Lecture 19: Tue Oct 20, 2020

Reminder:

- Homework 8 due Thursday

Lecture

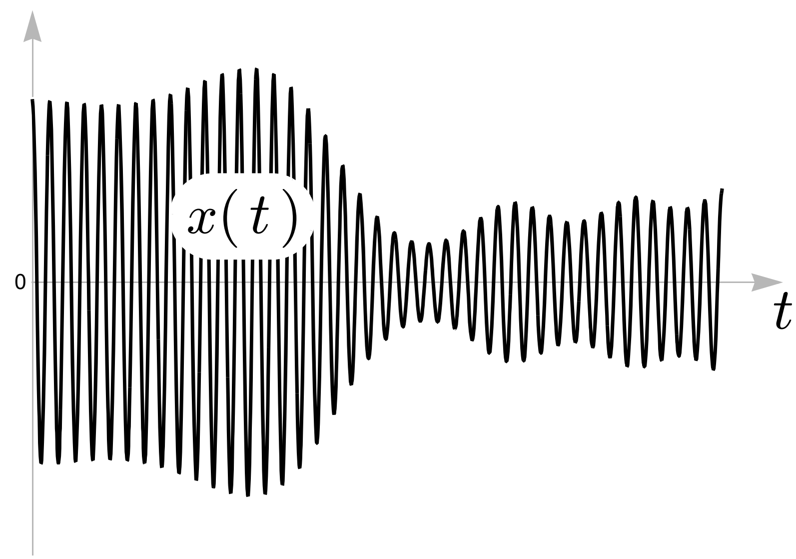
- up and down conversion
- I & Q components
- A frequency-domain view of downconversion
- A frequency-domain view of sampling: introduction

3084 Reading

I & Q

What class of signals can be written like this (the sum of 2 AM signals)?

$$x(t) = \boxed{} \cos(2\pi f_0 t) - \boxed{} \sin(2\pi f_0 t)$$



I & Q

What class of signals can be written like this (the sum of 2 AM signals)?

$$x(t) = \boxed{x_I(t)} \cos(2\pi f_0 t) - \boxed{x_Q(t)} \sin(2\pi f_0 t).$$

Any signal can be written like this, where

$$x_I(t) = \textit{in-phase} \text{ component of } x(t) \qquad \text{... w.r.t. } f_0$$

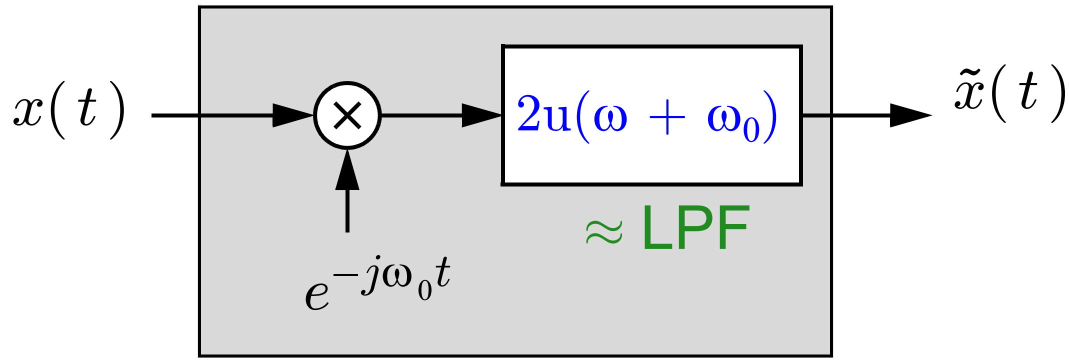
$$x_Q(t) = \textit{quadrature} \text{ component of } x(t) \qquad \text{... w.r.t. } f_0$$

are the real and imaginary parts of

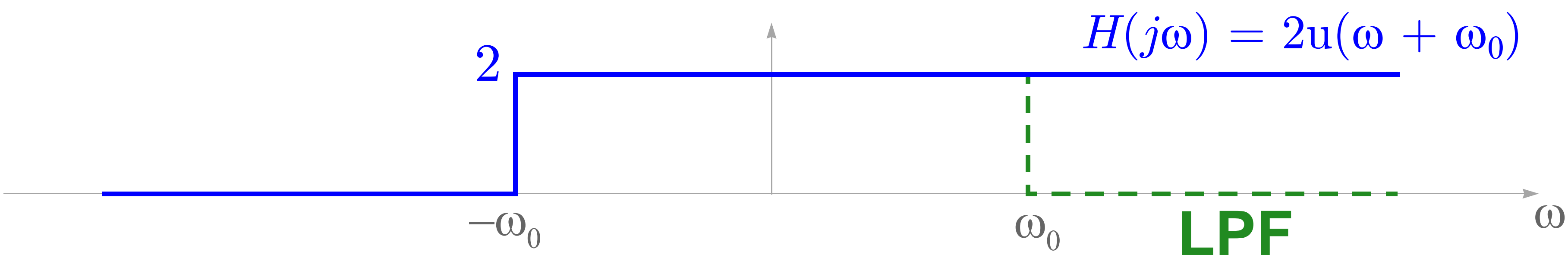
$$\tilde{x}(t) = \textit{complex envelope} \text{ of } x(t) \qquad \text{... w.r.t. } f_0$$

$$= x_I(t) + jx_Q(t).$$

Downconverter



Under mild conditions, we can substitute **LPF** for $2u(\omega + \omega_0)$:

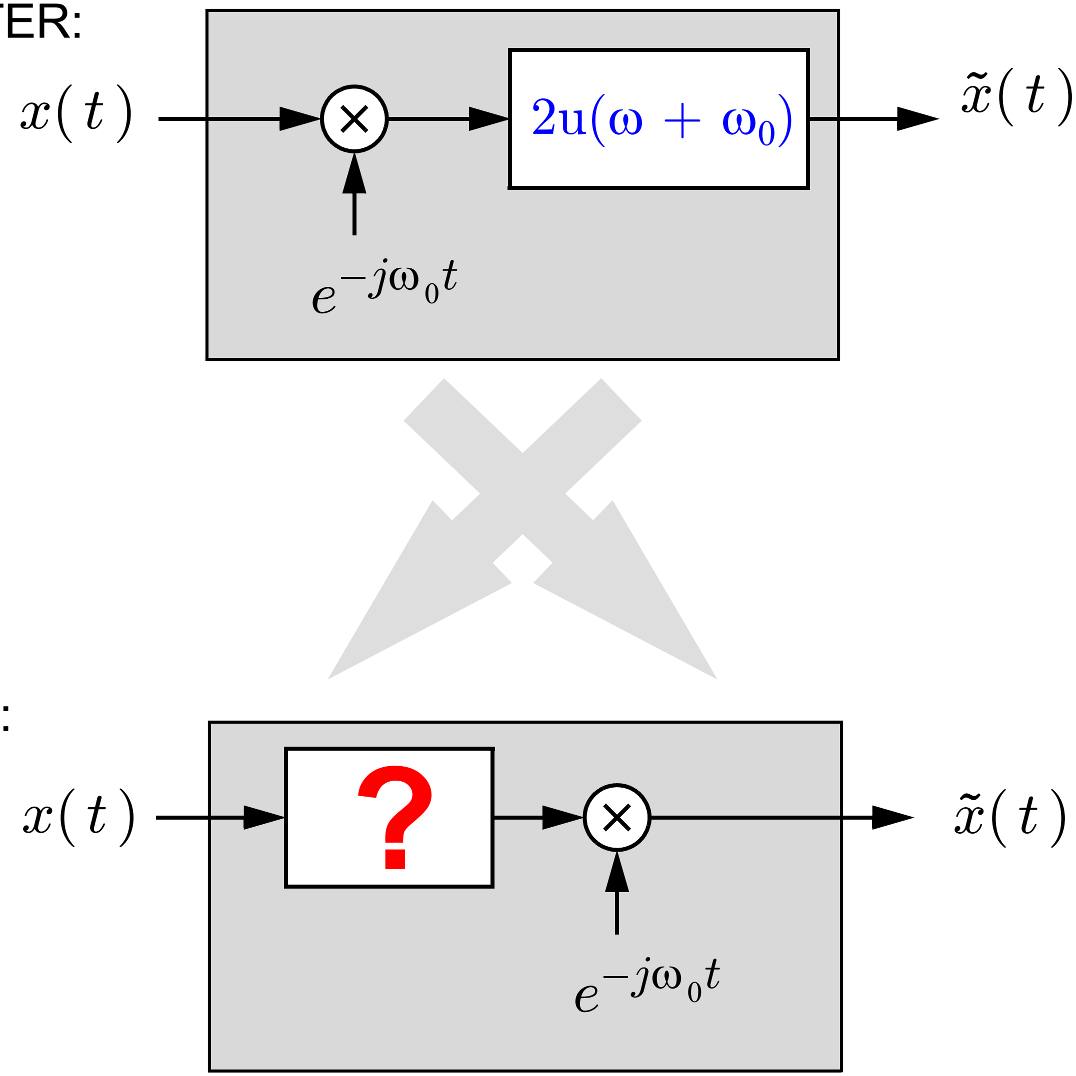


What are these conditions?

$X(f) = 0$ for $f > 2f_0$

Can we filter before frequency shift?

DOWNCONVERTER:



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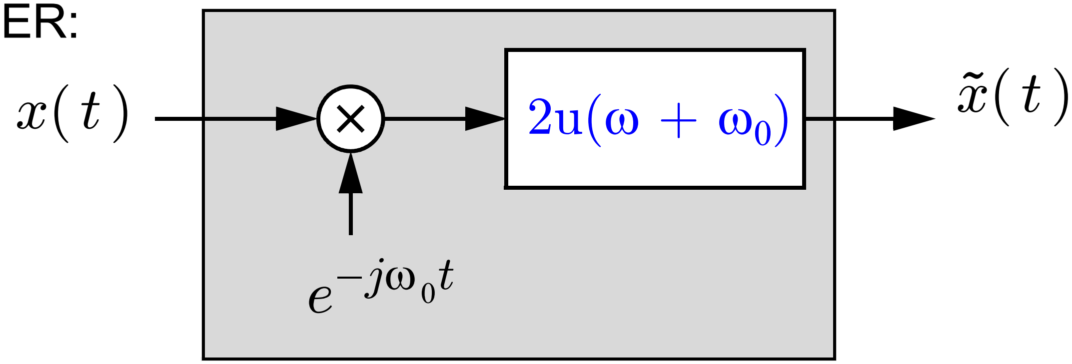
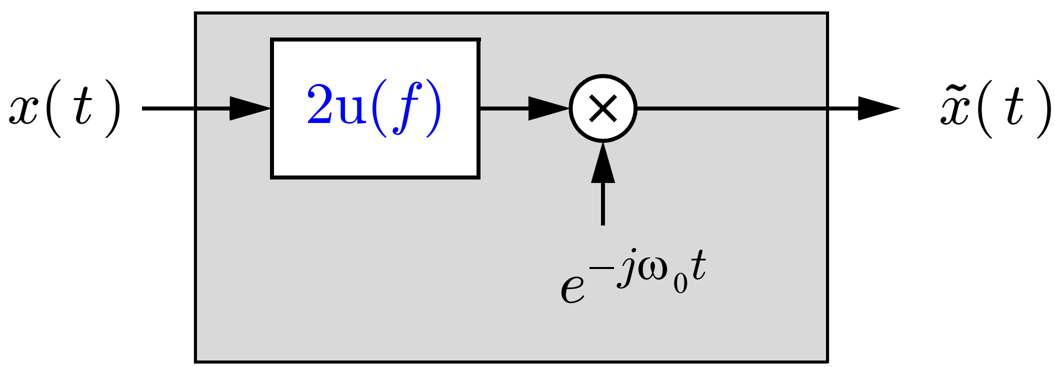
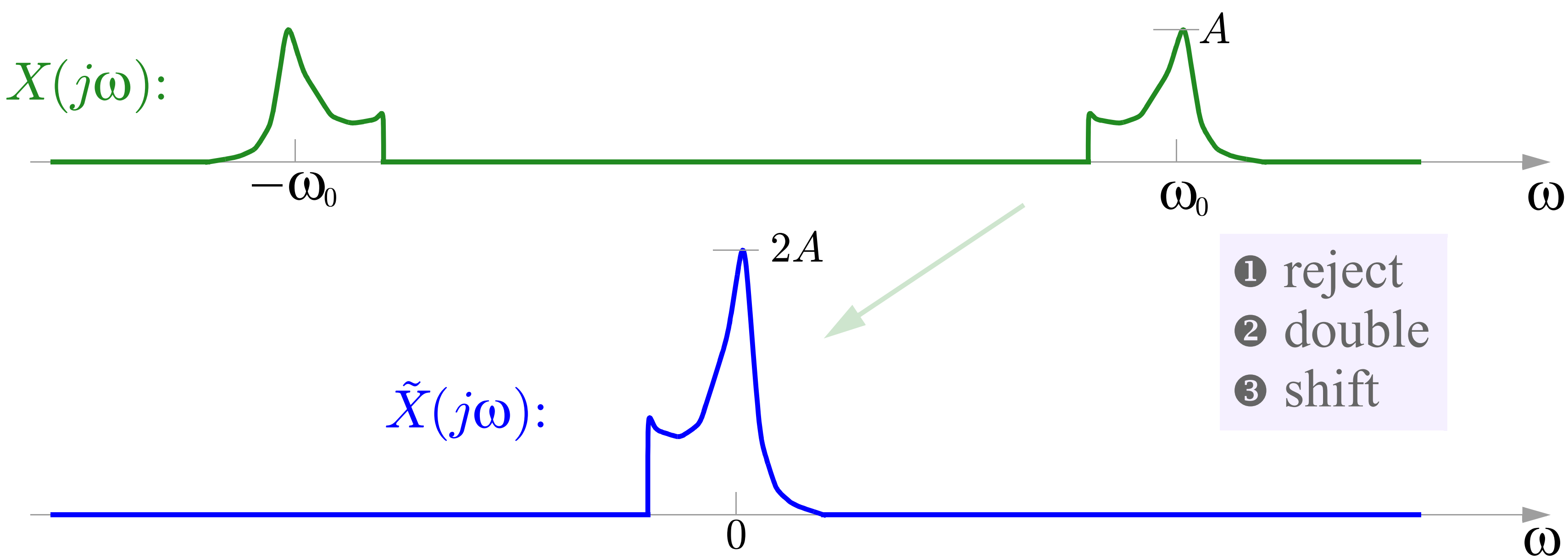
DOWNCONVERTER:



ALTERNATIVE:



What is FT of Complex Envelope?



Key properties of downconverter:

▷ invertible = info lossless. WLOG.

▷ baseband output easier to process, much lower sampling rate

For these reasons: 1st step at receiver is often **DOWNCONVERT**.

The Complex Envelope in 3 Steps

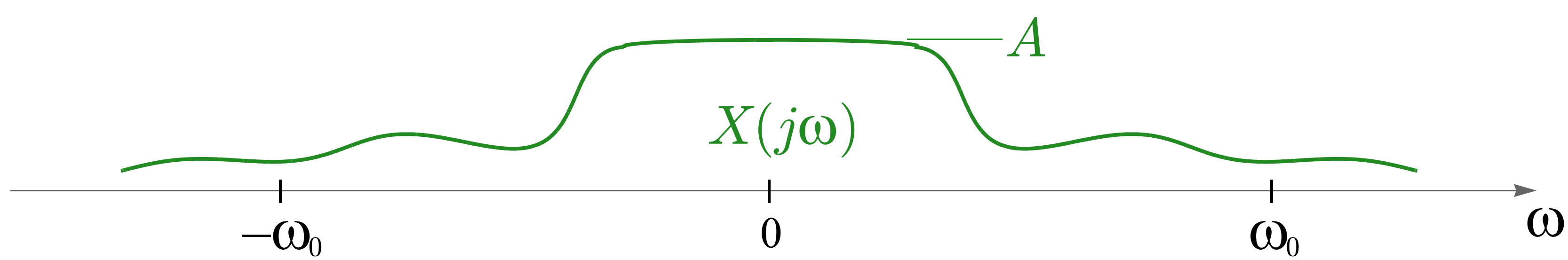
❶ reject

❷ double

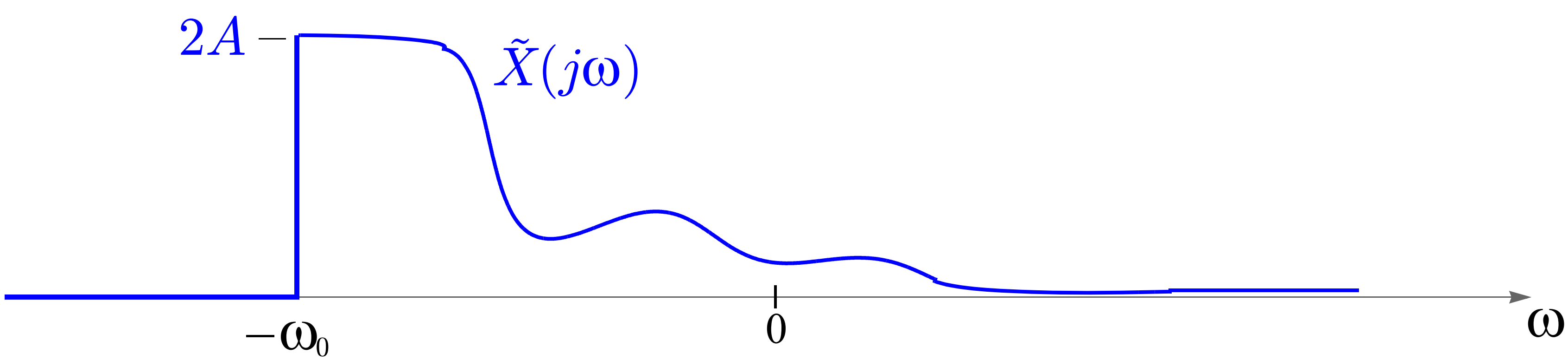
❸ shift

Signal need not be Concentrated at ω_0

In the frequency domain, the complex envelope of this:



Is this:



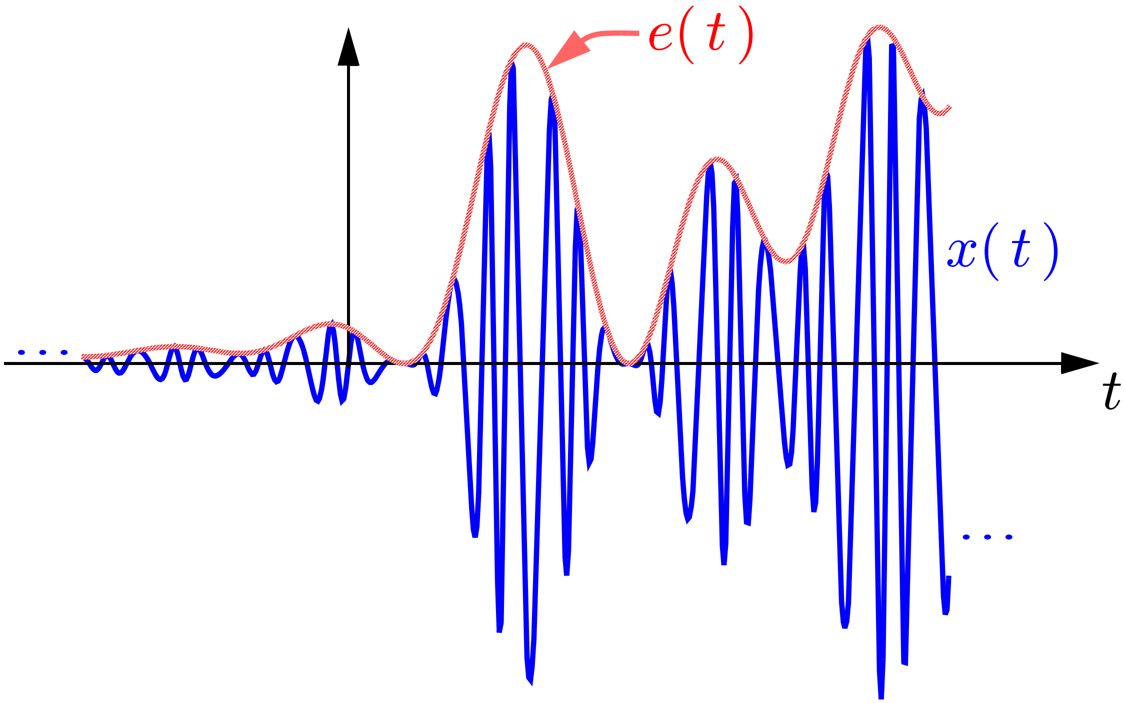
Implications of Complex Envelope

Any real signal $x(t)$ can be written *uniquely* in one of three forms:

C.E. representation: $x(t) = \mathrm{Re}\{\tilde{x}(t)e^{j\omega_0 t}\}$

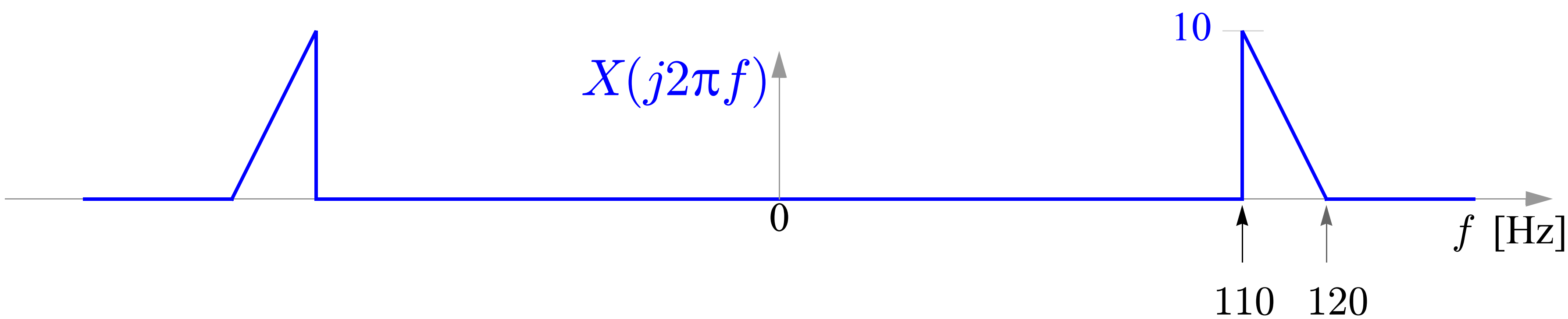
I &Q representation: $x(t) = x_I(t)\cos(\omega_0 t) - x_Q(t)\sin(\omega_0 t)$

E&P representation: $x(t) = e(t)\cos(\omega_0 t + \theta(t))$



Pop Quiz

Consider a signal $x(t)$ whose FT is shown below:



(a) Find its *in-phase* component $x_I(t)$

Pop Quiz

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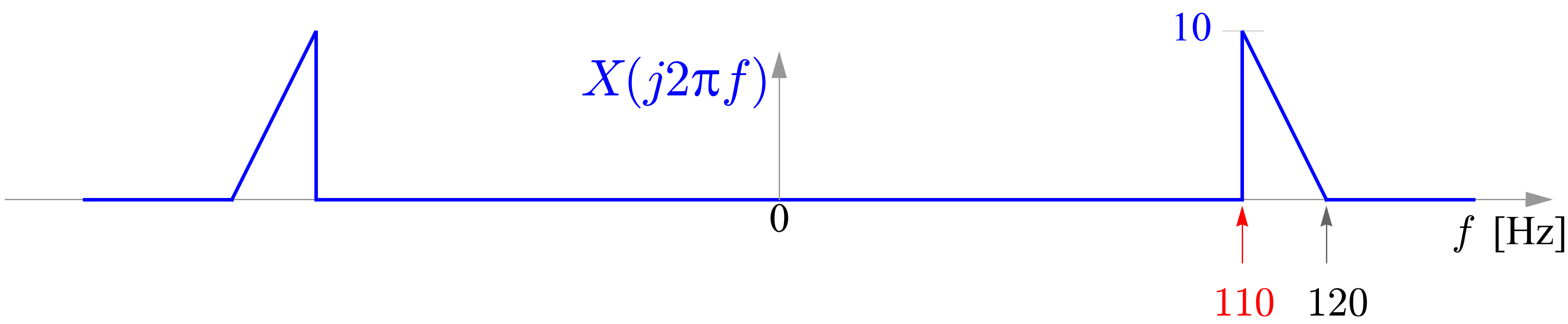

(a) Find its *in-phase* component $x_I(t)$ ... w.r.t. $f_0 = 110$ Hz.

Pop Quiz

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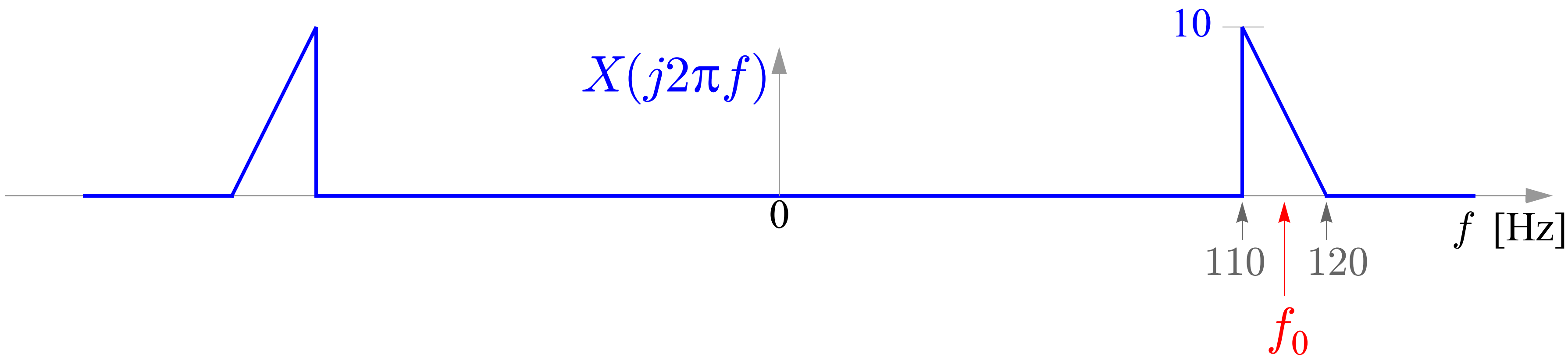
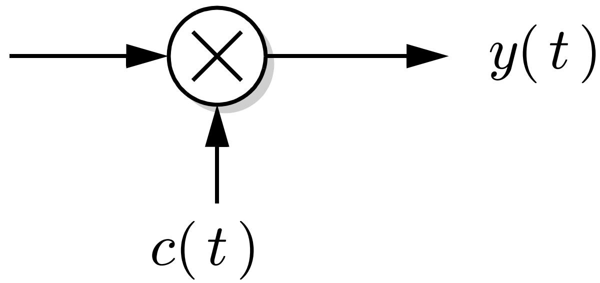


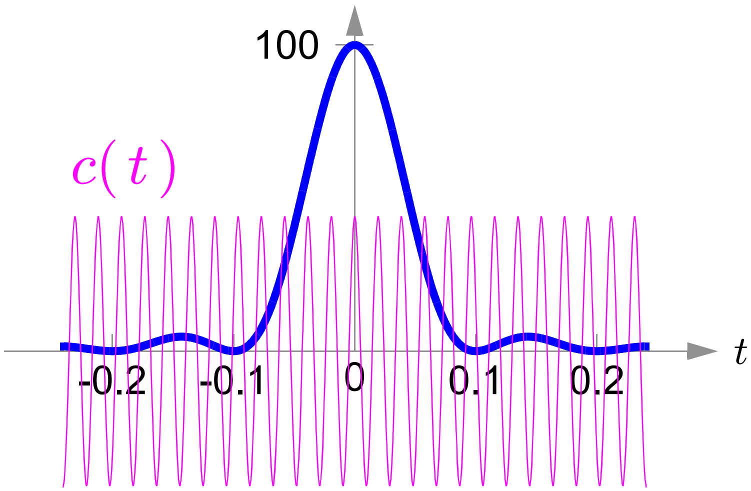
(a) Find its *in-phase* component $x_I(t)$ ... w.r.t. $f_0 = 110$ Hz.

(b) Find its *in-phase* component $x_I(t)$ ... w.r.t. $f_0 = 115$ Hz.

Pop Quiz: Modulation

$$x(t) = \left(\frac{\sin(10\pi t)}{\pi t}\right)^2$$





(a) Find $Y(j\omega)$ when $c(t) = \cos(100\pi t)$.

Pop Quiz: Modulation

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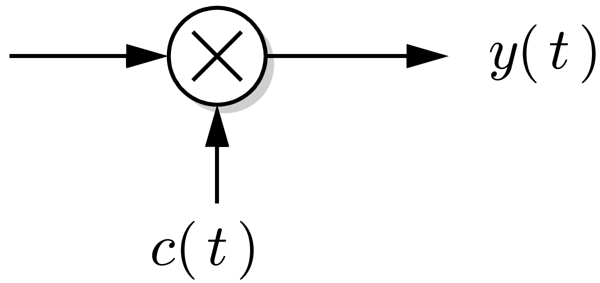
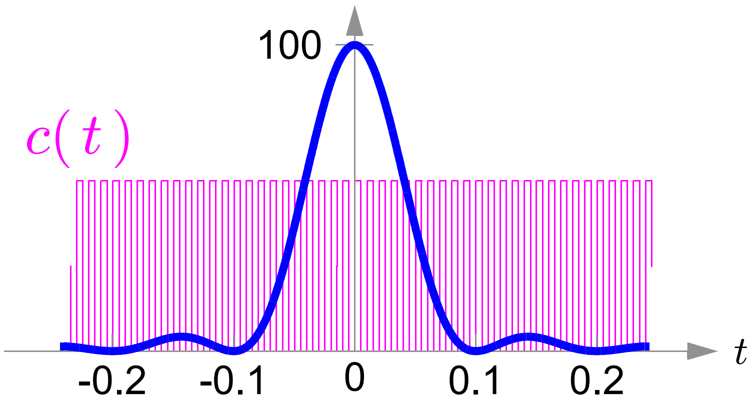
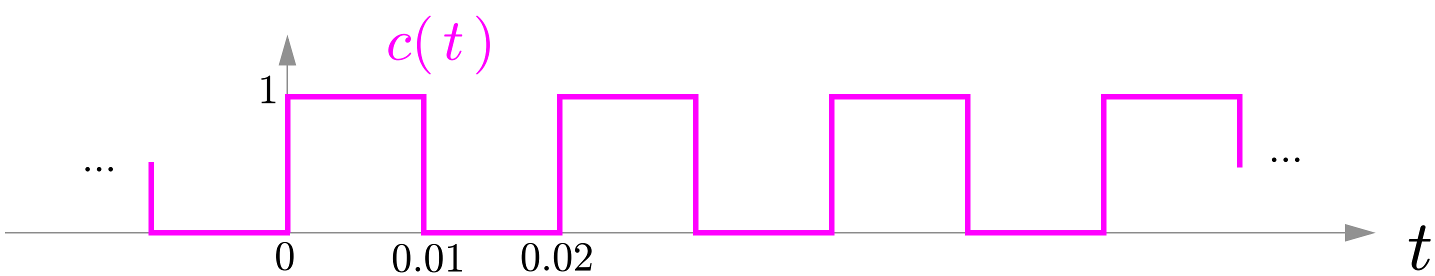


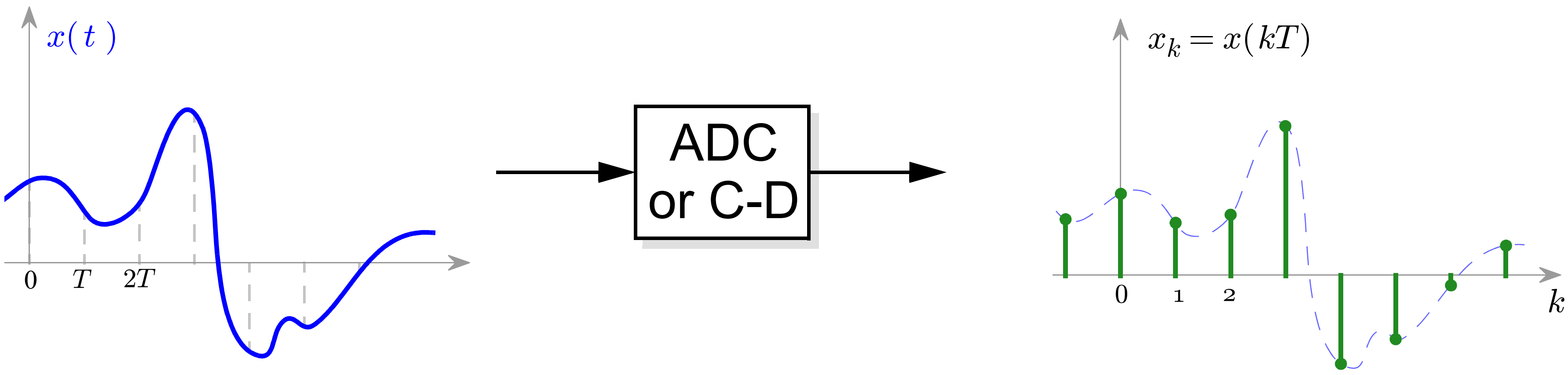


(a) Find $Y(j\omega)$ when $c(t) = \cos(100\pi t)$.

(b) Find $Y(j\omega)$ when $c(t)$ = 50%-duty-cycle square wave:



Impact of Sampling *in the Freq Domain*?



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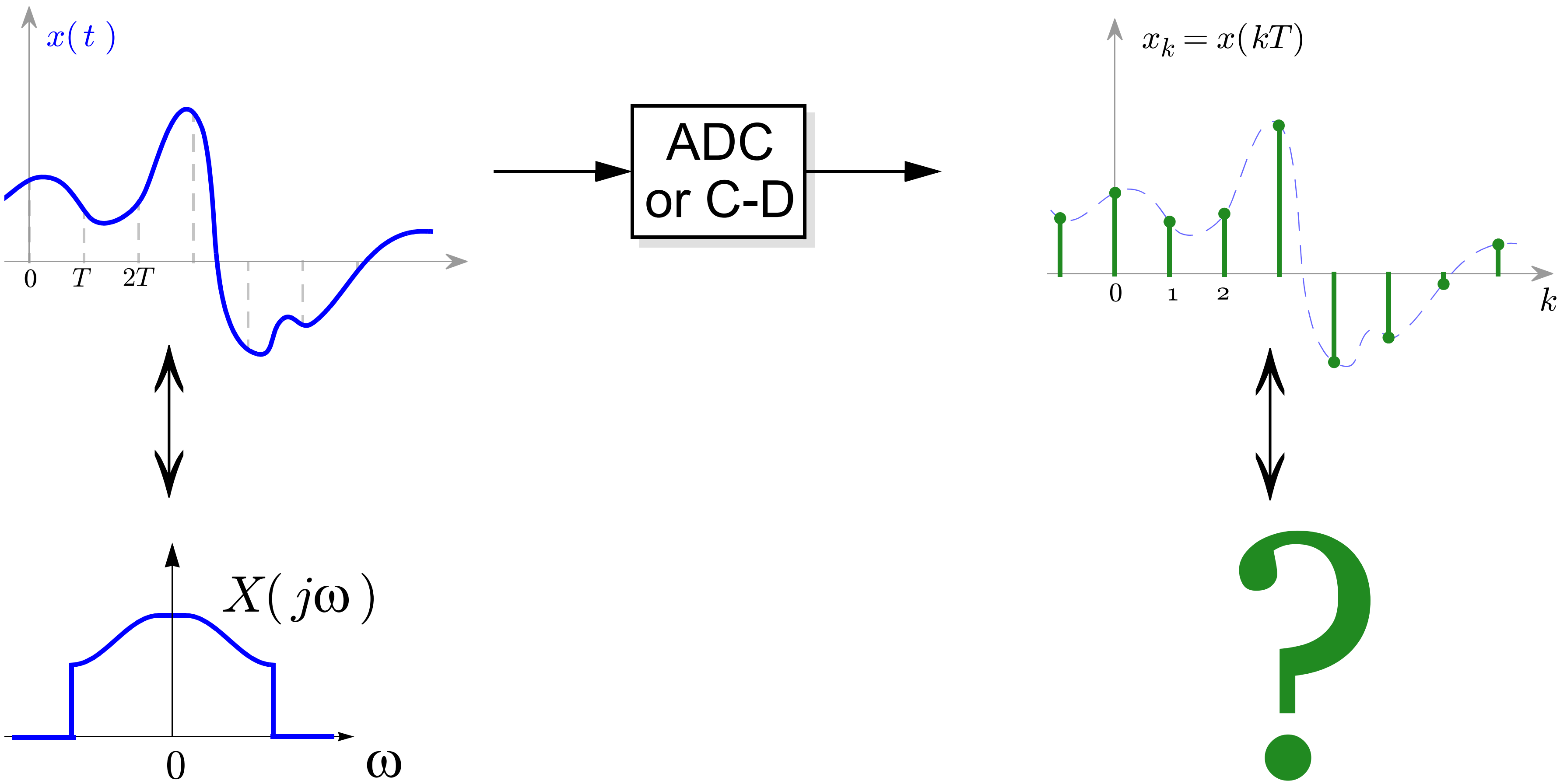
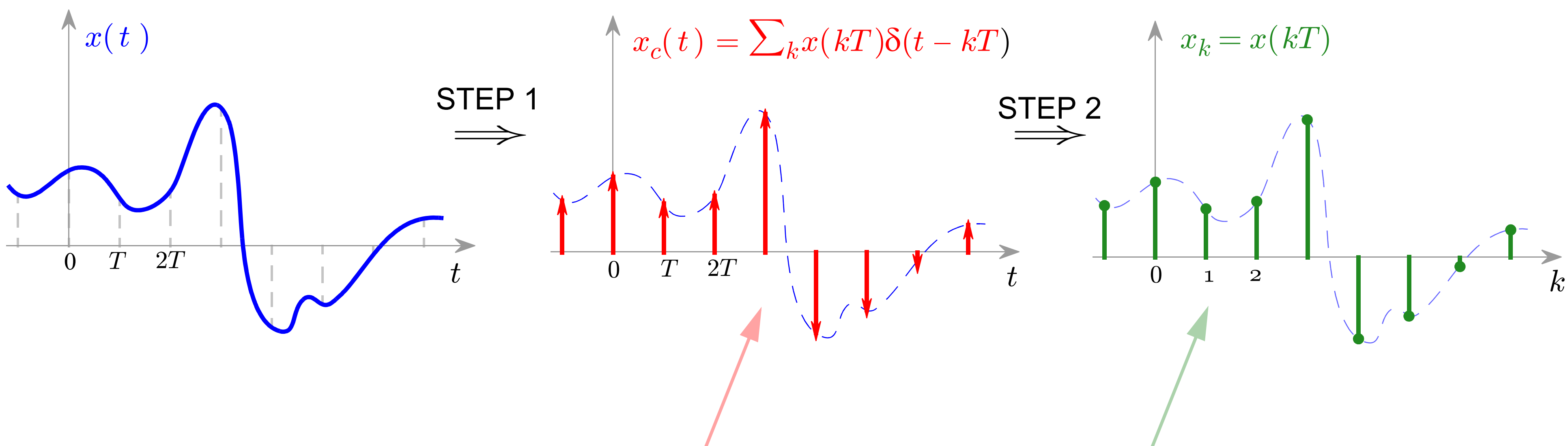


How is the FT *after* sampling related to the FT *before* sampling?

Sampling in 2 Steps

1. Multiply by $p(t) = \sum_k \delta(t - kT)$

2. Convert train of continous-time impulses to train of discrete-time impulses:



Compare the Fourier transform of these two signals.

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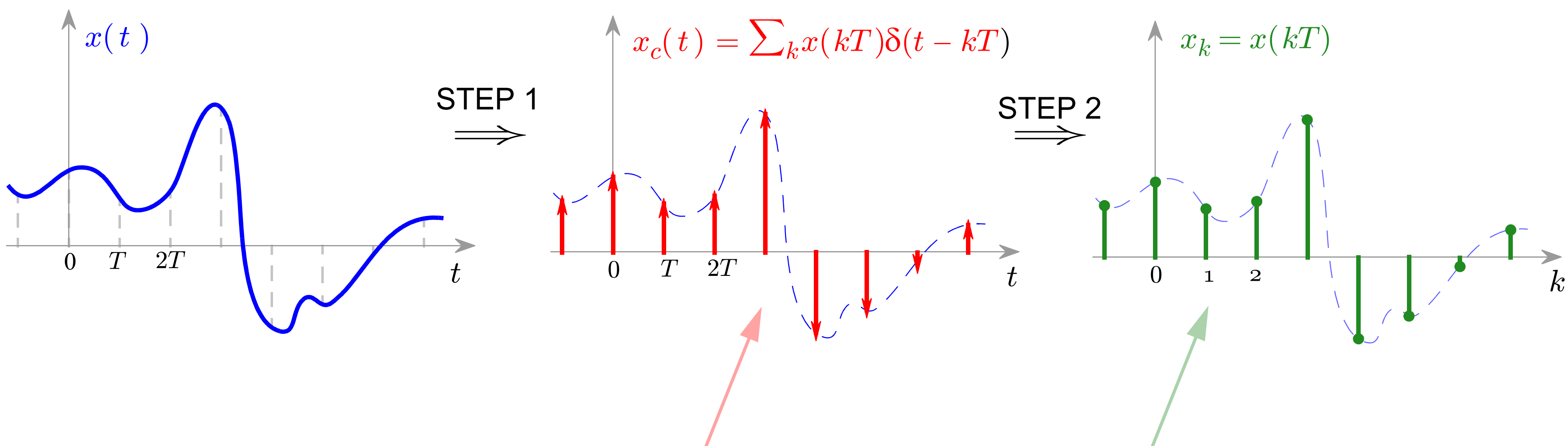

Compare the Fourier transform of these two signals.
They are identical!
Why?