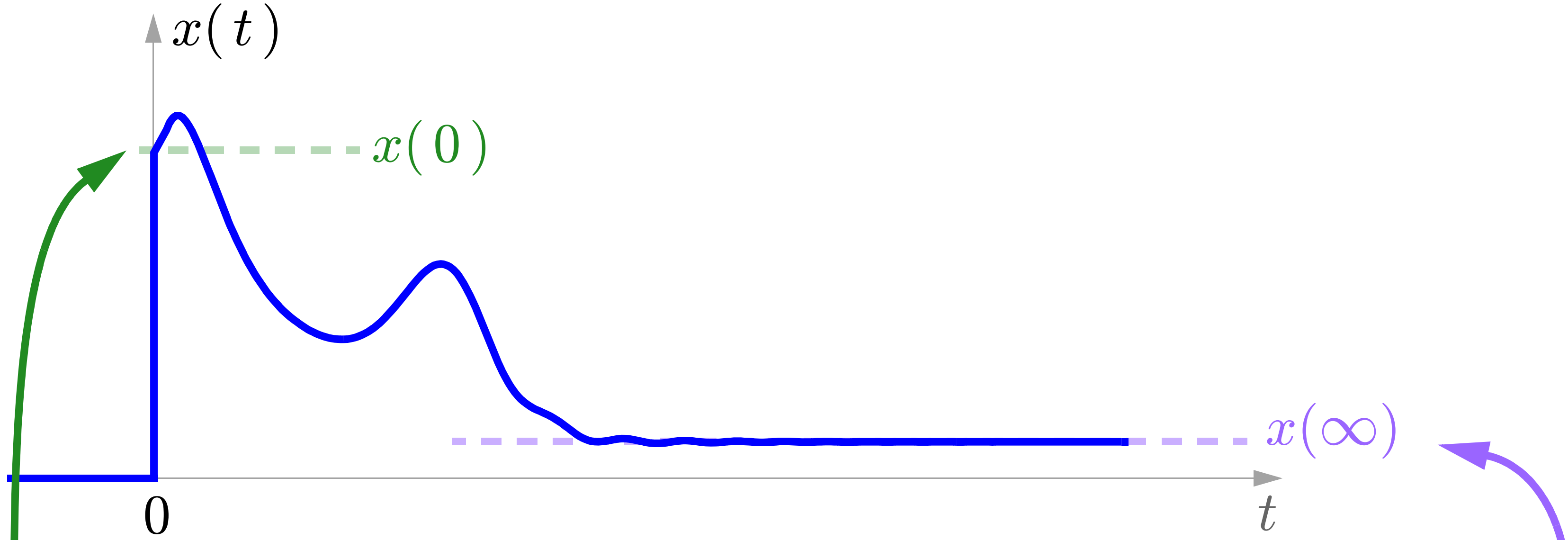
$$= \overset{0}{x(t)e^{-st}\Big|_{t=\infty}} - x(0) + sX(s)$$

Pop Quiz

If $X(s) = \dfrac{3s-12}{s^2+2s}$, find *initial value* $x(0)$ and *final value* $x(\infty)$:



Initial & Final-Value Theorems*



IVT:

$$x(0) = \lim_{s \to \infty} sX(s)$$

FVT:

$$x(\infty) = \lim_{s \to 0} sX(s)$$

Why? Because $\quad sX(s) = x(0) + \int_0^\infty \frac{d}{dt} x(t) e^{-st} dt \quad \ldots$

*when limits exist

Pop Quiz

What is $x(\infty)$ when $X(s) = \dfrac{1}{s^2+1}$?

Solving Differential Equations

Consider a system defined by:

$$\frac{d^2}{dt^2}y(t) + \frac{d}{dt}y(t) + 1.25y(t) = x(t) + \frac{d}{dt}x(t),$$

with initial conditions $y(0) = 6,\ \dot{y}(0) = -3$.

(a) Is the system LTI?

(b) Is the system stable?

(c) Find the “zero-input response:” i.e., find $y(t)$ when $x(t) = 0$.

(c) Find the “step response:” i.e., find $y(t)$ when $x(t) = u(t)$.

Solution

Take Laplace transform of both sides of diff-eq:

$$\Rightarrow \qquad Q(s)Y(s) - P_{\text{init}}(s) = P(s)X(s)$$

$$\Rightarrow \qquad Y(s) = \underbrace{\frac{P_{\text{init}}(s)}{Q(s)}}_{\substack{\text{zero-input}\\\text{response}}} + \underbrace{H(s)X(s)}_{\substack{\text{zero-state}\\\text{response}}}$$

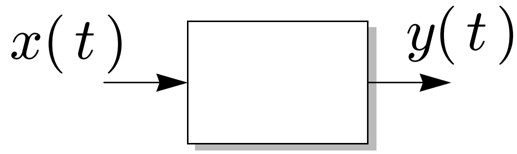
where $\qquad H(s) = \dfrac{P(s)}{Q(s)} =$

Zero-Input Response?
Step Response?

Solution (c)

Write $Y(\, s\,) =$

5 Ways to Specify an LTI System

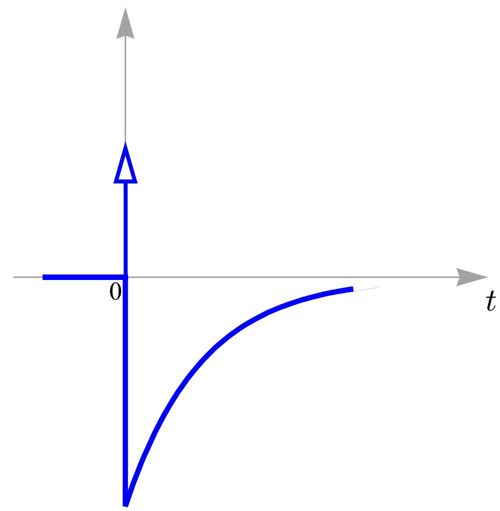


1. impulse response $h(t)$
2. differential equation, e.g. $\quad y(t) = \frac{d}{dt}x(t) + 3x(t-1)$
3. frequency response $H(j\omega)$
4. system transfer function $H(s)$
5. pole-zero plot

Key skill (e.g. homeworks and exams): *translate from any one to any other*

Pop Quiz

Let $h(t) = \delta(t) - 2\pi e^{-\pi t}$:



- Find $H(s)$
- Sketch pole-zero plot
- Sketch $|H(j\omega)|$ using geometry of pole-zero plot

Pop Quiz

Find $x(t)$ when $$X(s) = \frac{3s+6}{s^2+4s+29}$$

Pop Quiz

Find $x(t)$ when $$X(s) = \frac{3s + 6}{s^2 + 4s + 29}$$

```
>> roots([1 4 29])
ans =
  -2 + 5i
  -2 - 5i
```

Pop Quiz

Find $x(t)$ when: $X(s) = \dfrac{3s+6}{s^2+4s+29}$

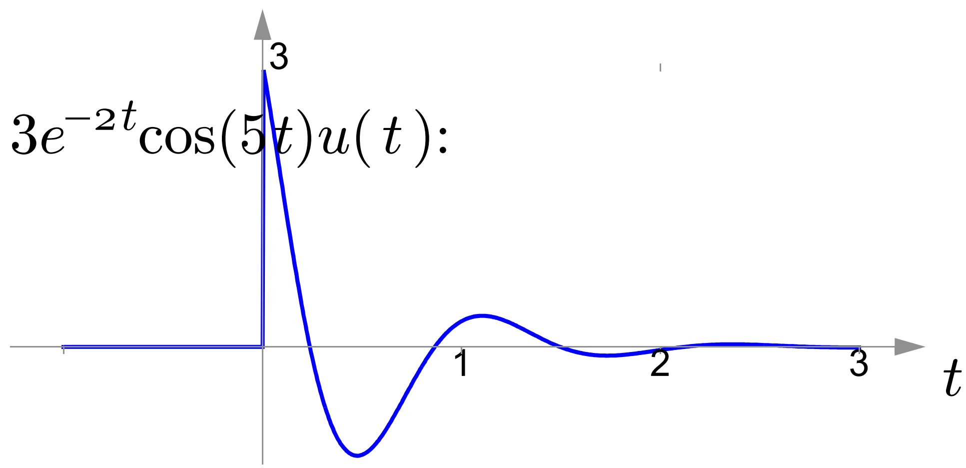
```
>> roots([1 4 29])
ans =
  -2 + 5i
  -2 - 5i
```

1. Brute force: $X(s) = \dfrac{A}{s+2-5j} + \dfrac{B}{s+2+5j}$ ...

2. Easier to complete square:

$$X(s) = \frac{3s+6}{s^2+4s+29} = 3\frac{(s+2)}{(s+2)^2+5^2}$$

$\Rightarrow x(t) = 3e^{-2t}\cos(5t)u(t)$:



Coming Next:

2nd-Order Systems