

Lecture 23: Tue Nov 3, 2020

Reminder:

- Homework 10 due Thursday

Lecture

- inverse Laplace via PFE
- Translating between equivalent representations:
 - ▷ pole-zero plot
 - ▷ impulse response
 - ▷ frequency response
 - ▷ transfer function

Quiz 2 in 9 Days

Coverage:

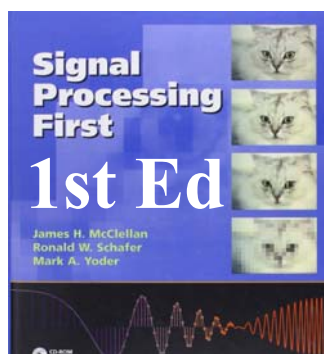
- lectures 10 – 23
- homeworks 6 – 10
- Chapters 7 – 10
of 3084 PDF book
- Chapters 4, 11 & 12
of 2026 SP1st (1st Ed)
- Youtube module 18 (lec 15)
through module 33 (lec 31)

:10	Thu Sep 17	More Fourier series, intro to Fourier transform
:11	Tue Sep 22	The Fourier Transform: famous pairs
:12	Thu Sep 24	Fourier Transform properties
:13	Tue Sep 29	Fourier transform: using the tables
:14	Thu Oct 1	QUIZ 1
:15	Tue Oct 6	MF in the frequency domain; Parseval's relation
:16	Thu Oct 8	Time-bandwidth product, the Gaussian pulse, AM modulation and demod
:17	Tue Oct 13	AM demodulation, downconversion, I&Q
:18	Thu Oct 15	up and downconversion, complex envelopes, I & Q, envelope & phase
:19	Tue Oct 20	Review I & Q; downconversion in the frequency domain
:20	Thu Oct 22	The sampling theorem revisited; intro to Laplace
:21	Tue Oct 27	The Laplace transform
:22	Thu Oct 29	Laplace: Tables 1 and 2
:23	Tue Nov 3	Translating between pole/zero, $H(s)$, $h(t)$; inverse Laplace; PFE

Coverage from 3084 PDF

7	Fourier Transforms	49
7.1	Motivation	49
7.2	A key observation	50
7.3	Your first Fourier transform: decaying exponential	50
7.3.1	Frequency response example	50
7.4	Your first Fourier transform property: time shift	50
7.5	Your second Fourier transform: delta functions	50
7.5.1	Sanity check	50
7.6	Your second Fourier transform property: derivatives in time	50
7.7	Rectangular boxcar functions	50
7.7.1	Fourier transform of a symmetric rectangular boxcar	50
7.7.2	Inverse Fourier transform of single symmetric boxcar	50
7.7.3	Observations about our boxcar examples	50
7.8	Fourier transforms of deltas and sinusoids	50
7.9	Fourier transform of periodic signals	50
8	Modulation	59
8.1	Fourier view of filtering	59
8.1.1	Filtering by an ideal lowpass filter	60
8.2	Modulation property of Fourier transforms	60
8.2.1	Modulation by a complex sinusoid	61
8.3	Double Side Band Amplitude Modulation	61
8.3.1	DSBAM transmission	61
8.3.2	DSBAM reception	62
8.3.3	Practical matters	63
8.4	Baseband representations of bandlimited signals	64
8.5	Example: Fourier transform of decaying sinusoids	66
9	Sampling and Periodicity	67
9.1	Sampling time-domain signals	67
9.1.1	A Warm-Up Question	67
9.1.2	Sampling: from ECE2026 to ECE3084	67
9.1.3	A mathematical model for sampling	68
9.1.4	Practical reconstruction from samples	71
9.2	Deriving the DTFT and IDTFT from the CTFT and ICTFT	73
9.3	Fourier series reimaged as frequency-domain sampling	75
9.3.1	A quick “sanity check”	76
9.4	The grand beauty of the duality of sampling and periodicity	77
10	Laplace Transforms	81
10.1	Introducing the Laplace transform	81
10.1.1	Beyond Fourier	81
10.1.2	Examples	82
10.2	Key properties of the Laplace transform	83
10.2.1	Linearity	83
10.2.2	Taking derivatives	84
10.2.3	Integration	84
10.2.4	Time delays	85
10.3	The initial and final value theorems	86
10.3.1	Examples	86
10.4	Partial fraction expansions (PFEs)	87
10.4.1	First PFE example	87
10.4.2	An example with distinct real and imaginary roots	88
10.4.3	An example with complex roots	89
10.4.4	Residue method with repeated roots	89
10.5	PFEs of Improper Fractions	91
10.6	Laplace and differential equations	92
10.6.1	First-order system example	92
10.6.2	Another first-order system example	93
10.7	Transfer functions	97
10.7.1	Input-output systems	98
10.7.2	Stability	102

Coverage from SP First



4 Sampling and Aliasing

71

4-1	Sampling	71
4-1.1	Sampling Sinusoidal Signals	73
4-1.2	The Concept of Aliasing	75
4-1.3	Spectrum of a Discrete-Time Signal	76
4-1.4	The Sampling Theorem	77
4-1.5	Ideal Reconstruction	78
4-2	Spectrum View of Sampling and Reconstruction	79
4-2.1	Spectrum of a Discrete-Time Signal Obtained by Sampling	79
4-2.2	Over-Sampling	79
4-2.3	Aliasing Due to Under-Sampling	81
4-2.4	Folding Due to Under-Sampling	82
4-2.5	Maximum Reconstructed Frequency	83
4-3	Strobe Demonstration	84
4-3.1	Spectrum Interpretation	87
4-4	Discrete-to-Continuous Conversion	88
4-4.1	Interpolation with Pulses	88
4-4.2	Zero-Order Hold Interpolation	89
4-4.3	Linear Interpolation	90
4-4.4	Cubic Spline Interpolation	90
4-4.5	Over-Sampling Aids Interpolation	91
4-4.6	Ideal Bandlimited Interpolation	92
4-5	The Sampling Theorem	93
4-6	Summary and Links	94
4-7	Problems	96

11 Continuous-Time Fourier Transform

307

11-1	Definition of the Fourier Transform	308
11-2	Fourier Transform and the Spectrum	310
11-2.1	Limit of the Fourier Series	310
11-3	Existence and Convergence of the Fourier Transform	312
11-4	Examples of Fourier Transform Pairs	313
11-4.1	Right-Sided Real Exponential Signals	313
11-4.1.1	Bandwidth and Decay Rate	314
11-4.2	Rectangular Pulse Signals	314
11-4.3	Bandlimited Signals	316
11-4.4	Impulse in Time or Frequency	317
11-4.5	Sinusoids	318
11-4.6	Periodic Signals	319
11-5	Properties of Fourier Transform Pairs	322
11-5.1	The Scaling Property	322
11-5.2	Symmetry Properties of Fourier Transform Pairs	324
11-6	The Convolution Property	326
11-6.1	Frequency Response	326
11-6.2	Fourier Transform of a Convolution	327
11-6.3	Examples of the Use of the Convolution Property	328
11-6.3.1	Convolution of Two Bandlimited Functions	328
11-6.3.2	Product of Two Sinc Functions	329
11-6.3.3	Partial Fraction Expansions	330
11-7	Basic LTI Systems	332
11-7.1	Time Delay	332
11-7.2	Differentiation	333
11-7.3	Systems Described by Differential Equations	334
11-8	The Multiplication Property	335
11-8.1	The General Signal Multiplication Property	335
11-8.2	The Frequency Shifting Property	336
11-9	Table of Fourier Transform Properties and Pairs	337
11-10	Using the Fourier Transform for Multipath Analysis	337
11-11	Summary	341
11-12	Problems	342

12 Filtering, Modulation, and Sampling

346

12-1	Linear Time-Invariant Systems	346
12-1.1	Cascade and Parallel Configurations	347
12-1.2	Ideal Delay	348
12-1.3	Frequency Selective Filters	351
12-1.3.1	Ideal Lowpass Filter	351
12-1.3.2	Other Ideal Frequency Selective Filters	352
12-1.4	Example of Filtering in the Frequency-Domain	353
12-1.5	Compensation for the Effect of an LTI Filter	355
12-2	Sinewave Amplitude Modulation	358
12-2.1	Double-Sideband Amplitude Modulation	358
12-2.2	DSBAM with Transmitted Carrier (DSBAM-TC)	362
12-2.3	Frequency Division Multiplexing	366
12-3	Sampling and Reconstruction	368
12-3.1	The Sampling Theorem and Aliasing	368
12-3.2	Bandlimited Signal Reconstruction	370
12-3.3	Bandlimited Interpolation	372
12-3.4	Ideal C-to-D and D-to-C Converters	373
12-3.5	The Discrete-Time Fourier Transform	375
12-3.6	The Inverse DTFT	376
12-3.7	Discrete-Time Filtering of Continuous-Time Signals	377
12-4	Summary	380

Coverage from Youtube

Modules 18 - 33:

Handouts for ECE 3084

- [syllabus](#)
- [3084 book](#)

Supplemental Material

- [Step and Delta functions](https://math.mit.edu/~stoon/18.031/) (from <https://math.mit.edu/~stoon/18.031/>)
- [Summer 2026 ECE3084 video lecture playlist \(youtube\)](#)
- [ECE3084 Resource Page](#): videos and lecture notes.
- [MIT Class](#): Videos, lecture notes, and problems.

ECE3084 Signals and Systems
53 videos • 2,060 views • Last updated on Aug 15, 2020

These are video lectures from the Summer 2020 offering of ECE3084: Signals and Systems at Georgia Tech, brought to you as a result of Covid-19 forcing Tech to operate in "distance learning" mode.

Lantertronics [SUBSCRIBE](#)

Module	Video Title	Duration	Channel
18	ECE3084 Lecture 15: Intro to Fourier Transforms (Ex: Decaying Exponential) (Signals & Systems, 2020)	12:34	Lantertronics
19	ECE3084 Lecture 16: Revisiting Frequency Response (Signals & Systems, Summer 2020, Georgia Tech)	9:09	Lantertronics
20	ECE3084 Lecture 17: Fourier Transforms of Impulses & Sinusoids (Signals & Systems, 2020, GA Tech)	13:18	Lantertronics
21	ECE3084 Lecture 18: The Fourier Time-Delay Property (Signals & Systems, Summer 2020, Georgia Tech)	7:00	Lantertronics
22	ECE3084 Lecture 19: The Fourier Time-Derivative Property (Signals & Systems, Summer 2020, GA Tech)	7:25	Lantertronics
23	ECE3084 Lecture 20: The Fourier Frequency-Shift Property (Signals & Systems, Summer 2020, GA Tech)	12:00	Lantertronics
24	ECE3084 Lecture 21: Fourier Transforms of Boxcars and Sinc Functions (Signals & Systems, 2020, GT)	18:36	Lantertronics
25	ECE3084 Lecture 22: Fourier Transforms of Periodic Signals (Signals & Systems, Summer 2020, GA Tech)	5:31	Lantertronics
26	ECE3084 Lecture 23: Amplitude Modulation for Communication, UPDATED VERSION (Signals & Systems)	21:02	Lantertronics
27	ECE3084 Lecture 24: Sampling in Time -- Replication in Frequency (Signals & Systems, 2020, GA Tech)	21:25	Lantertronics
28	ECE3084 Lecture 25: Replication in Time -- Sampling in Frequency (Signals & Systems, 2020, GA Tech)	16:23	Lantertronics
29	ECE3084 Lecture 27: Intro to Laplace Transforms (Exponentials & Sinusoids) (Signals & Systems, 2020)	20:10	Lantertronics
30	ECE3084 Lecture 28: The Laplace Time-Delay Property & Transforms of Impulses (Summer 2020, GA Tech)	16:23	Lantertronics
31	ECE3084 Lecture 29: The Laplace Time-Integral Property (Signals & Systems, Summer 2020, GA Tech)	6:22	Lantertronics
32	ECE3084 Lecture 30: Laplace Transforms of Polynomials (Ramps, etc.) (Signals & Systems, Summer 2020)	6:45	Lantertronics
33	ECE3084 Lecture 31: The Laplace "Frequency-Shift" Property (Signals & Systems, Summer 2020, GA Tech)	4:44	Lantertronics

Table 1: Pairs

Impulse	$\delta(t)$	1	✓
Delayed Impulse	$\delta(t - t_0), \quad t_0 \geq 0$	e^{-st_0}	
Step	$u(t)$	$\frac{1}{s}$	✓
Rectangular Pulse	$u(t) - u(t - T), \quad T > 0$	$\frac{1 - e^{-sT}}{s}$	✓
Ramp	$tu(t)$	$\frac{1}{s^2}$	✓
Polynomial	$t^k u(t), \quad k \geq 0$	$\frac{k!}{s^{k+1}}$	✓
Exponential	$e^{-at}u(t)$	$\frac{1}{s + a}$	✓
Polynomial $\times$ Exponential	$t^k e^{-at}u(t)$	$\frac{k!}{(s + a)^{k+1}}$	✓
Cosine	$\cos(\omega_0 t)u(t)$	$\frac{s}{s^2 + \omega_0^2}$	✓
Sine	$\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	✓
Exponential $\times$ Cosine	$e^{-at} \cos(\omega_0 t)u(t)$	$\frac{s + a}{(s + a)^2 + \omega_0^2}$	
Exponential $\times$ Sine	$e^{-at} \sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(s + a)^2 + \omega_0^2}$	

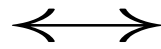
Table 2: Properties

Linearity	$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(s) + \beta X_2(s)$	✓
Right-Shift	$x(t - t_0), \quad t_0 \geq 0$	$e^{-st_0} X(s)$	✓
Time Scaling	$x(at), \quad a > 0$	$\frac{1}{a} X\left(\frac{s}{a}\right)$	
First Derivative	$\dot{x}(t) = \frac{d}{dt}x(t)$	$sX(s) - x(0)$	
Second Derivative	$\ddot{x}(t) = \frac{d^2}{dt^2}x(t)$	$s^2X(s) - sx(0) - \dot{x}(0)$	
Integration	$\int_0^t x(\tau) d\tau$	$\frac{X(s)}{s}$	✓
Modulation	$x(t)e^{at}$	$X(s - a)$	✓
Convolution	$x(t) * h(t)$	$X(s)H(s)$	✓
Final Value Theorem	$\lim_{t \rightarrow \infty} x(t), \quad (\text{if limit exists})$	$\lim_{s \rightarrow 0} sX(s)$	
Initial Value Theorem	$\lim_{t \rightarrow 0} x(t), \quad (\text{if limit exists})$	$\lim_{s \rightarrow \infty} sX(s)$	

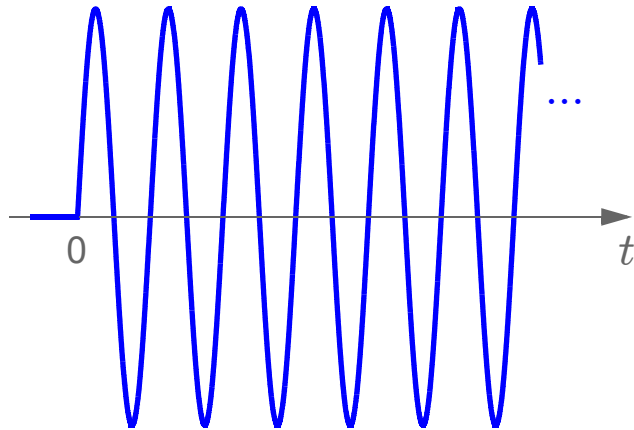
s-domain differentiation $-tx(t) \quad \frac{d}{ds}X(s) \quad \checkmark$

Pop Quiz

$$h(t) = \sin(t)u(t)$$

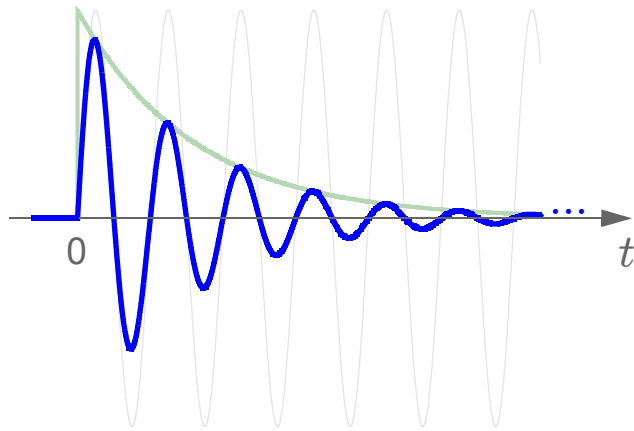


Pole-zero plot?



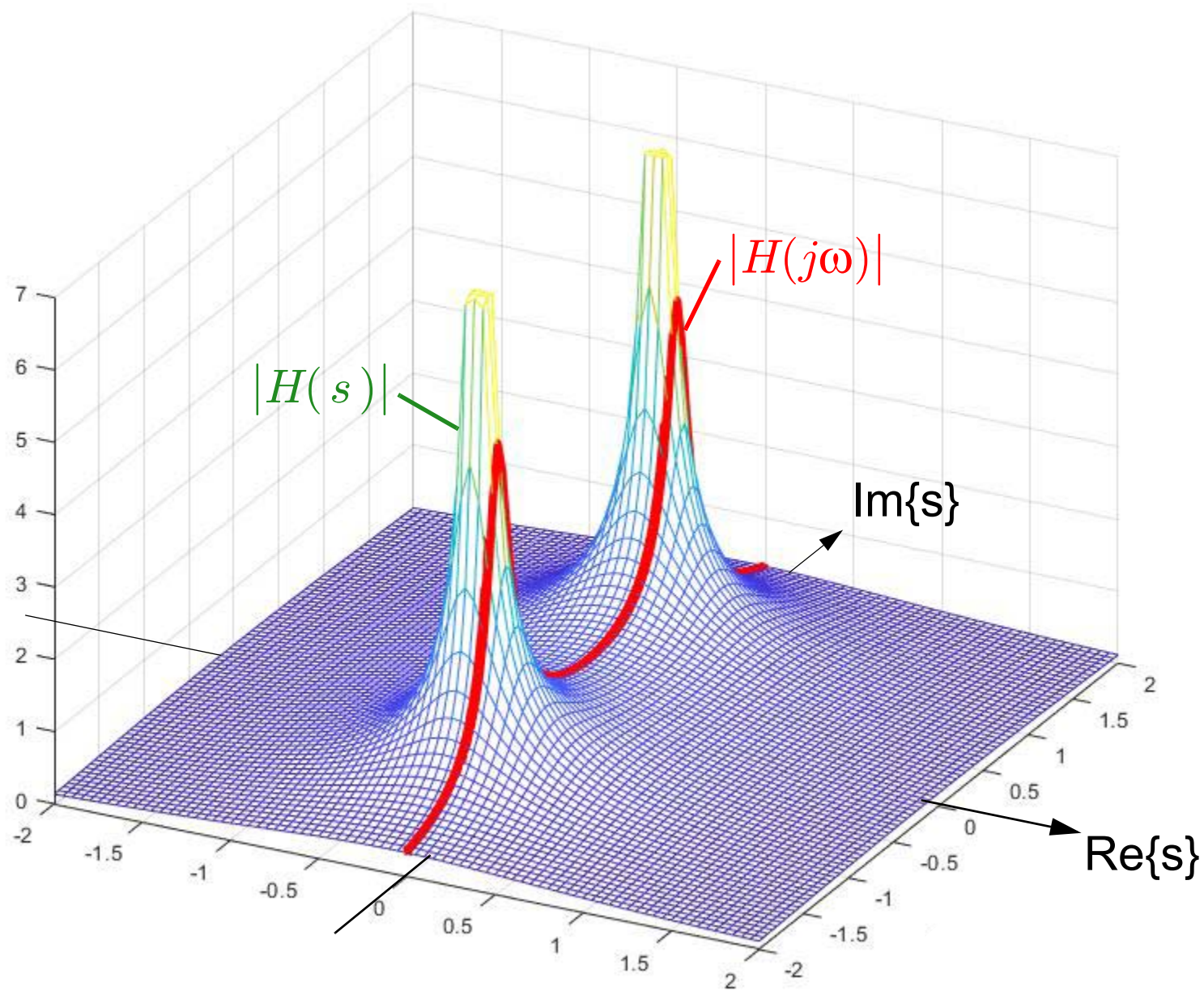
Pop Quiz

$$h(t) = e^{-0.1t} \sin(t) u(t) \quad \longleftrightarrow \quad \text{Pole-zero plot?}$$



$$H(s)?$$

$$\text{Magnitude response } |H(j\omega)|?$$

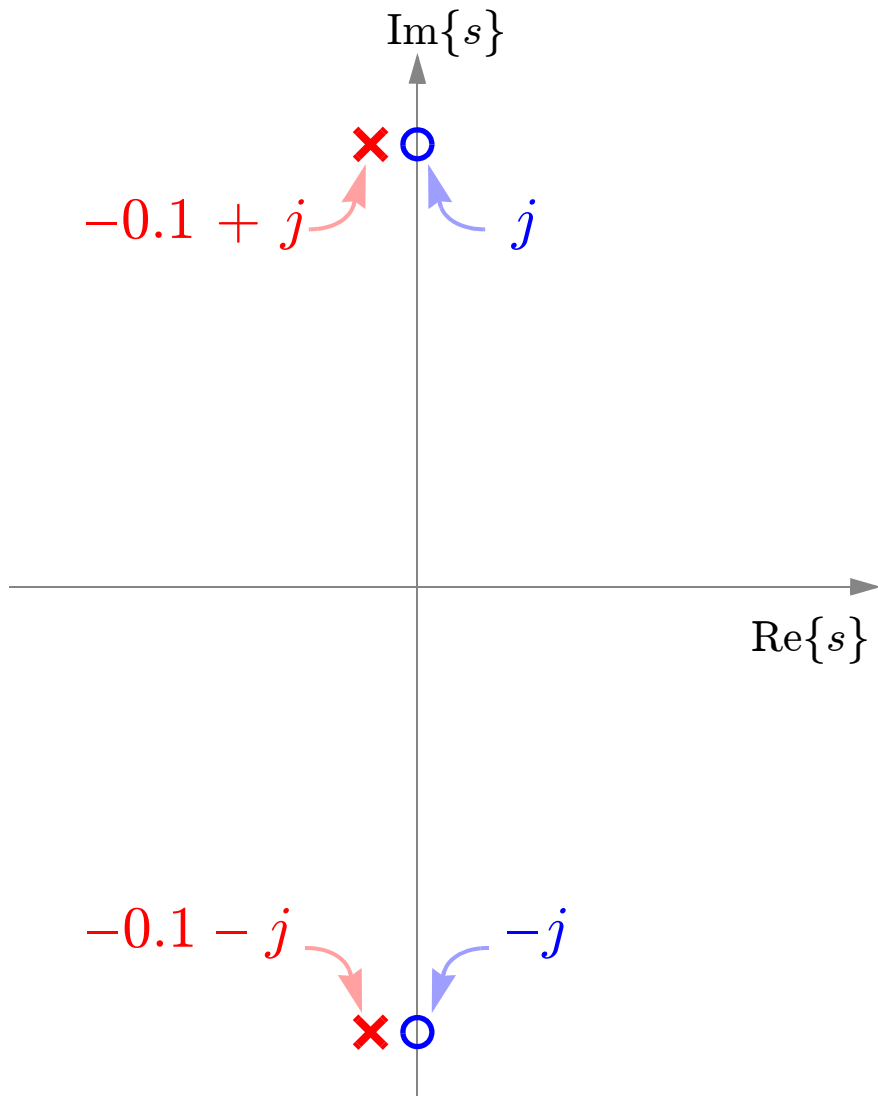


What Kind of Filter is This?

$$H(s) = \frac{s^2 + 1}{s^2 + 0.2s + 1.01}$$

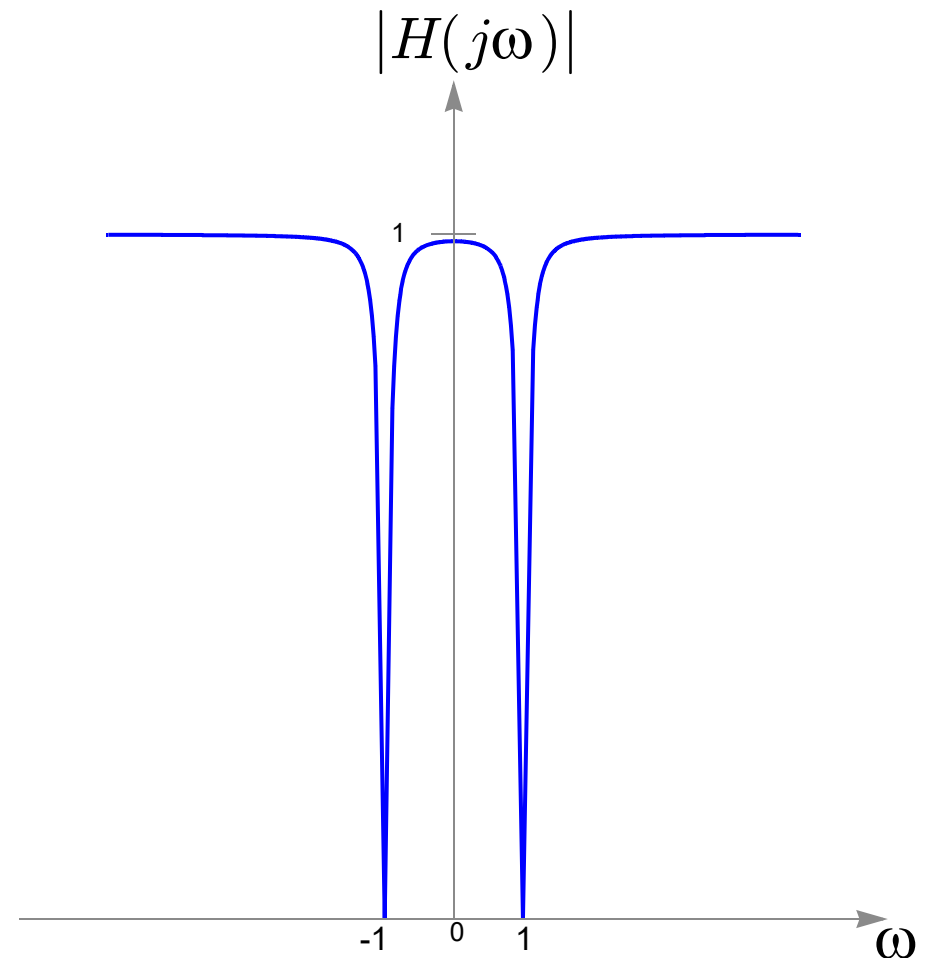
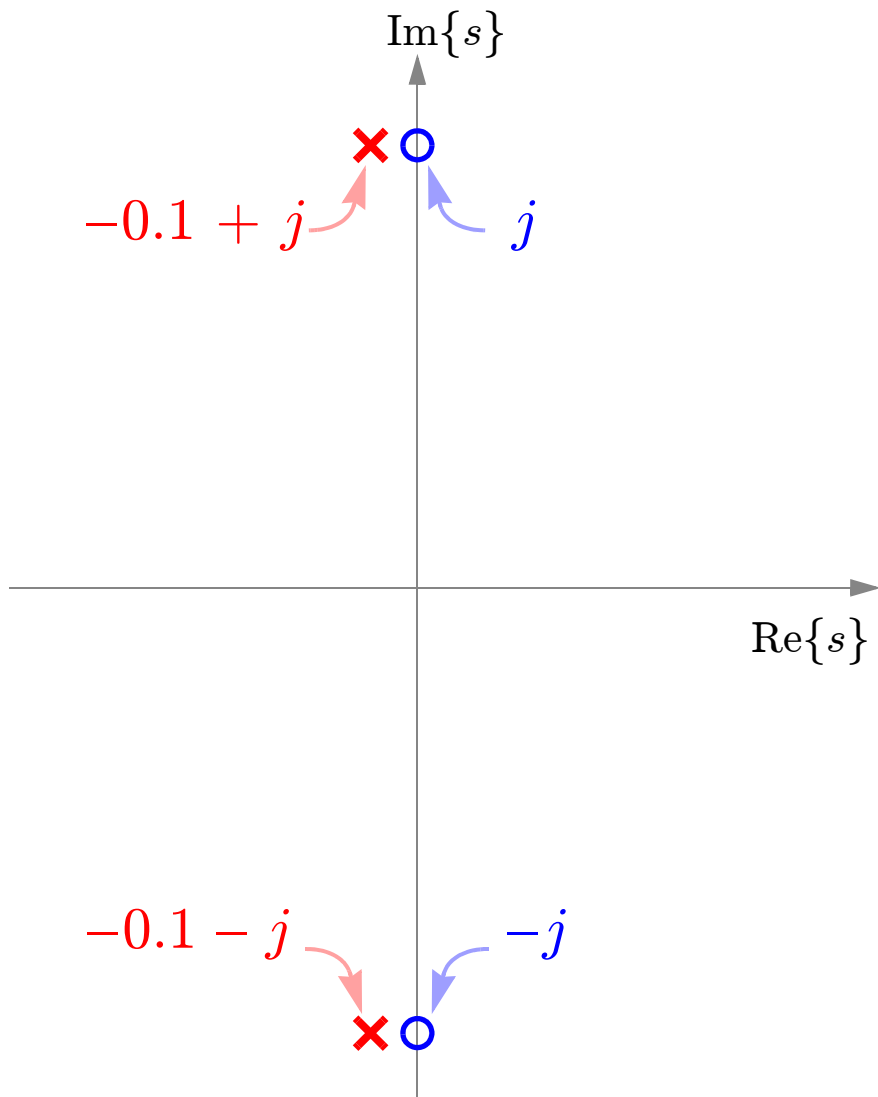
What Kind of Filter is This?

$$H(s) = \frac{s^2 + 1}{s^2 + 0.2s + 1.01}$$

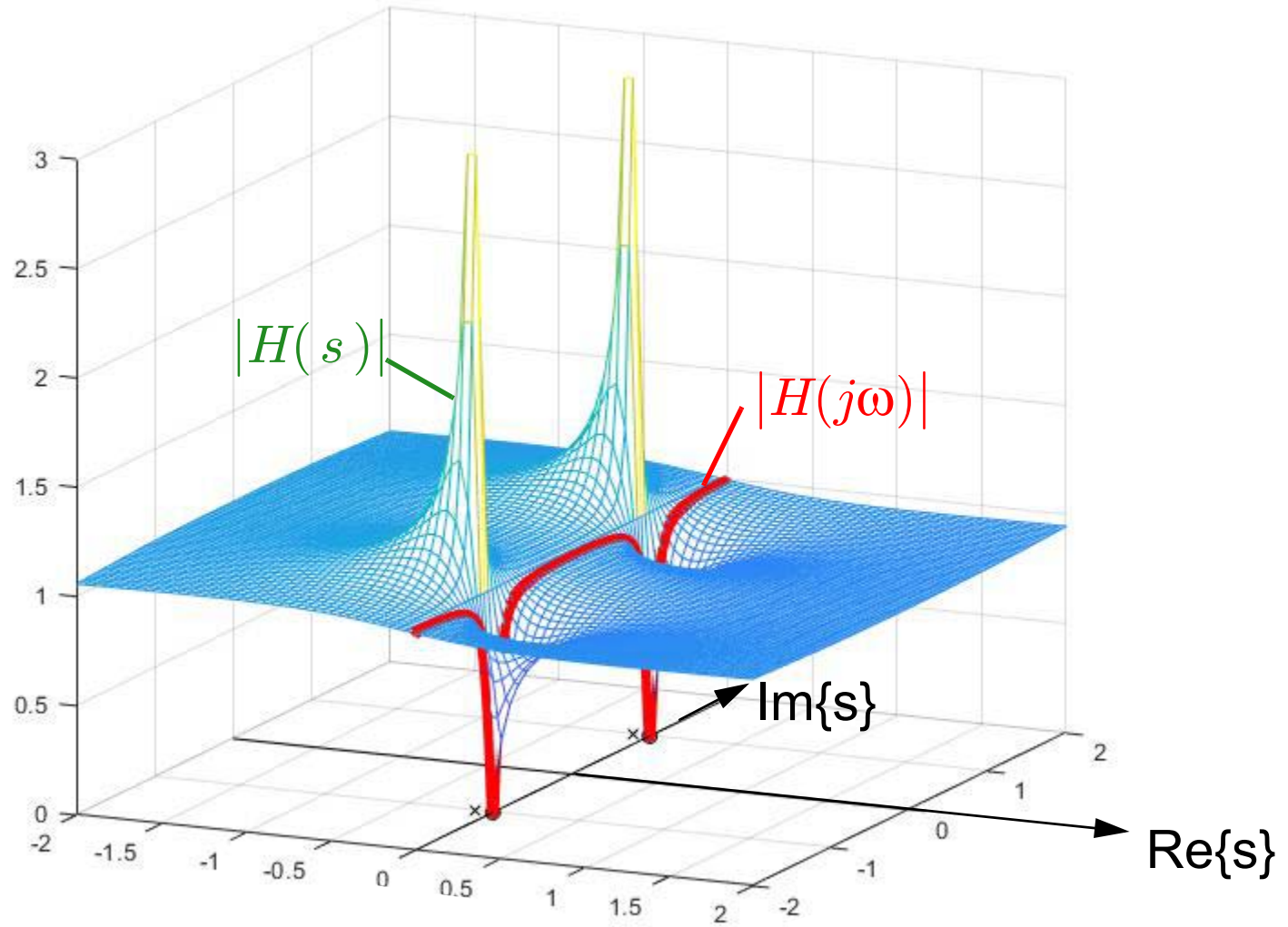


What Kind of Filter is This?

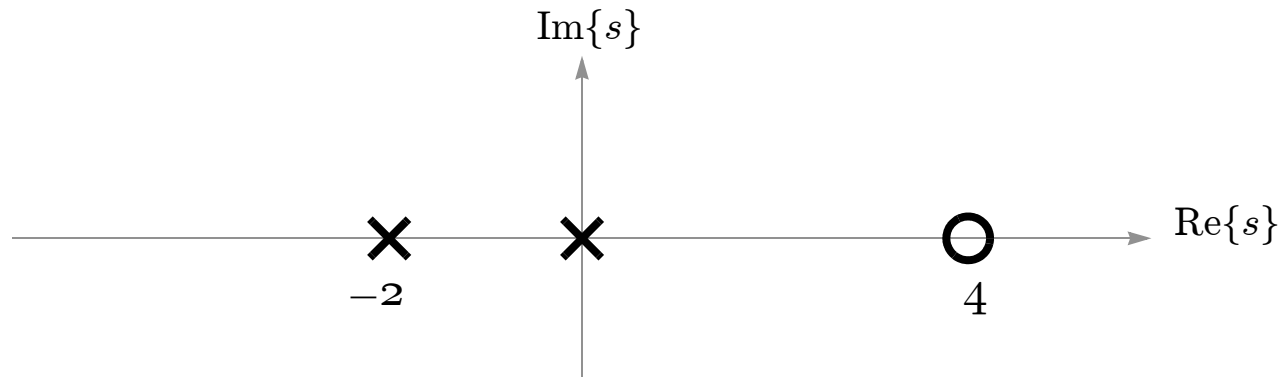
$$H(s) = \frac{s^2 + 1}{s^2 + 0.2s + 1.01}$$



$\Rightarrow$ “notch” filter

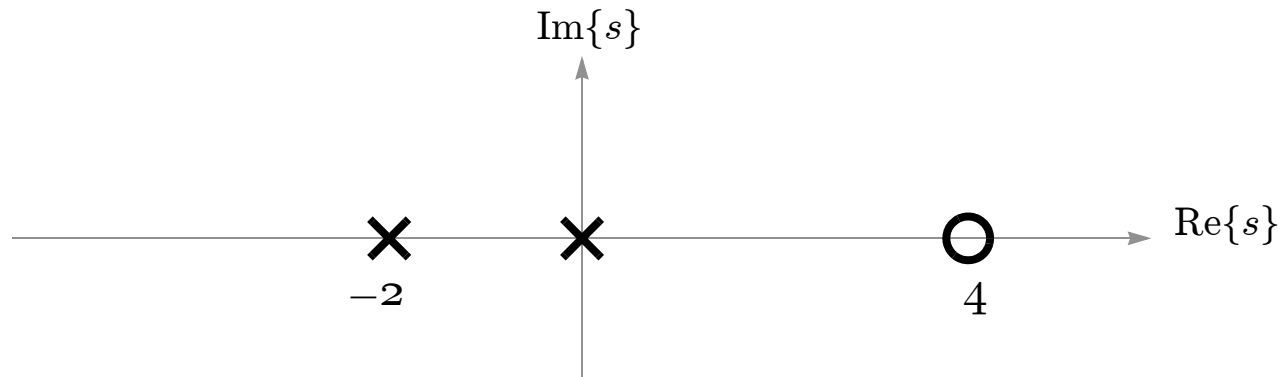


From Pole-Zero Plot to $h(t)$



Find $h(t)$.

From Pole-Zero Plot to $h(t)$



Find $h(t)$.

$$H(s) = \frac{s - 4}{s(s + 2)} \dots \text{How to take inverse Laplace transform?}$$

Inverse Laplace Transform

Contour integral not needed for rational $X(s) = \frac{P(s)}{Q(s)}$;

Instead, use PFE and tables.

Assume $\text{order } P(s) < N = \text{order } Q(s)$.

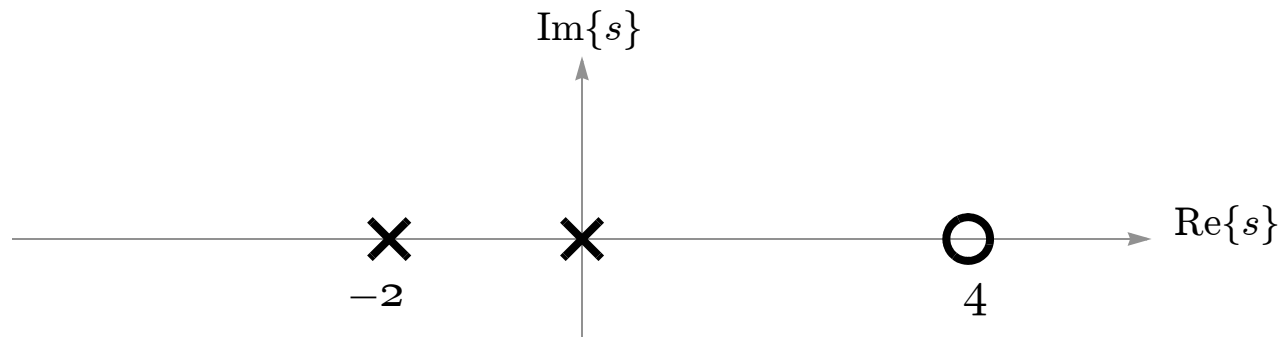
(If not, do polynomial division and apply PFE only to remainder.)

Case 1: All poles distinct:

$$X(s) = \frac{P(s)}{Q(s)} = \frac{A_1}{s - p_1} + \frac{A_2}{s - p_2} + \dots + \frac{A_N}{s - p_N}$$

Use “residue” or “clear denom” strategy to find coefficients.

From Pole-Zero Plot to $h(t)$



Find $h(t)$.

$$H(s) = \frac{s-4}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} \quad \text{PARTIAL FRACTION EXPANSION:}$$

- clear denominator $\Rightarrow s - 4 = A(s + 2) + Bs$
- equate powers of $s^0 \Rightarrow A = -2$
- equate powers of $s^1 \Rightarrow A + B = 1 \Rightarrow B = 3$
- Laplace pairs table $\Rightarrow h(t) = -2u(t) + 3e^{-2t}u(t)$.

```
>> [r, p] = residue([1 -4],[1 2 0])  
  
r =  
  
    3  
   -2  
  
p =  
  
   -2  
    0
```

Another Residue Example

$\text{order}_{\text{num}} < \text{order}_{\text{den}}$, no repeated roots:

$$\begin{aligned} X(s) &= \frac{(s+5)(s+6)}{(s+1)(s+2)(s+3)(s+4)} \\ &= \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3} + \frac{D}{s+4} \end{aligned}$$

where:

$$A = (s+1)X(s)|_{s=-1} = \frac{(s+5)(s+6)}{\cancel{(s+1)}(s+2)(s+3)(s+4)}|_{s=-1} = \frac{20}{6}$$

$$B = (s+2)X(s)|_{s=-2} = \frac{(s+5)(s+6)}{(s+1)\cancel{(s+2)}(s+3)(s+4)}|_{s=-2} = -6$$

$$C = (s+3)X(s)|_{s=-3} = \frac{(s+5)(s+6)}{(s+1)(s+2)\cancel{(s+3)}(s+4)}|_{s=-3} = 3$$

$$D = (s+4)X(s)|_{s=-4} = \frac{(s+5)(s+6)}{(s+1)(s+2)(s+3)\cancel{(s+4)}}|_{s=-4} = -\frac{1}{3}$$

Pop Quiz

Find $x(t)$ when $X(s) = \frac{1}{s^2 + s + 1}$

Pop Quiz

```
>> roots([1 1 1])
```

```
ans =
```

```
-0.5 + 0.8660i
```

```
-0.5 - 0.8660i
```

Find $x(t)$ when $X(s) = \frac{1}{s^2 + s + 1}$

$$= \frac{1}{(s + 0.5 - \frac{\sqrt{3}}{2}j)(s + 0.5 + \frac{\sqrt{3}}{2}j)}$$

$$= \frac{\frac{-j}{\sqrt{3}}}{s + 0.5 - \frac{\sqrt{3}}{2}j} + \frac{\frac{j}{\sqrt{3}}}{s + 0.5 + \frac{\sqrt{3}}{2}j} \quad \text{(PFE)}$$

From table

$$\Rightarrow x(t) = \dots$$

There is an easier way!

There is an Easier Way!

1. Complete the square:

$$\Rightarrow X(s) = \frac{1}{s^2 + s + 1} = \frac{1}{(s + 0.5)^2 + 0.75}.$$

2. Use modulation property.
(Or just use table 2, it's already there!)

Either way, rewrite $X(s) = \frac{1}{(s + 0.5)^2 + 0.75} = \frac{1}{\omega_0} \frac{\omega_0}{(s + 0.5)^2 + \omega_0^2}$

$$\Rightarrow x(t) = \frac{1}{\omega_0} e^{-0.5t} \cos(\omega_0 t) u(t)$$

$$= \frac{2}{\sqrt{3}} e^{-0.5t} \cos\left(\frac{\sqrt{3}}{2} t\right) u(t)$$