Lecture 13: fpTue Sep 29, 2020

Reminder:

- Quiz 1 is Thursday

Lecture

- the Fourier transform:
  - ▹ more properties (table 2)
  - ▹ using table 1 and table 2

Quiz 1: Thursday

Rules:

- open book/notes
- Google not recommended
- Use Piazza to check for typos
- Otherwise: No communication (electronic or otherwise) with anyone
- See piazza post for logistics

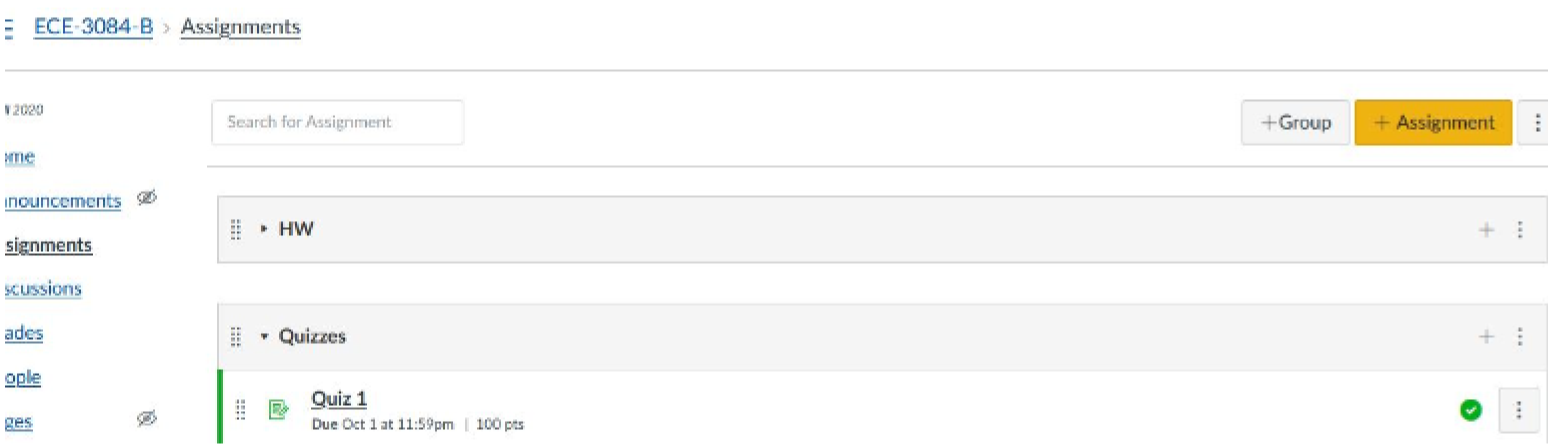
Coverage:

- lectures 1 – 10
- homeworks 1 – 5
- Chapters 1 – 6 of 3084 PDF book
- Chapters 3, 9, 10 of 2026 SP1st (1^{st} Ed)
- Youtube modules 1 – 17

PDF	Date	Topic
lec1	Tue Aug 18	Intro and Motivation; the unit step function
lec2	Thu Aug 20	The Dirac impulse function
lec3	Tue Aug 25	Signal properties: even, odd, energy, power
lec4	Thu Aug 27	Systems and their properties: linear, time-invariance
lec5	Tue Sep 1	LTI systems, impulse response, the convolution integral
lec6	Thu Sep 3	Convolution: properties, examples
lec7	Tue Sep 8	The RADAR problem: crosscorrelation, autocorrelation, the matched filte
lec8	Thu Sep 10	Review RADAR and the MF, intro to frequency response
lec9	Tue Sep 15	Frequency response and Fourier series
lec10	Thu Sep 17	More Fourier series, intro to Fourier transform

Piazza

- A PDF for quiz 1 will be made available to you at 10:00pm EDT on Wednesday, September 30.
- The **submission deadline** is Thursday October 1, at 11:59pm EDT.
- Somewhere in that 26 hour window, you need to set aside roughly 75 minutes to complete the exam, plus whatever time you need to scan and upload.
- Let N be the number of pages in the quiz PDF.
  - If you have a printer: Print it out, work directly on the printout.
  - If you do not have a printer
    - Take out N blank pages and show your work and answers on the corresponding page, in the corresponding space
    - Do not attach extra sheets! Only the first N pages will be graded.
    - For example, if the first page asks only for your name, write only your name on the first page.
    - If the top half of page 3 is part (b) of Problem 2, then limit the top half of your page 3 to Prob. 2(b).
    - If page 5 contains Prob. 4 and nothing else, then what you write on page 5 should only relate to Prob. 4.
    - Please draw answer boxes on each page that align with their positions on the quiz PDF. And place your answers inside.
- Scan the N pages into a single PDF document.
  - Ensure they are scanned in order: the first page of the scanned document should match the first page of the quiz PDF, etc.
  - Before the deadline: upload the PDF to the "Assignments > Quiz 1" page on **canvas**:

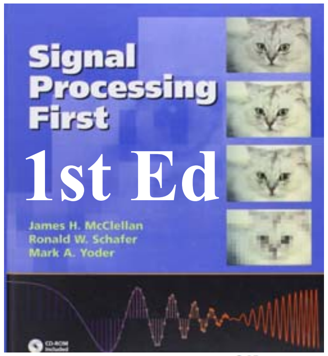


Late Penalty 1% per minute

Coverage from 3084 PDF

Coverage from SP First

9 Continuous-Time Signals and LTI Systems 245



10 Frequency Response 285

3 Spectrum Representation 36

Coverage from Youtube

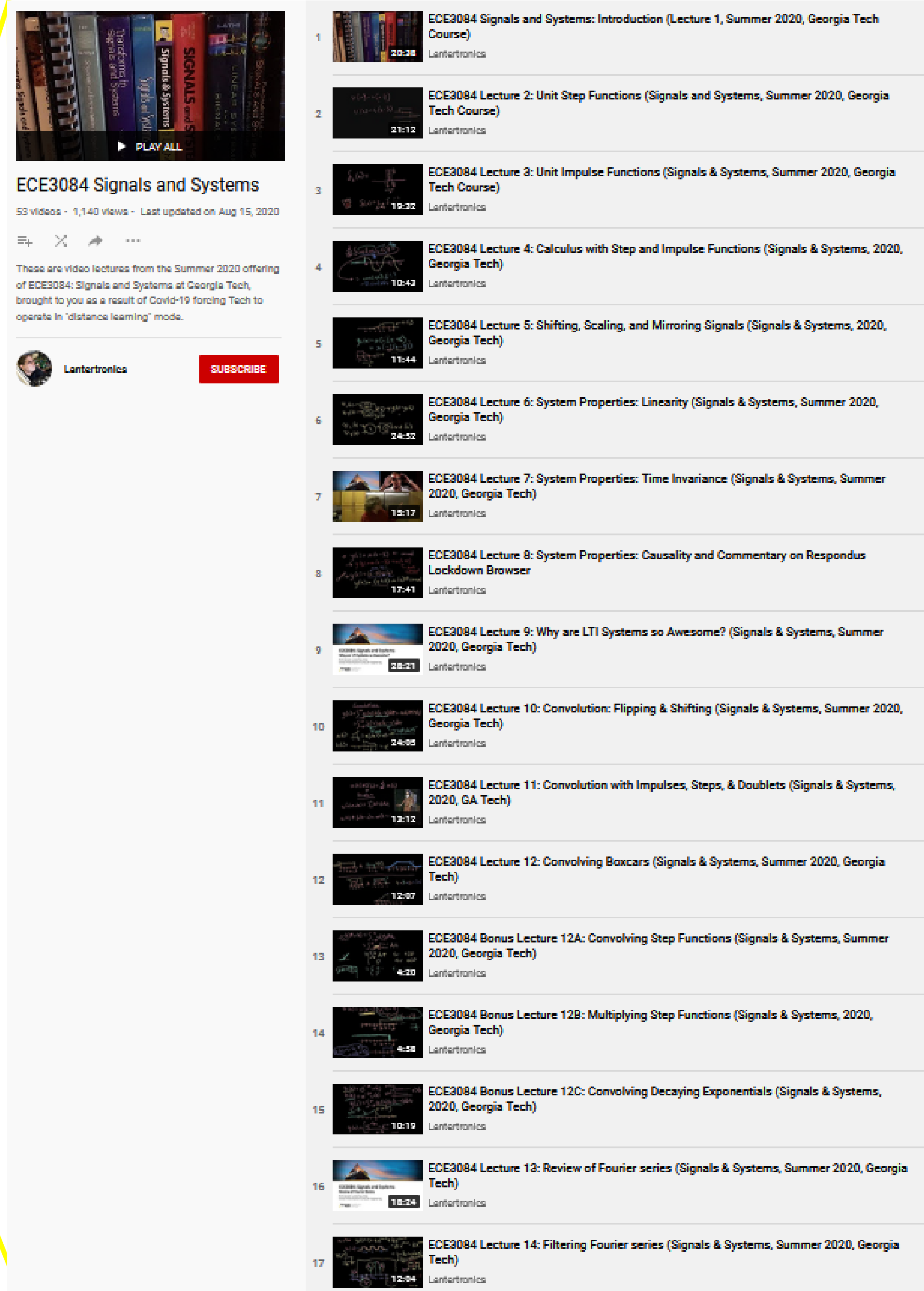
Modules 1 - 17:

Handouts for ECE 3084

- syllabus
- 3084 book

Supplemental Material

- Step and Delta functions (from https://math.mit.edu/~stoopn/18.031/)
- Summer 2026 ECE3084 video lecture playlist (youtube)
- ECE3084 Resource Page: Videos and lecture notes.
- MIT Class: Videos, lecture notes, and problems.



Review Session Today

From piazza:

Exam Review Session

Hey everyone!

Ahead of your exam this week, I'll be holding a review session this Tuesday at 5pm. I'll probably work through one of the practice tests that are posted online but feel free to come with questions. This will be a very casual review.

9/29 @ 5PM @ https://bluejeans.com/1982829984

Best,
Max

edit: Changed time from 4pm to 5pm

Fourier Transform Pair

Forward $$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Inverse $$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

TABLE 1

Table of Fourier Transform Pairs		
	Time-Domain: $x(t)$	*Frequency-Domain:* $X(j\omega)$
Right-sided exponential	$e^{-at}u(t) \quad (a > 0)$	$\frac{1}{a + j\omega}$ ✓
Left-sided exponential	$e^{bt}u(-t) \quad (b > 0)$	$\frac{1}{b - j\omega}$ ✓
Square pulse	$[u(t + T/2) - u(t - T/2)]$	$\frac{\sin(\omega T/2)}{\omega/2}$ ✓
"sinc" function	$\frac{\sin(\omega_0 t)}{\pi t}$	$[u(\omega + \omega_0) - u(\omega - \omega_0)]$ ✓
Impulse	$\delta(t)$	1 ✓
Shifted impulse	$\delta(t - t_0)$	$e^{-j\omega t_0}$ ✓
Complex exponential	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$ ✓
General cosine	$A\cos(\omega_0 t + \phi)$	$\pi A e^{j\phi}\delta(\omega - \omega_0) + \pi A e^{-j\phi}\delta(\omega + \omega_0)$ ✓
Cosine	$\cos(\omega_0 t)$	$\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$ ✓
Sine	$\sin(\omega_0 t)$	$-j\pi\delta(\omega - \omega_0) + j\pi\delta(\omega + \omega_0)$
General periodic signal	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$\sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$ ✓
Impulse train	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k/T)$ ✓

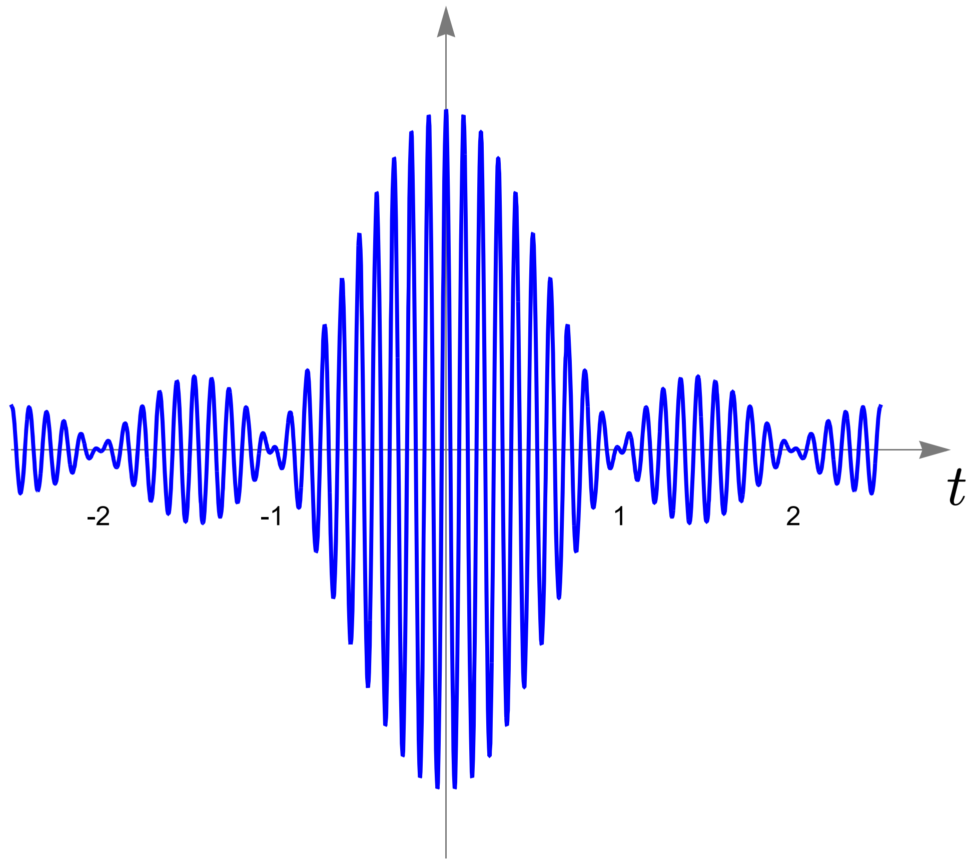
TABLE 2

Table of Fourier Transform Properties

	Time-Domain $x(t)$	*Frequency-Domain* $X(j\omega)$	
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$	✓
Conjugation	$x^*(t)$	$X^*(-j\omega)$	
Time-Reversal	$x(-t)$	$X(-j\omega)$	✓
Scaling	$f(at)$	$\frac{1}{\|a\|}X(j(\omega/a))$	
Delay	$x(t - t_d)$	$e^{-j\omega t_d}X(j\omega)$	✓
Modulation	$x(t)e^{j\omega_0 t}$	$X(j(\omega - \omega_0))$	✓
Modulation	$x(t)\cos(\omega_0 t)$	$\frac{1}{2}X(j(\omega - \omega_0)) + \frac{1}{2}X(j(\omega + \omega_0))$	
Differentiation	$\frac{d^k x(t)}{dt^k}$	$(j\omega)^k X(j\omega)$	
Convolution	$x(t) * h(t)$	$X(j\omega)H(j\omega)$	✓
Multiplication	$x(t)p(t)$	$\frac{1}{2\pi}X(j\omega) * P(j\omega)$	

Motivate Modulation Property

Find frequency response when $h(t) = \cos(20\pi t)\dfrac{\sin \pi t}{\pi t}$:



More FT Properties

- time stretch: $y(t) = x(at) \quad \longleftrightarrow \quad Y(j\omega) = \frac{1}{|a|} X(-j\omega/a)$

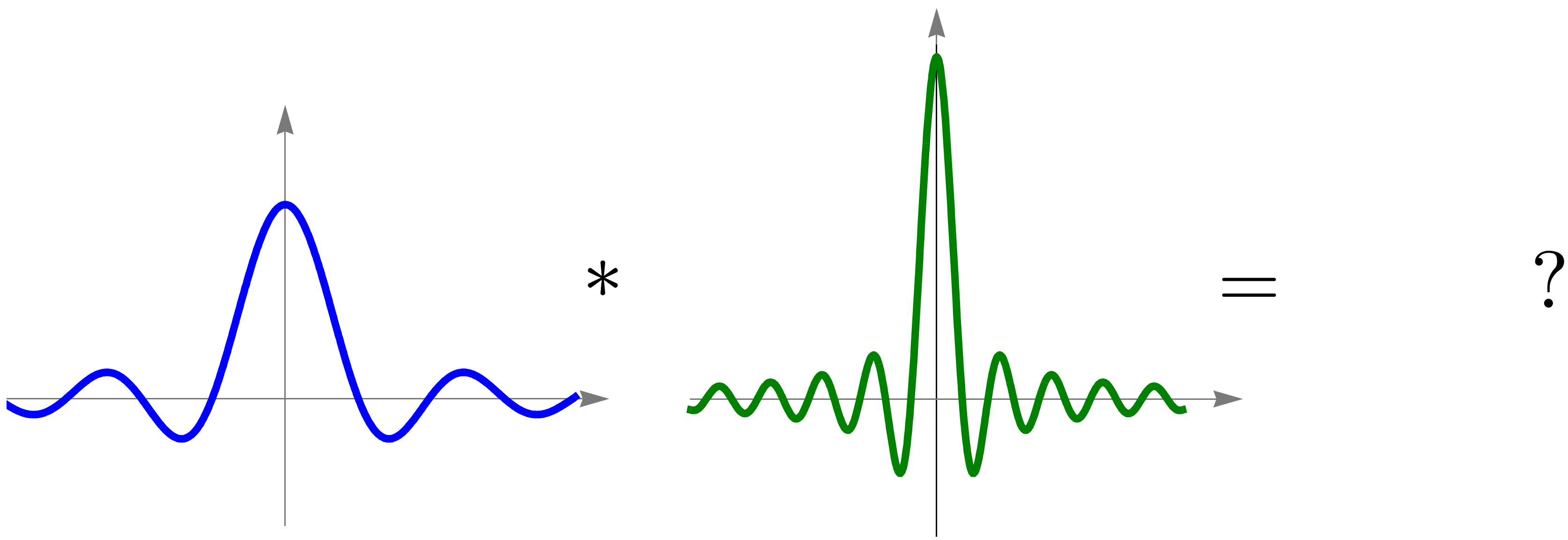
- FT of $x^*(t)$?
- FT of $x(-t)$?

Implications:

- real signals $\Rightarrow$
- real even signals $\Rightarrow$
- real odd signals $\Rightarrow$

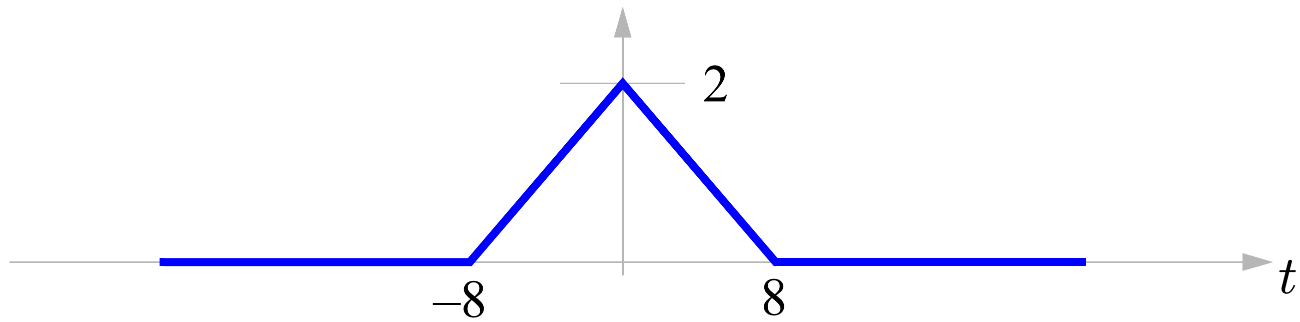
Pop Quiz:

Find the convolution of two different sinc functions:



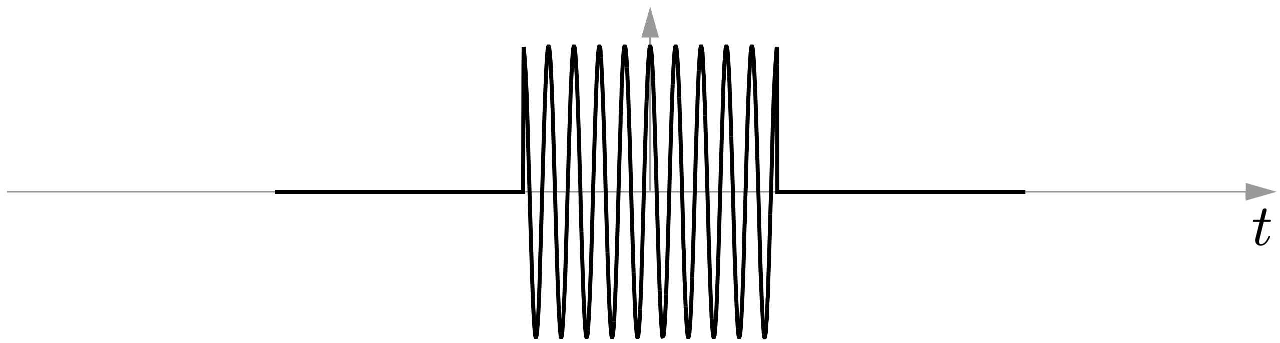
Pop Quiz

Find F.T. of this triangle:



Pop Quiz

- (Multiplication) — Find FT of $x(t) = c(t)r(t)$
$$= \cos(10\pi t)(u(t+1) - u(t-1)):$$



A New Route to FS Coefficients

- Write a periodic signal as:

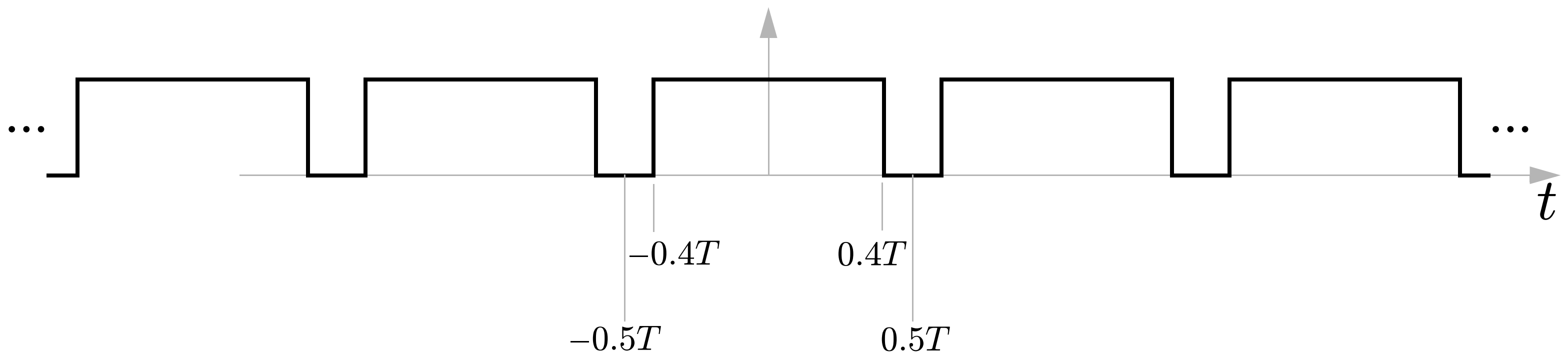
$$x(t) = \sum_k g(t - kT)$$

- View it as convolution $x(t) = g(t) * p(t)$, where $p(t)$ is impulse train
- Use convolution property

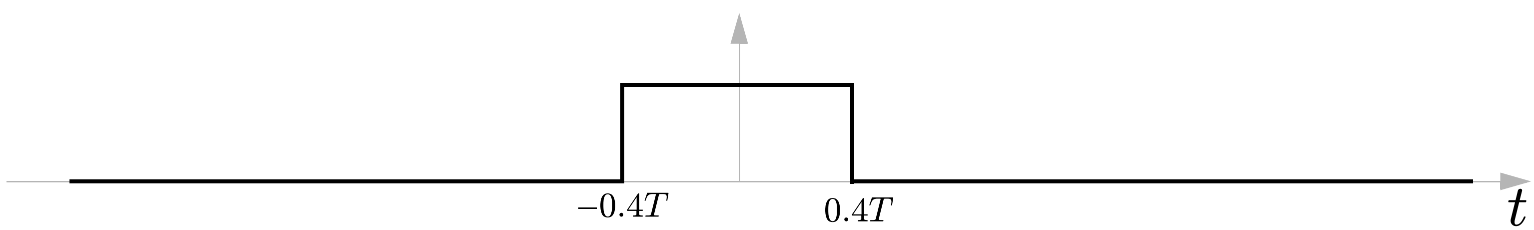
$$\Rightarrow \quad a_k = \frac{1}{T} G\left(\frac{jk2\pi}{T}\right)$$

In words: Sampling the FT of one period yields the FS coeffs.

Example: Find FS coeffs of 80% Duty

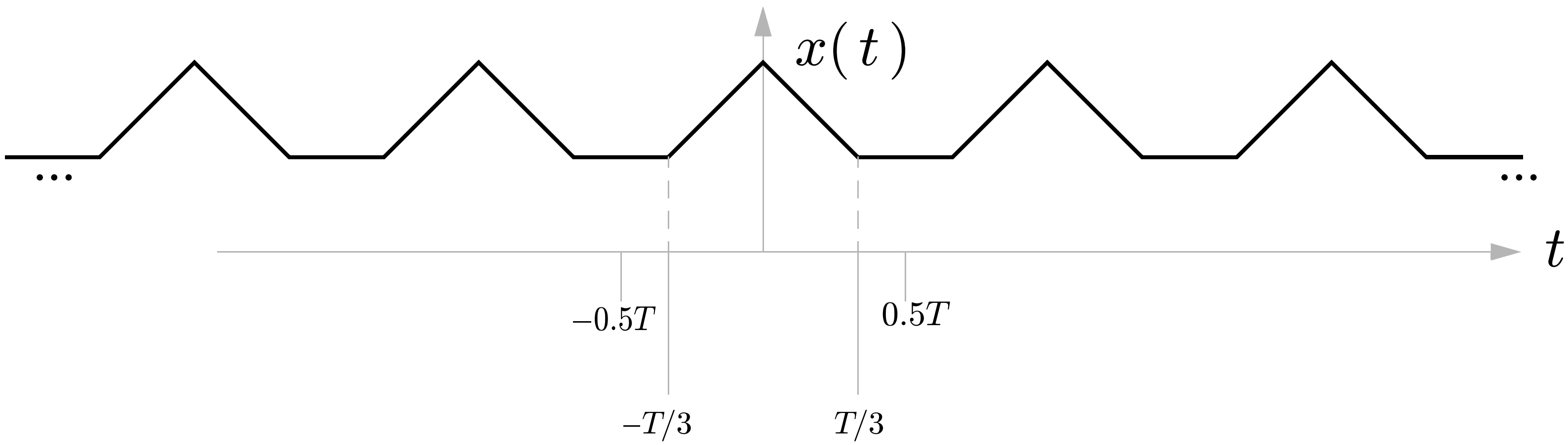


Define $g(t)$.

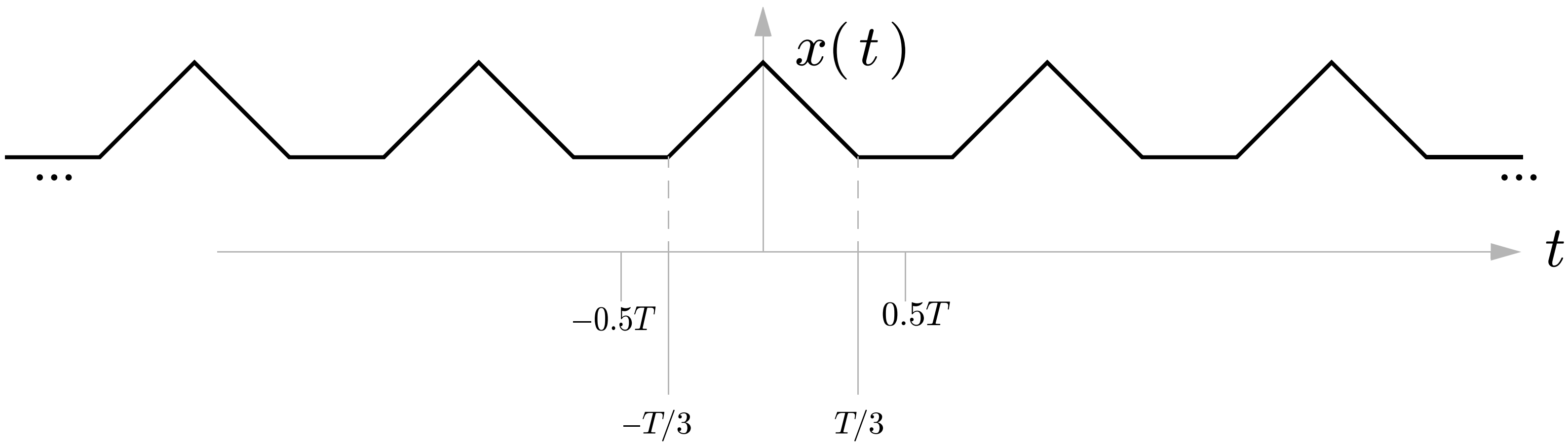


Find a_k by sampling $G(j\omega)$.

Not So Obvious Example:

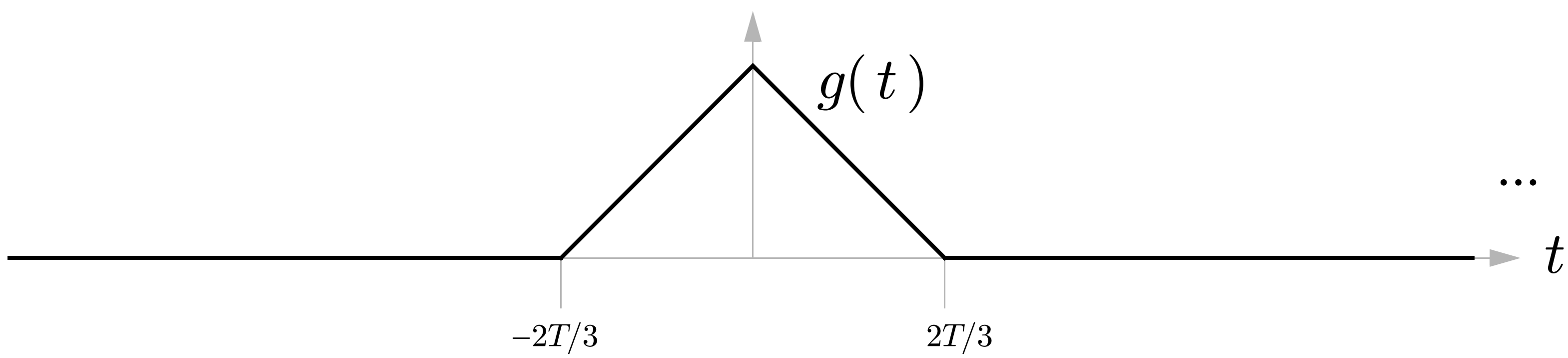


Not So Obvious Example:



To get FS coeffs the old way: must integrate above, 4 time regions! Not fun.

Instead: View $x(t) = \sum_k g(t - kT)$, where $x(t)$ is the following triangle:



But $G(j\omega) = \text{sinc}^2$.

Therefore, the FS coeffs $\{a_k\}$ can be found by sampling sinc^2