

Lecture 13: fpTue Sep 29, 2020

Reminder:

- Quiz 1 is Thursday

Lecture

- the Fourier transform:
 - ▷ more properties (table 2)
 - ▷ using table 1 and table 2

Quiz 1: Thursday

Rules:

- open book/notes
- Google not recommended
- Use Piazza to check for typos
- Otherwise: No communication (electronic or otherwise) with anyone
- See piazza post for logistics

Coverage:

- lectures 1 – 10
- homeworks 1 – 5
- Chapters 1 – 6 of 3084 PDF book
- Chapters 3, 9, 10 of 2026 SP1st (1st Ed)
- Youtube modules 1 – 17

PDF	Date	Topic
lec1	Tue Aug 18	Intro and Motivation; the unit step function
lec2	Thu Aug 20	The Dirac impulse function
lec3	Tue Aug 25	Signal properties: even, odd, energy, power
lec4	Thu Aug 27	Systems and their properties: linear, time-invariance
lec5	Tue Sep 1	LTI systems, impulse response, the convolution integral
lec6	Thu Sep 3	Convolution: properties, examples
lec7	Tue Sep 8	The RADAR problem: crosscorrelation, autocorrelation, the matched filter
lec8	Thu Sep 10	Review RADAR and the MF, intro to frequency response
lec9	Tue Sep 15	Frequency response and Fourier series
lec10	Thu Sep 17	More Fourier series, intro to Fourier transform

Piazza

- A PDF for quiz 1 will be made available to you at 10:00pm EDT on Wednesday, September 30.
- The **submission deadline** is Thursday October 1, at 11:59pm EDT.
- Somewhere in that 26 hour window, you need to set aside roughly 75 minutes to complete the exam, plus whatever time you need to scan and upload.
- Let N be the number of pages in the quiz PDF.
 - If you have a printer: Print it out, work directly on the printout.
 - If you do not have a printer
 - Take out N blank pages and show your work and answers on the corresponding page, in the corresponding space
 - Do not attach extra sheets! Only the first N pages will be graded.
 - For example, if the first page asks only for your name, write only your name on the first page.
 - If the top half of page 3 is part (b) of Problem 2, then limit the top half of your page 3 to Prob. 2(b).
 - If page 5 contains Prob. 4 and nothing else, then what you write on page 5 should only relate to Prob. 4.
 - Please draw answer boxes on each page that align with their positions on the quiz PDF. And place your answers inside.
- Scan the N pages into a single PDF document.
 - Ensure they are scanned in order: the first page of the scanned document should match the first page of the quiz PDF, etc.
 - Before the deadline: upload the PDF to the "Assignments > Quiz 1" page on **canvas**:

☰ [ECE-3084-B](#) > [Assignments](#)

▼ 2020

[Home](#)

[Announcements](#)

[Assignments](#)

[Discussions](#)

[Pages](#)

[People](#)

[Resources](#)

Search for Assignment

+ Group

+ Assignment

⋮

⋮

▶ HW

+

⋮

⋮

▶ Quizzes

+

⋮

⋮

 Quiz 1

Due Oct 1 at 11:59pm | 100 pts

✓

⋮

Late Penalty 1% per minute

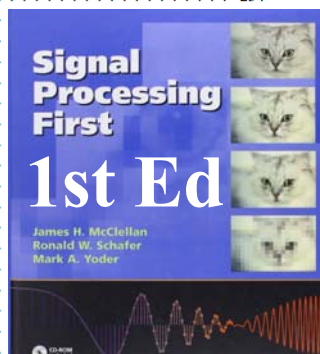
Coverage from 3084 PDF

Preface	iii
0.1 The <i>DSP First</i> Legacy	iii
0.2 Forward to the Past	iv
0.3 Putting Educational Eggs in Many Baskets – Not Just One	v
0.4 For <i>All</i> Electrical Engineers	v
0.5 Tough Choices	vi
1 What are Signals?	1
1.1 Convenient continuous-time signals	1
1.1.1 Unit step functions	1
1.1.2 Delta “functions”	3
1.1.3 Calculus with Dirac deltas and unit steps	4
1.2 Shifting, flipping and scaling continuous-time signals in time	4
1.3 Under the hood: what professors don’t want to talk about	5
2 What are Systems?	9
2.1 System properties	10
2.1.1 Linearity	10
2.1.2 Time-invariance	10
2.1.3 Causality	10
2.1.4 Examples of systems and their properties	10
2.2 Concluding thoughts	10
2.2.1 Linearity and time-invariance as approximations	10
2.2.2 Contemplations on causality	10
2.2.3 How these properties play out in practice in a typical “signals and systems” course	10
3 Why are LTI Systems so Important?	11
3.1 Review of convolution for discrete-time signals	11
3.2 Convolution for continuous-time signals	11
3.3 Review of frequency response of discrete-time systems	11
3.4 Frequency response of continuous-time systems	11
3.5 Connection to Fourier transforms	11
4 More on Continuous-Time Convolution	21
4.1 The convolution integral	21
4.2 Properties of convolution	22
4.3 Convolution examples	23
4.4 Some final comments	28
5 Cross-Correlation and Matched Filtering	29
5.1 Cross-correlation properties	30
5.2 Cross-correlation examples	31
5.3 Matched filter implementation	31
5.4 Delay estimation	32
5.5 Causal concerns	33
5.6 A caveat	34
5.7 Under the hood: squared-error metrics and correlation processing	34
6 Review of Fourier Series	37
6.1 Fourier synthesis sum and analysis integral	37
6.2 System response to a periodic signal	38
6.3 Properties of Fourier series	38
6.4 Fourier series of a symmetric “square wave”	39
6.4.1 Lowpass filtering the square wave	40
6.5 What makes Fourier series tick?	41
6.6 Under the hood	43

Coverage from SP First

9 Continuous-Time Signals and LTI Systems 245

9-1	Continuous-Time Signals	246
9-1.1	Two-Sided Infinite-Length Signals	246
9-1.2	One-Sided Signals	247
9-1.3	Finite-Length Signals	248
9-2	The Unit Impulse	248
9-2.1	Sampling Property of the Impulse	250
9-2.2	Mathematical Rigor	252
9-2.3	Engineering Reality	252
9-2.4	Derivative of the Unit Step	252
9-3	Continuous-Time Systems	254
9-3.1	Some Basic Continuous-Time Systems	
9-3.2	Continuous-Time Outputs	
9-3.3	Analogous Discrete-Time Systems	
9-4	Linear Time-Invariant Systems	
9-4.1	Time-Invariance	
9-4.2	Linearity	
9-4.3	The Convolution Integral	
9-4.4	Properties of Convolution	
9-5	Impulse Responses of Basic LTI Systems	
9-5.1	Integrator	
9-5.2	Differentiator	
9-5.3	Ideal Delay	
9-6	Convolution of Impulses	
9-7	Evaluating Convolution Integrals	
9-7.1	Delayed Unit-Step Input	
9-7.2	Evaluation of Discrete Convolution	
9-7.3	Square-Pulse Input	
9-7.4	Very Narrow Square Pulse Input	269
9-7.5	Discussion of Convolution Examples	270
9-8	Properties of LTI Systems	270
9-8.1	Cascade and Parallel Combinations	270
9-8.2	Differentiation and Integration of Convolution	272
9-8.3	Stability and Causality	273
9-9	Using Convolution to Remove Multipath Distortion	276
9-10	Summary	278
9-11	Problems	279



10 Frequency Response 285

10-1	The Frequency Response Function for LTI Systems	285
10-1.1	Plotting the Frequency Response	287
10-1.1.1	Logarithmic Plot	288
10-1.2	Magnitude and Phase Changes	288
10-2	Response to Real Sinusoidal Signals	289
10-2.1	Cosine Inputs	290
10-2.2	Symmetry of $H(j\omega)$	290
10-2.3	Response to a General Sum of Sinusoids	293
10-2.4	Periodic Input Signals	294
10-3	Ideal Filters	295
10-3.1	Ideal Delay System	295
10-3.2	Ideal Lowpass Filter	296
10-3.3	Ideal Highpass Filter	297
10-3.4	Ideal Bandpass Filter	297
10-4	Application of Ideal Filters	298
10-5	Time-Domain or Frequency-Domain?	300
10-6	Summary/Future	301
10-7	Problems	302

3 Spectrum Representation 36

3-1	The Spectrum of a Sum of Sinusoids	36
3-1.1	Notation Change	38
3-1.2	Graphical Plot of the Spectrum	38
3-2	Beat Notes	39
3-2.1	Multiplication of Sinusoids	39
3-2.2	Beat Note Waveform	40
3-2.3	Amplitude Modulation	41
3-3	Periodic Waveforms	43
3-3.1	Synthetic Vowel	44
3-3.2	Example of a Nonperiodic Signal	45
3-4	Fourier Series	47
3-4.1	Fourier Series: Analysis	48
3-4.2	Fourier Series Derivation	48
3-5	Spectrum of the Fourier Series	50
3-6	Fourier Analysis of Periodic Signals	51
3-6.1	The Square Wave	52
3-6.1.1	DC Value of a Square Wave	53
3-6.2	Spectrum for a Square Wave	53
3-6.3	Synthesis of a Square Wave	54
3-6.4	Triangle Wave	55
3-6.5	Synthesis of a Triangle Wave	56
3-6.6	Convergence of Fourier Synthesis	57

Coverage from Youtube

Modules 1 - 17:

Handouts for ECE 3084

- [syllabus](#)
- [3084 book](#)

Supplemental Material

- [Step and Delta functions](https://math.mit.edu/~stoon/18.031/) (from <https://math.mit.edu/~stoon/18.031/>)
- [Summer 2026 ECE3084 video lecture playlist \(youtube\)](#)
- [ECE3084 Resource Page](#): videos and lecture notes.
- [MIT Class](#): Videos, lecture notes, and problems.

ECE3084 Signals and Systems

53 videos · 1,140 views · Last updated on Aug 15, 2020

These are video lectures from the Summer 2020 offering of ECE3084: Signals and Systems at Georgia Tech, brought to you as a result of Covid-19 forcing Tech to operate in "distance learning" mode.

Lanterntronics [SUBSCRIBE](#)

- 1 ECE3084 Signals and Systems: Introduction (Lecture 1, Summer 2020, Georgia Tech Course) 20:38
- 2 ECE3084 Lecture 2: Unit Step Functions (Signals and Systems, Summer 2020, Georgia Tech Course) 21:12
- 3 ECE3084 Lecture 3: Unit Impulse Functions (Signals & Systems, Summer 2020, Georgia Tech Course) 19:22
- 4 ECE3084 Lecture 4: Calculus with Step and Impulse Functions (Signals & Systems, 2020, Georgia Tech) 10:43
- 5 ECE3084 Lecture 5: Shifting, Scaling, and Mirroring Signals (Signals & Systems, 2020, Georgia Tech) 11:44
- 6 ECE3084 Lecture 6: System Properties: Linearity (Signals & Systems, Summer 2020, Georgia Tech) 24:52
- 7 ECE3084 Lecture 7: System Properties: Time Invariance (Signals & Systems, Summer 2020, Georgia Tech) 15:17
- 8 ECE3084 Lecture 8: System Properties: Causality and Commentary on Respondus Lockdown Browser 17:41
- 9 ECE3084 Lecture 9: Why are LTI Systems so Awesome? (Signals & Systems, Summer 2020, Georgia Tech) 28:21
- 10 ECE3084 Lecture 10: Convolution: Flipping & Shifting (Signals & Systems, Summer 2020, Georgia Tech) 24:05
- 11 ECE3084 Lecture 11: Convolution with Impulses, Steps, & Doublets (Signals & Systems, 2020, GA Tech) 12:12
- 12 ECE3084 Lecture 12: Convolution with Boxcars (Signals & Systems, Summer 2020, Georgia Tech) 12:07
- 13 ECE3084 Bonus Lecture 12A: Convolution with Step Functions (Signals & Systems, Summer 2020, Georgia Tech) 4:20
- 14 ECE3084 Bonus Lecture 12B: Multiplying Step Functions (Signals & Systems, 2020, Georgia Tech) 4:58
- 15 ECE3084 Bonus Lecture 12C: Convolution with Decaying Exponentials (Signals & Systems, 2020, Georgia Tech) 10:19
- 16 ECE3084 Lecture 13: Review of Fourier series (Signals & Systems, Summer 2020, Georgia Tech) 18:24
- 17 ECE3084 Lecture 14: Filtering Fourier series (Signals & Systems, Summer 2020, Georgia Tech) 12:04

Review Session Today

From piazza:

Exam Review Session

Hey everyone!

Ahead of your exam this week, I'll be holding a review session this Tuesday at 5pm. I'll probably work through one of the practice tests that are posted online but feel free to come with questions. This will be a very casual review.

9/29 @ 5PM @ <https://bluejeans.com/1982829984>

Best,
Max

edit: Changed time from 4pm to 5pm

Fourier Transform Pair

Forward $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

Inverse $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$

TABLE 1

Table of Fourier Transform Pairs

	<i>Time-Domain: $x(t)$</i>	<i>Frequency-Domain: $X(j\omega)$</i>	
Right-sided exponential	$e^{-at}u(t) \quad (a > 0)$	$\frac{1}{a + j\omega}$	✓
Left-sided exponential	$e^{bt}u(-t) \quad (b > 0)$	$\frac{1}{b - j\omega}$	✓
Square pulse	$[u(t + T/2) - u(t - T/2)]$	$\frac{\sin(\omega T/2)}{\omega/2}$	✓
“sinc” function	$\frac{\sin(\omega_0 t)}{\pi t}$	$[u(\omega + \omega_0) - u(\omega - \omega_0)]$	✓
Impulse	$\delta(t)$	1	✓
Shifted impulse	$\delta(t - t_0)$	$e^{-j\omega t_0}$	✓
Complex exponential	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	✓
General cosine	$A \cos(\omega_0 t + \phi)$	$\pi A e^{j\phi} \delta(\omega - \omega_0) + \pi A e^{-j\phi} \delta(\omega + \omega_0)$	✓
Cosine	$\cos(\omega_0 t)$	$\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$	✓
Sine	$\sin(\omega_0 t)$	$-j\pi\delta(\omega - \omega_0) + j\pi\delta(\omega + \omega_0)$	
General periodic signal	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$\sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$	✓
Impulse train	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k/T)$	✓

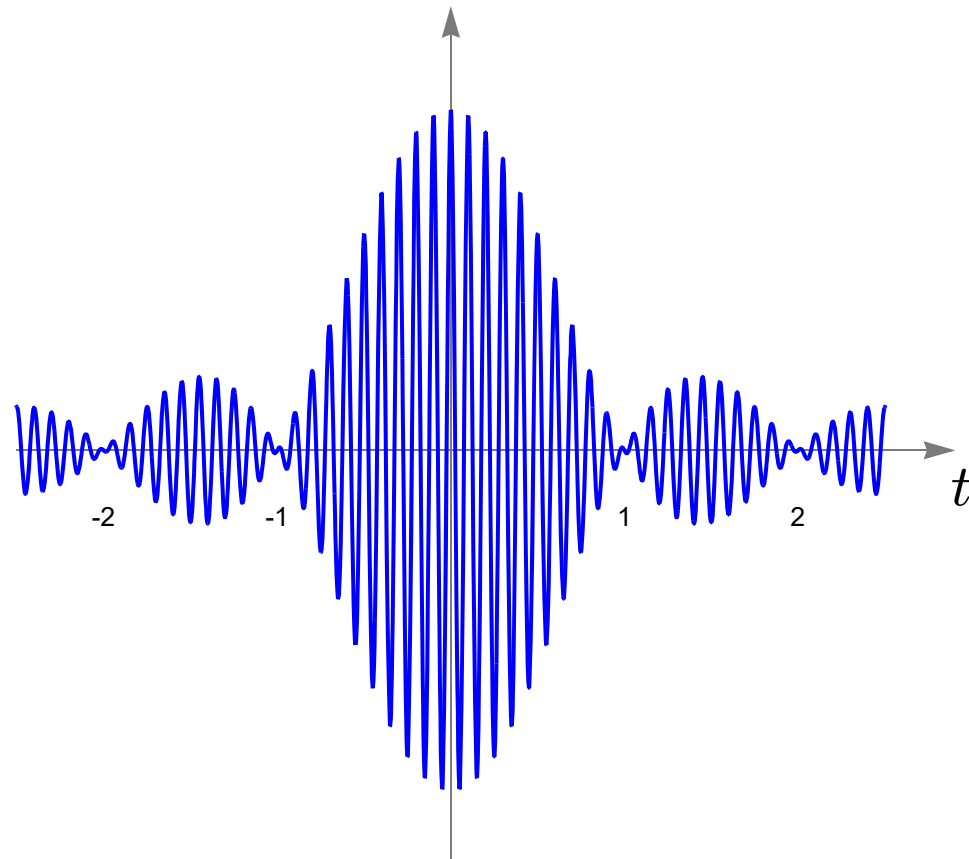
Table of Fourier Transform Properties

TABLE 2

	<i>Time-Domain $x(t)$</i>	<i>Frequency-Domain $X(j\omega)$</i>
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$ ✓
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time-Reversal	$x(-t)$	$X(-j\omega)$ ✓
Scaling	$f(at)$	$\frac{1}{ a }X(j(\omega/a))$
Delay	$x(t - t_d)$	$e^{-j\omega t_d}X(j\omega)$ ✓
Modulation	$x(t)e^{j\omega_0 t}$	$X(j(\omega - \omega_0))$ ✓
Modulation	$x(t) \cos(\omega_0 t)$	$\frac{1}{2}X(j(\omega - \omega_0)) + \frac{1}{2}X(j(\omega + \omega_0))$
Differentiation	$\frac{d^k x(t)}{dt^k}$	$(j\omega)^k X(j\omega)$
Convolution	$x(t) * h(t)$	$X(j\omega)H(j\omega)$ ✓
Multiplication	$x(t)p(t)$	$\frac{1}{2\pi}X(j\omega) * P(j\omega)$

Motivate Modulation Property

Find frequency response when $h(t) = \cos(20\pi t) \frac{\sin \pi t}{\pi t}$:



More FT Properties

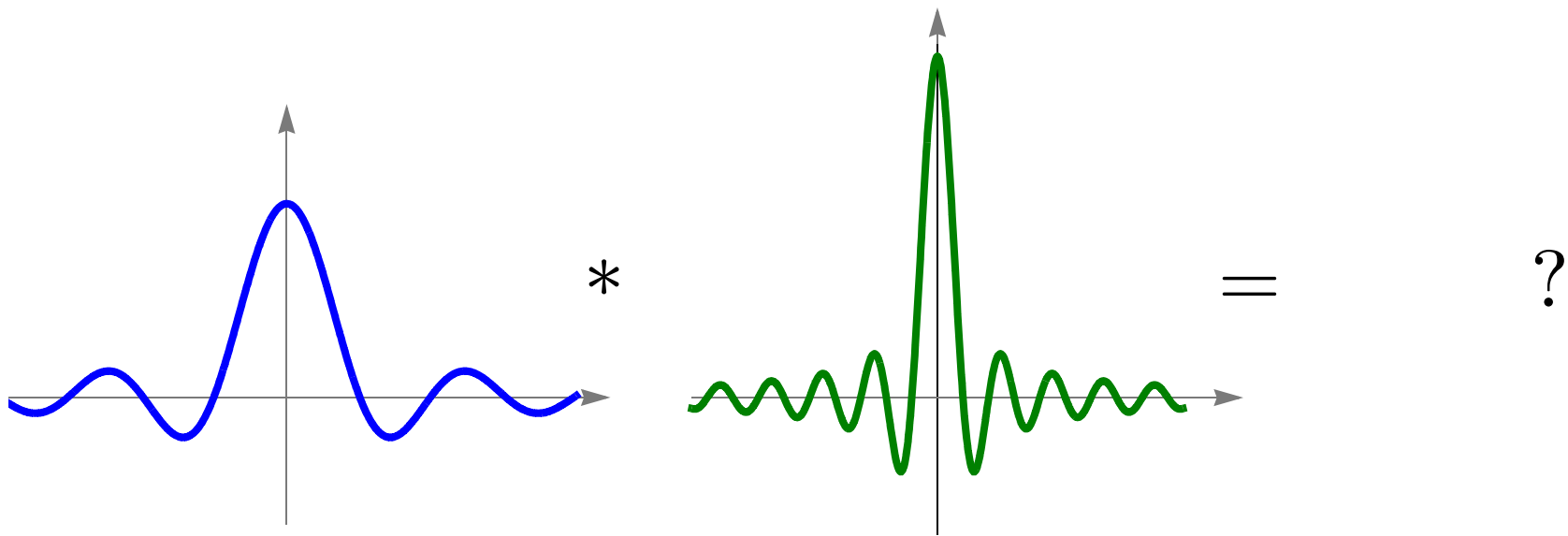
- time stretch: $y(t) = x(at) \iff Y(j\omega) = \frac{1}{|a|} X(-j\omega/a)$
- FT of $x^*(t)$?
- FT of $x(-t)$?

Implications:

- real signals \Rightarrow
- real even signals \Rightarrow
- real odd signals \Rightarrow

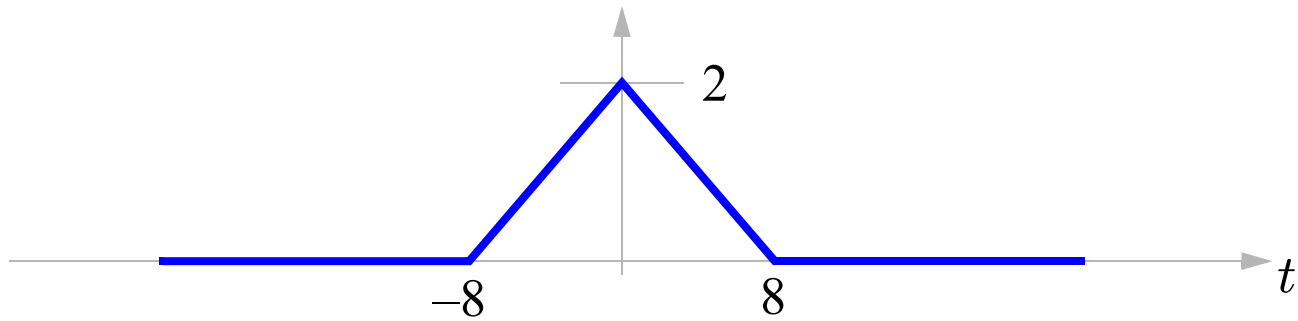
Pop Quiz:

Find the convolution of two different sinc functions:



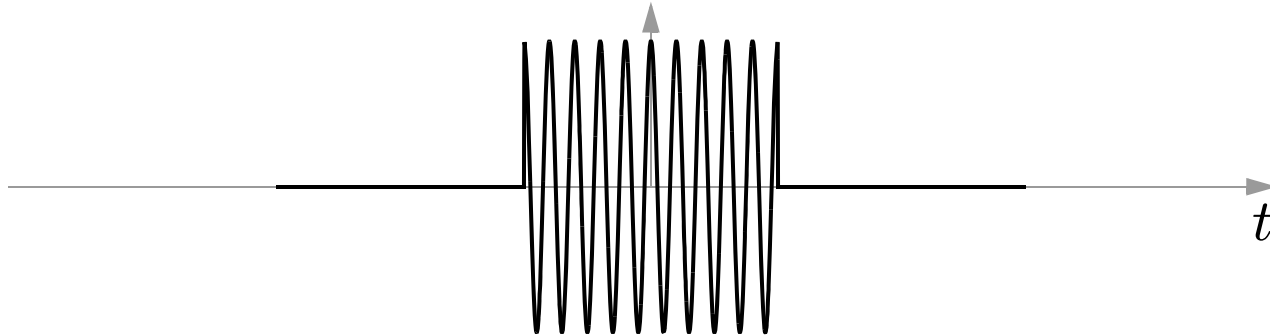
Pop Quiz

Find F.T. of this triangle:



Pop Quiz

- (Multiplication) — Find FT of $x(t) = c(t)r(t)$
 $= \cos(10\pi t)(u(t+1) - u(t-1))$:



A New Route to FS Coefficients

- Write a periodic signal as:

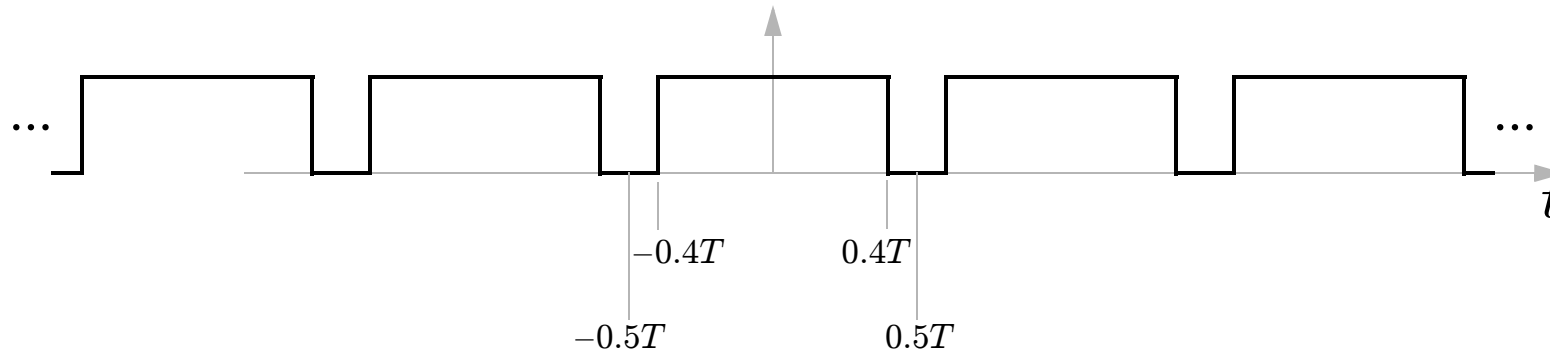
$$x(t) = \sum_k g(t - kT)$$

- View it as convolution $x(t) = g(t) * p(t)$, where $p(t)$ is impulse train
- Use convolution property

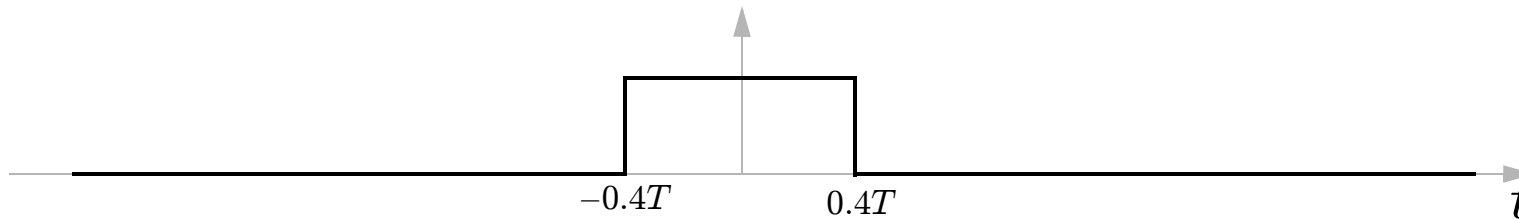
$$\Rightarrow a_k = \frac{1}{T} G\left(\frac{jk2\pi}{T}\right)$$

In words: Sampling the FT of one period yields the FS coeffs.

Example: Find FS coeffs of 80% Duty

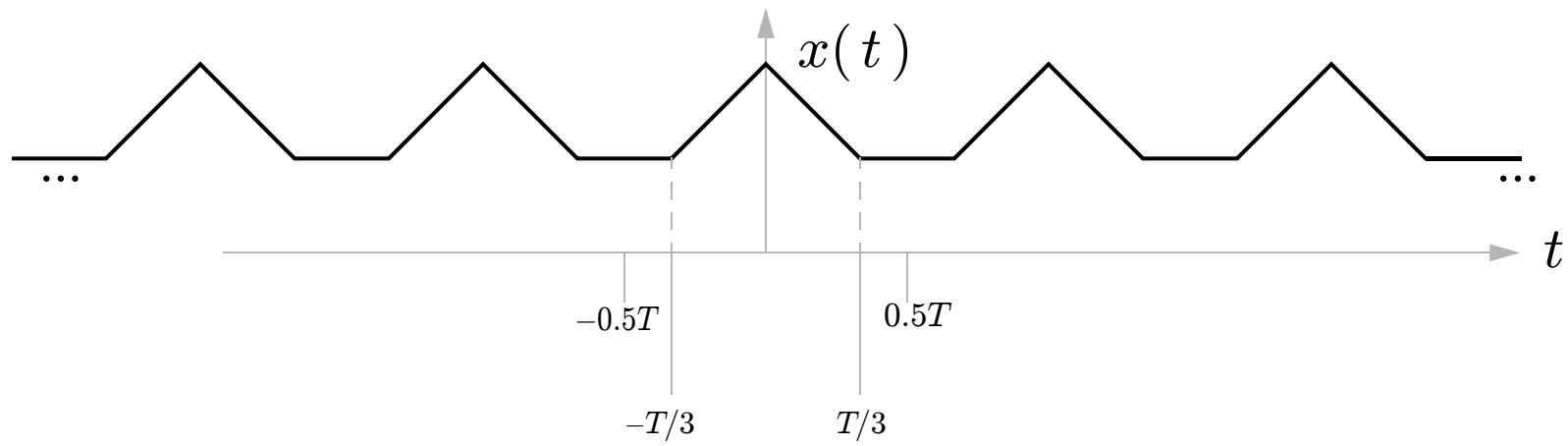


Define $g(t)$.

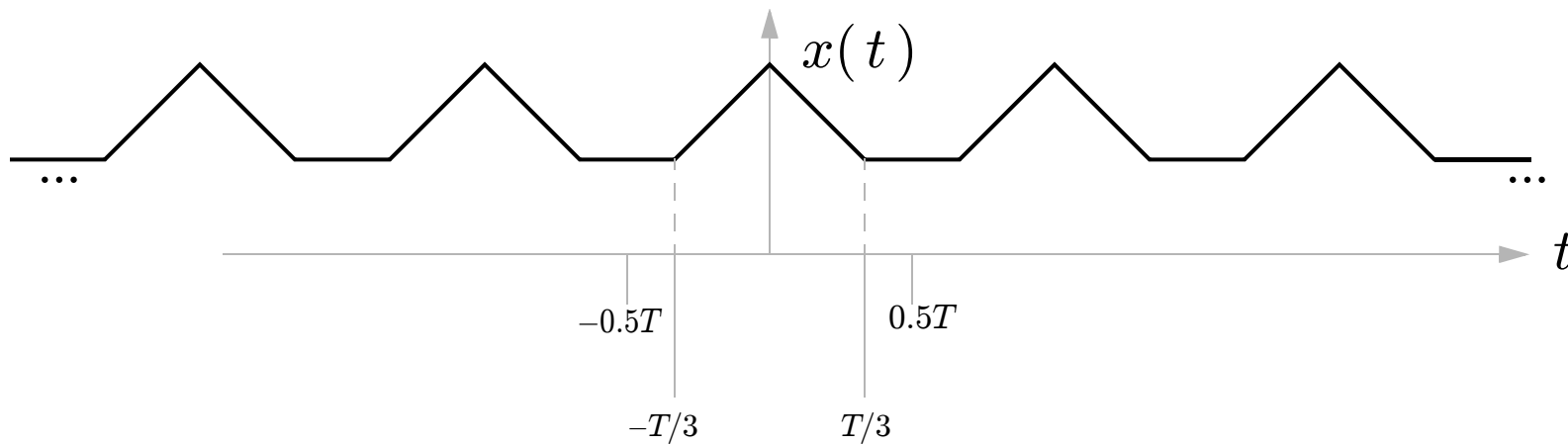


Find a_k by sampling $G(j\omega)$.

Not So Obvious Example:

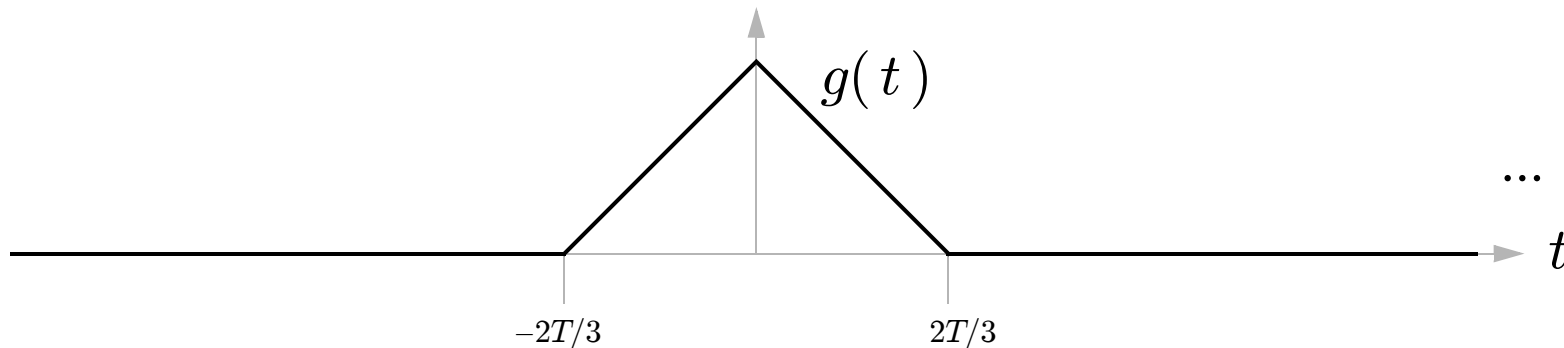


Not So Obvious Example:



To get FS coeffs the old way: must integrate above, 4 time regions! Not fun.

Instead: View $x(t) = \sum_k g(t - kT)$, where $g(t)$ is the following triangle:



But $G(j\omega) = \text{sinc}^2$.

Therefore, the FS coeffs $\{a_k\}$ can be found by sampling sinc^2