

Lecture 2: Thu Aug 20, 2020

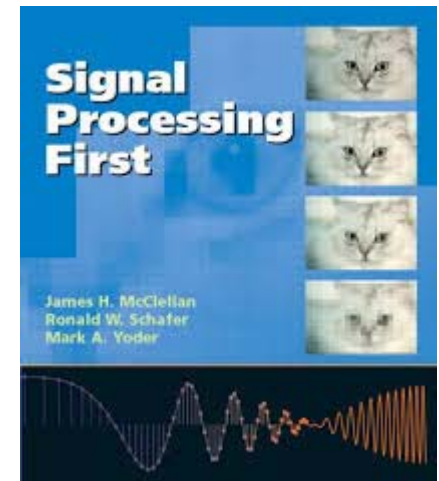
Lecture

- Dirac impulse “function”
- Calculus for $\delta(t)$

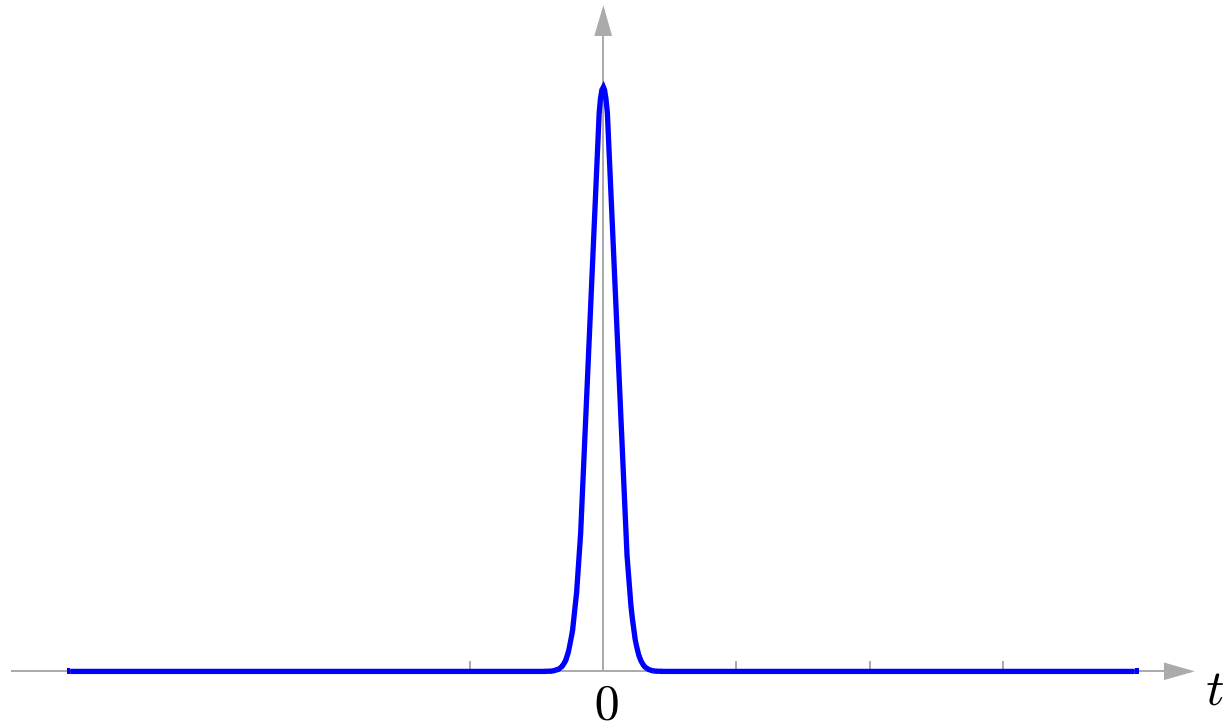
Reading Assignment

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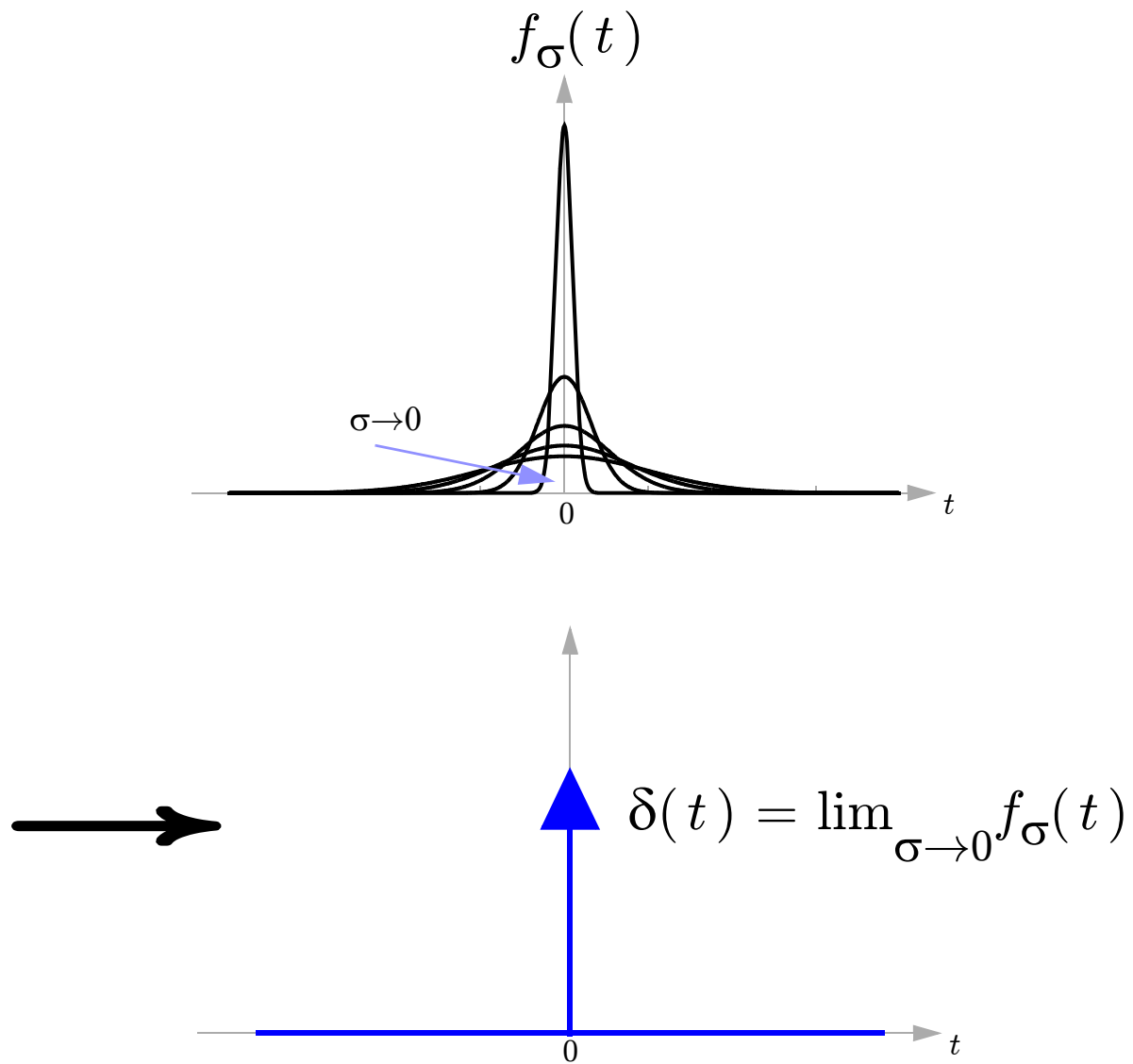
A Short-Duration Signal at Time Zero



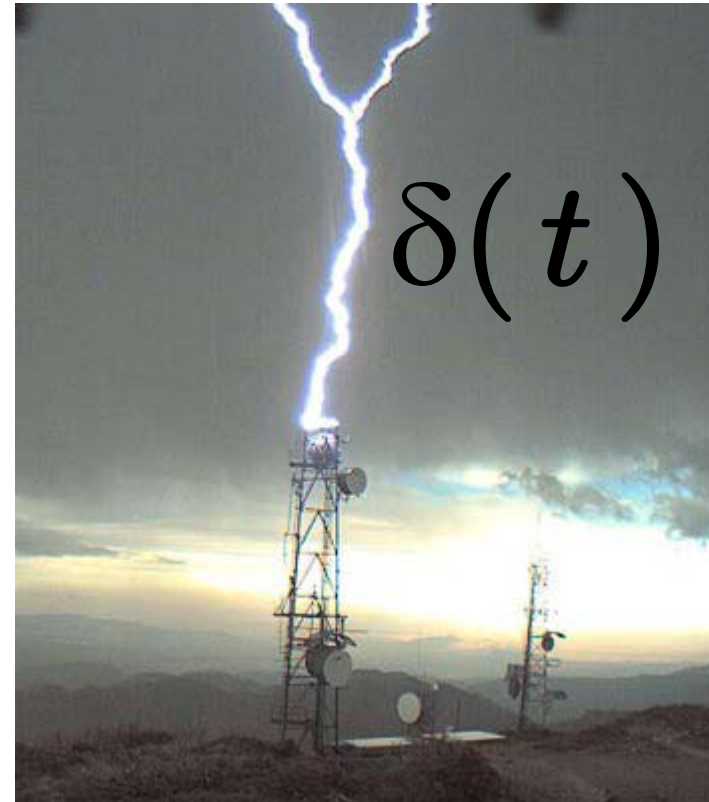
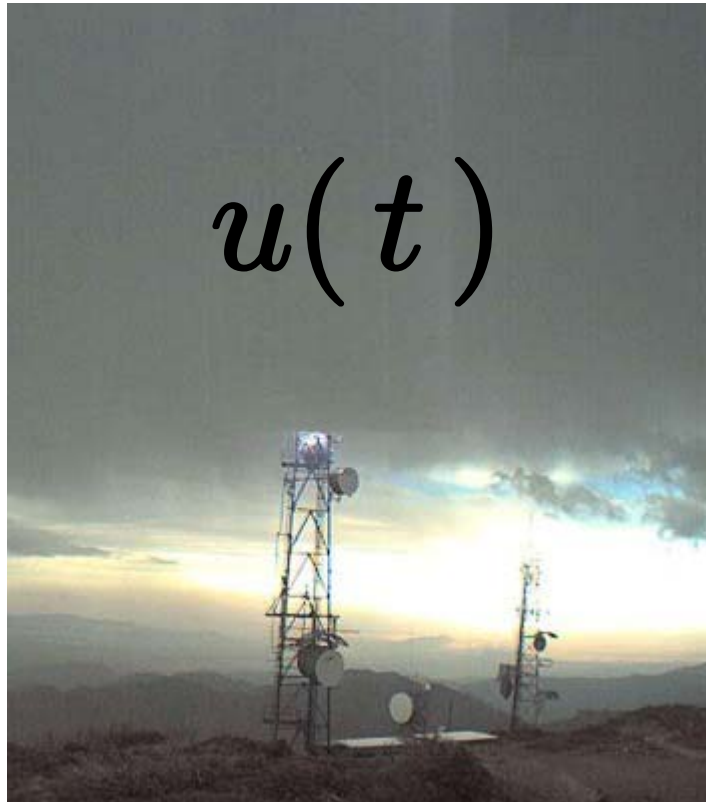
How to model mathematically?

Dirac Impulse “Function”

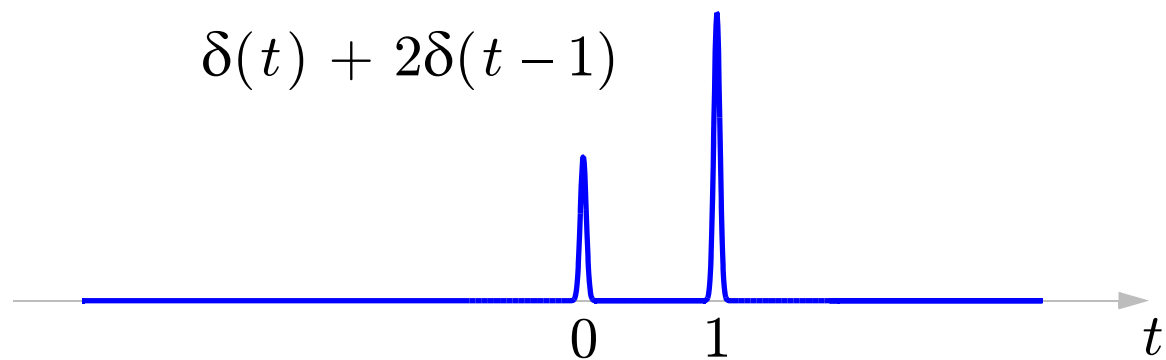
An idealization of an incredibly short pulse that integrates to one:



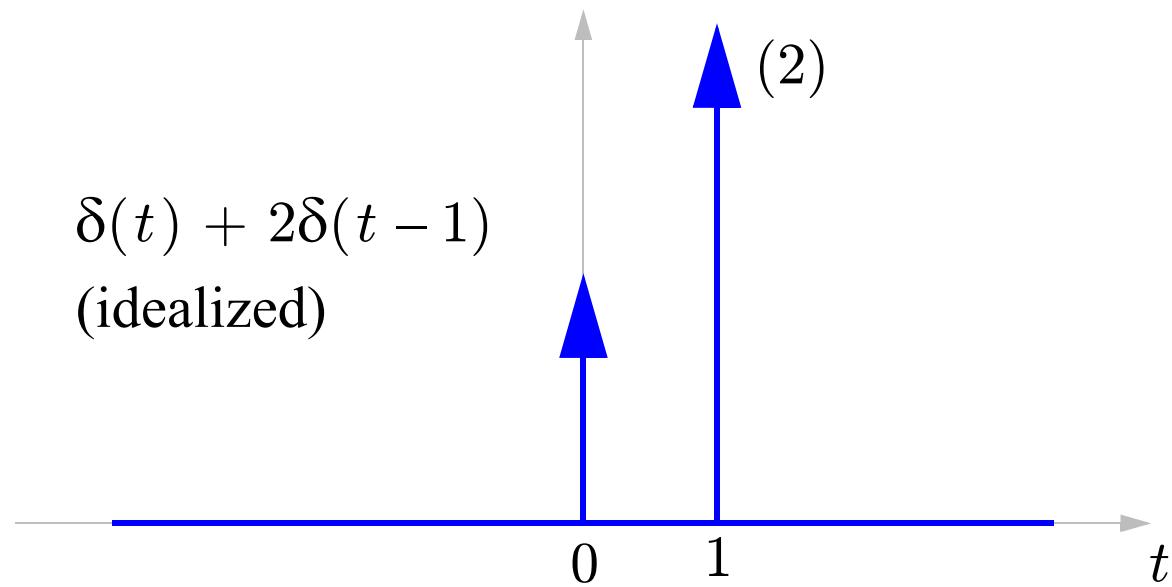
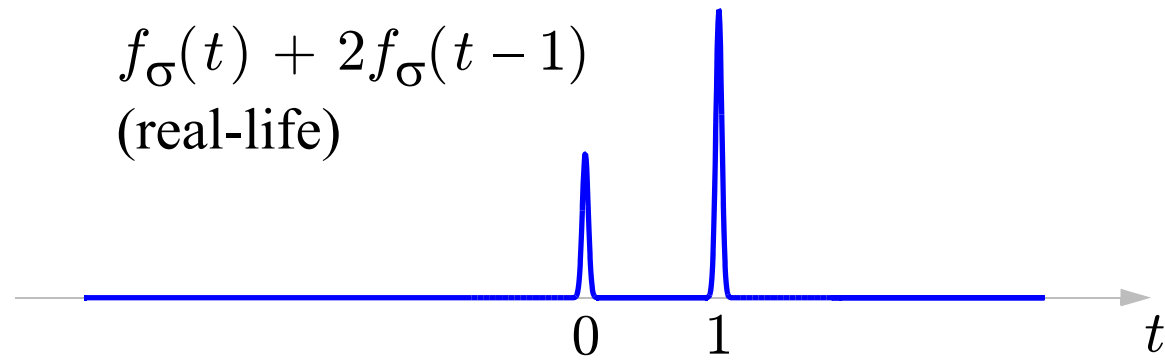
The Difference Between the Unit Step and the Dirac Impulse



How to Plot

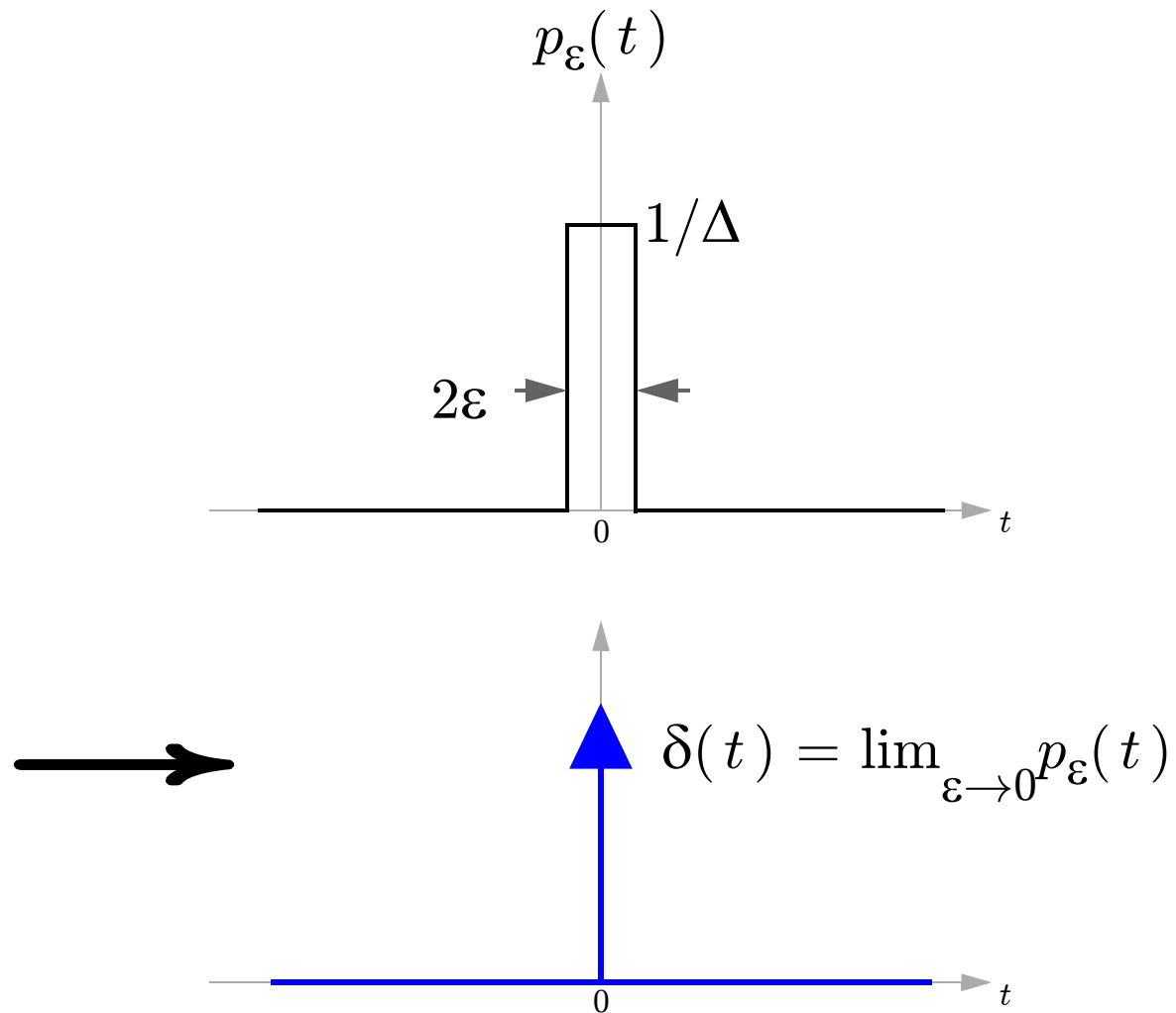


How to Plot



Another Limiting Functions

An idealization of a incredibly short rectangular pulse that integrates to one:



Properties of Dirac Delta “Function”

- zero for all $t \neq 0$, yet integrates to one: $\int_{-\infty}^{\infty} \delta(t) dt = 1$

- Integrates to unit step: $u(t) = \int_{-\infty}^t \delta(\tau) d\tau$

- derivative of unit step: $\delta(t) = \frac{d}{dt} u(t)$

- **Sampling** property:

Multiplying *anything* by a delta function yields a *scaled* delta function:

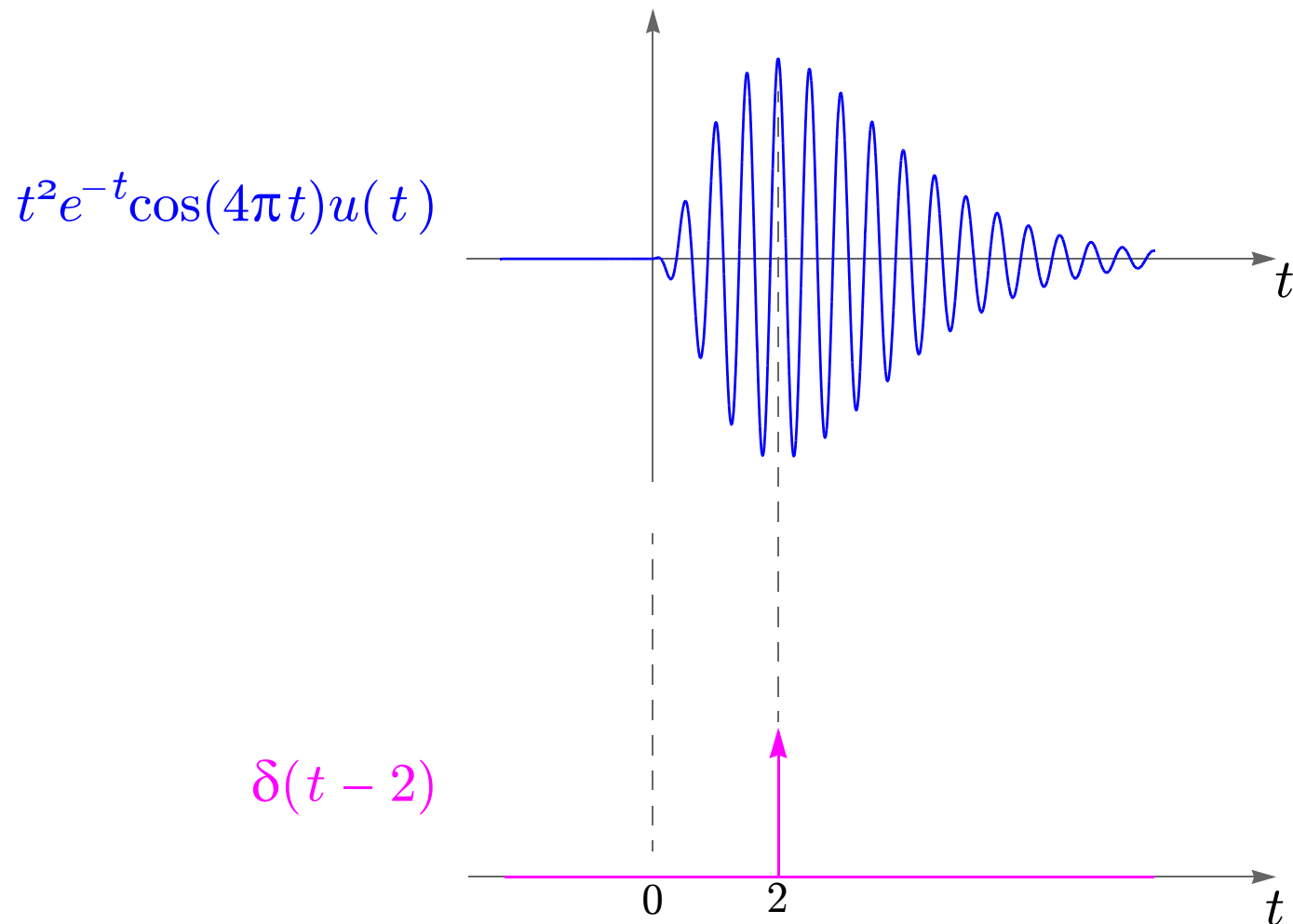
$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$$

- **Sifting** property:

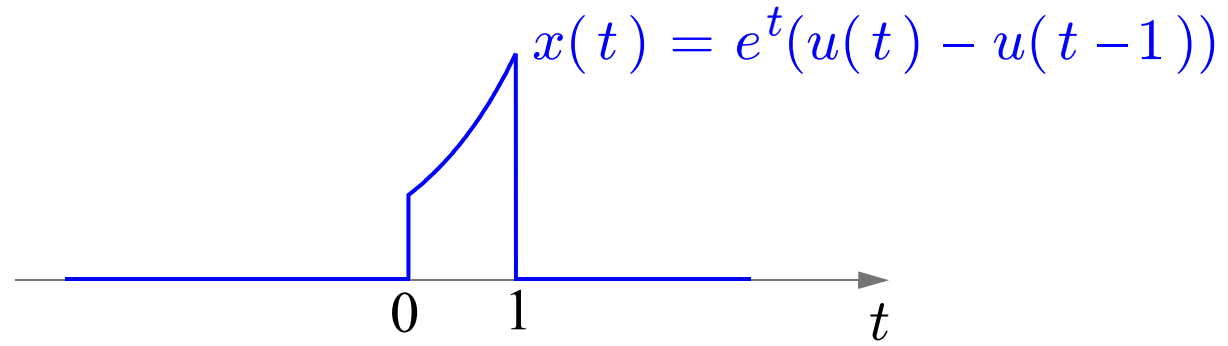
$$\int_{-\infty}^{\infty} x(t)\delta(t - t_0) dt = x(t_0)$$

Pop Quiz 1

Simplify the product $x(t) = t^2 e^{-t} \cos(4\pi t) u(t) \delta(t - 2)$:



Pop Quiz 2



Find derivative $\frac{d}{dt}x(t) =$