GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #4

DATE: 22-Apr-05 COURSE: ECE-2025

NAME: ______________________ GT #: ______________
LAST, FIRST (ex: gtz123a)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Chang)		L06:Thur-Noon (Ingram)
	L07:Tues-1:30pm (Chang)		L08:Thurs-1:30pm (Zhou)
L01:M-3pm (Williams)	L09:Tues-3pm (Casinovi)	L02:W-3pm (Juang)	L10:Thur-3pm (Zhou)
L03:M-4:30pm (Casinovi)	L11:Tues-4:30pm (Casinovi)	L04:W-4:30pm (Juang)	GTSav: (Moore)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
No Rec	–3	

PROBLEM sp-05-Q.4.1:

In each of the following cases, simplify the expression ***as much as possible.*** Provide some **explanation** or intermediate steps for each answer. *Note:* Star $*$ is the convolution operator.

(a) $\left\{e^{-(t-4)}u(t-4)\right\} * h(t) = 2e^{-t}u(t)$ (find $h(t)$ that satisfies this equation)

(b) $\displaystyle\int_{-\infty}^{\infty} u(t+2)u(7-t)dt$

(c) $\displaystyle\int_{-\infty}^{t-5} \delta(\tau-1)d\tau$

(d) $(\delta(t-1)+\delta(t-2)) * (\delta(t)-\delta(t-1))$

(e) $\left.\dfrac{\sin(4\omega)}{8\omega}\right|_{\omega=0}$ (i.e., evaluate at $\omega = 0$)

PROBLEM sp-05-Q.4.2:

In each of the following cases, determine the (inverse or forward) Fourier transform. Give your answer as a plot, or a simple formula that is ***real-valued.***

Explain each answer (briefly) by stating which property and/or transform pair you used.

(a) Find $x(t)$ when $X(j\omega) = 14\cos(3\omega)$.

(b) Find $s(t)$ when $S(j\omega) = j75\pi\,\delta(\omega + 8\pi) - j75\pi\,\delta(\omega - 8\pi)$.

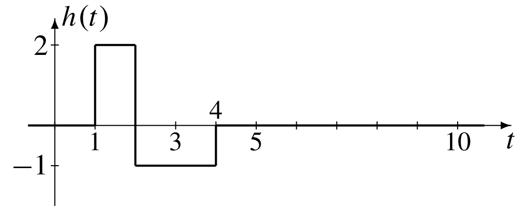
(c) Find $r(t)$ when $R(j\omega) = \dfrac{12}{4 + j2\omega}$.

(d) Find $H(j\omega)$ when $h(t) = \dfrac{\sin(3\pi(t - \frac{1}{2}))}{2t - 1}$.

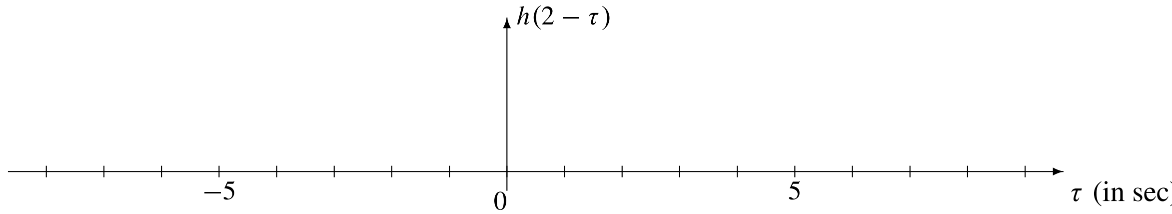
(e) Sketch the magnitude of $H(j\omega)$ obtained in the previous part, i.e., $|H(j\omega)|$ versus ω.

PROBLEM sp-05-Q.4.3:

A linear time-invariant system has this impulse response:



(a) Plot $h(t-\tau)$ versus τ, for $t = 2$. Label your plot carefully.



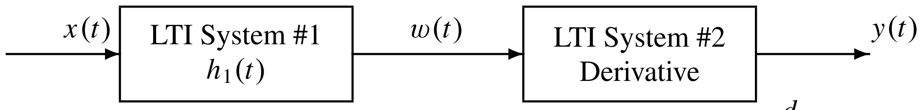
(b) If the input signal is the unit-step signal, $x(t) = u(t)$, use the convolution integral to find $y(2)$; i.e., the value of the output signal, $y(t) = x(t) * h(t)$, when $t = 2$.

(c) When the input signal is $x(t) = u(t)$ and the impulse response is $h(t)$ given above, it turns out that the output is **zero** for $t < T_1$ and for $t > T_2$. Find the values of T_1 and T_2. **Explain** your answers. *You may "flip and shift" either $x(t)$ or $h(t)$, whichever leads to the easiest explanation.*

$T_1 =$ ☐ $T_2 =$ ☐

PROBLEM sp-05-Q.4.4:

A cascade of linear time-invariant systems is depicted by the following block diagram:



$$h_1(t) = 20e^{-3(t-5)}u(t-5) \qquad\qquad y(t) = \frac{d}{dt}w(t)$$

(a) If the input to the first system is a sinusoid:

$$x(t) = 100\cos(3t)$$

Determine the output of the ***first system***, $w(t)$. Give your answer in the ***simplest possible form.***

(b) If the input to the first system is a shifted unit-step signal, $x(t) = u(t+5)$, determine the overall output signal, $y(t)$.

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SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #4

DATE: 22-Apr-05 COURSE: ECE-2025

NAME: **ANSWER KEY** GT #: **VERSION #1**
LAST, FIRST (ex: gtz123a)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

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Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
No Rec	−3	

PROBLEM sp-05-Q.4.1:

In each of the following cases, simplify the expression **as much as possible** using the properties of the continuous-time unit impulse signal. Provide some **explanation** or intermediate steps for each answer.

(a) $\{e^{-(t-4)}u(t-4)\} * h(t) = 2e^{-t}u(t)$ (find $h(t)$ that satisfies this equation)

Need to multiply by 2 and shift the signal 4 secs to the left $\Rightarrow$ delay of -4 secs.

$h(t) = 2\delta(t+4)$

(b) $\int_{-\infty}^{\infty} u(t+2)u(7-t)dt = \int_{-2}^{7} 1\, dt = 9$

$u(t+2) = 0$ when $t+2 < 0$, or when $t < -2$.

$u(7-t) = 0$ when $7-t < 0$, or when $t > 7$

(c) $\int_{-\infty}^{t-5} \delta(\tau - 1)d\tau = u(\tau-1)\Big|_{-\infty}^{t-5} = u(t-6)$

(d) $(\delta(t-1) + \delta(t-2)) * (\delta(t) - \delta(t-1))$

$= \delta(t-1) * \delta(t) + \delta(t-2) * \delta(t) - \delta(t-1) * \delta(t-1) - \delta(t-2) * \delta(t-1)$

$= \delta(t-1) + \delta(t-2) - \delta(t-2) - \delta(t-3)$

$= \delta(t-1) - \delta(t-3)$

(e) $\left.\frac{\sin(4\omega)}{8\omega}\right|_{\omega=0}$ (i.e., evaluate at $\omega = 0$)

$\lim_{\omega \to 0} \frac{\sin(4\omega)}{8\omega} \approx \lim_{\omega \to 0} \frac{4\omega}{8\omega} = \frac{1}{2}$

Because $\sin(\epsilon) \approx \epsilon$ when $\epsilon \to 0$.

PROBLEM sp-05-Q.4.2:

In each of the following cases, determine the (inverse or forward) Fourier transform. Give your answer as a simple formula or a plot. **Explain** each answer by stating which property and transform pair you used.

(a) Find $x(t)$ when $X(j\omega) = 14\cos(3\omega)$. $= 7e^{j3\omega} + 7e^{-j3\omega}$

Use $e^{-j\omega t_d} \longleftrightarrow \delta(t-t_d)$

$$x(t) = 7\delta(t+3) + 7\delta(t-3)$$

(b) Find $s(t)$ when $S(j\omega) = j75\pi\,\delta(\omega + 8\pi) - j75\pi\,\delta(\omega - 8\pi)$.

Use $2\pi\delta(\omega-\omega_0) \longleftrightarrow e^{j\omega_0 t}$

$$s(t) = j\tfrac{75}{2}e^{-j8\pi t} - j\tfrac{75}{2}e^{j8\pi t} = 75\left\{\frac{e^{j8\pi t} - e^{-j8\pi t}}{2j}\right\}$$

$$= 75\sin(8\pi t)$$

(c) Find $r(t)$ when $R(j\omega) = \dfrac{12}{4 + j2\omega}$. $= \dfrac{6}{2+j\omega}$

Use $\dfrac{1}{a+j\omega} \longleftrightarrow e^{-at}u(t)$

$$\Rightarrow r(t) = 6e^{-2t}u(t)$$

(d) Find $H(j\omega)$ when $h(t) = \dfrac{\sin(3\pi(t - \frac{1}{2}))}{2t - 1}$. $= \dfrac{\pi}{2}\cdot\dfrac{\sin(3\pi(t-\frac{1}{2}))}{\pi(t-\frac{1}{2})}$

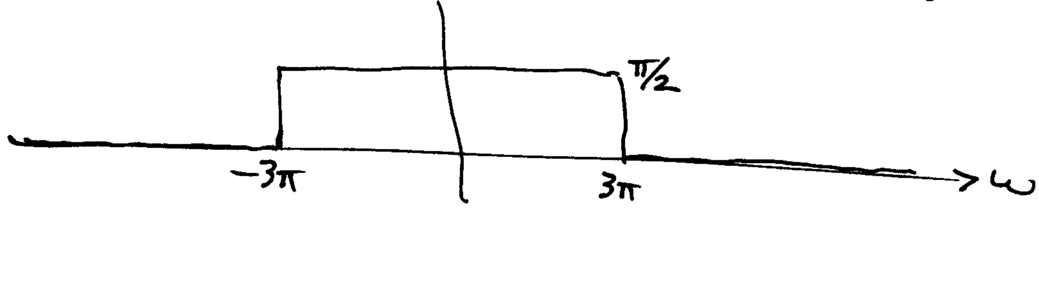
sinc $\longleftrightarrow$ rect

time-shift $\longleftrightarrow$ mult. by $e^{-j\omega t_d}$.

$$H(j\omega) = \frac{\pi}{2}e^{-j\omega/2}\left\{u(\omega+3\pi) - u(\omega-3\pi)\right\}$$

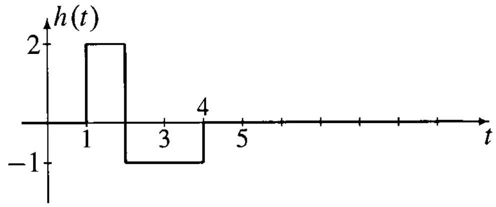
(e) Sketch the magnitude of $H(j\omega)$ from the previous part.

Magnitude of $e^{-j\omega/2}$ is one. Thus, we plot the mag of a rectangle.

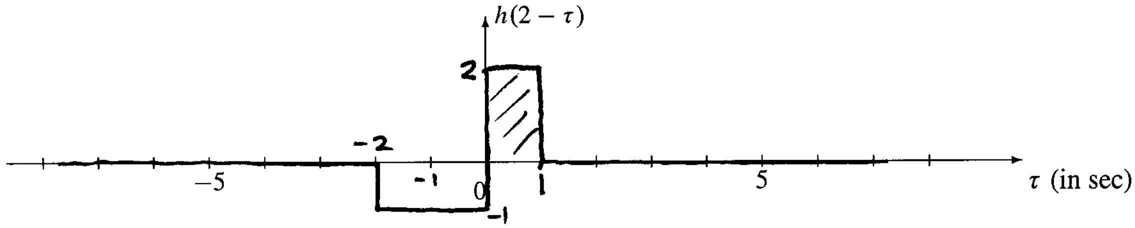


PROBLEM sp-05-Q.4.3:

A linear time-invariant system has this impulse response:



(a) Plot $h(t-\tau)$ versus τ, for $t = 2$. Label your plot carefully. Flip +1 to −1, then slide by 2.



(b) If the input signal is $x(t) = u(t)$, use the convolution integral to find $y(2)$; i.e., the value of the output signal $y(t) = x(t) * h(t)$ when $t = 2$.

$$y(2) = \int_{-\infty}^{\infty} u(\tau)\, h(2-\tau)\, d\tau = \int_{0}^{\infty} h(2-\tau)\, d\tau$$

In the figure of part (a) we must find the area for $\tau > 0$. It's the shaded part.

$$y(2) = 2$$

(c) When the input signal is $x(t) = u(t)$ and the impulse response is $h(t)$ given above, it turns out that the output is **zero** for $t < T_1$ and for $t > T_2$. Find the values of T_1 and T_2. **Explain** your answers. *You may "flip and shift" either $x(t)$ or $h(t)$, whichever leads to the easiest solution.*

$T_1 =$ 1 sec $\quad T_2 =$ 4 sec

If we write the convolution integral as:

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau$$

Then

$$y(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$$

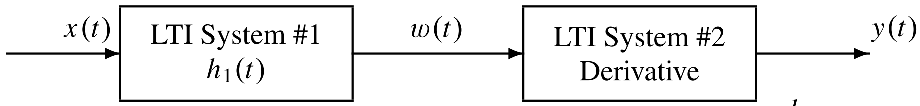
← This is the area in $h(t)$ from $-\infty$ up to t

There is zero area until $t = 1$

The total area of $h(t)$ is zero because the positive and negative parts cancel. We get the total area when $t \geq 4$.

PROBLEM sp-05-Q.4.4:

A cascade of linear time-invariant systems is depicted by the following block diagram:



$h_1(t) = 20e^{-3(t-5)}u(t-5)$ $\qquad$ $y(t) = \frac{d}{dt}w(t)$

(a) If the input to the first system is a sinusoid:

$$x(t) = 100\cos(3t)$$

Determine the output of the ***first system***, $w(t)$. Give your answer in the ***simplest possible form.***

Need to get the frequency response, and evaluate it at $\omega = 3$ rad/s. The frequency response is the Fourier transform of the impulse response. Thus

$$H(j\omega) = \frac{20e^{-j5\omega}}{3 + j\omega}$$

When evaluated at $\omega = 3$, we get

$$H(j3) = \frac{20e^{-j5(3)}}{3 + j3} = \frac{20}{3\sqrt{2}}e^{-j(\pi/4+5(3))} = 4.714e^{-j15.785} = 4.714e^{j3.064}$$

Thus, $y(t) = (100)(4.714)\cos(3t + 3.064) = 471.4\cos(3t + 3.064)$

(b) If the input to the first system is a shifted unit-step signal, $x(t) = u(t+5)$, determine the overall output signal, $y(t)$.

Since both systems are LTI, we can swap the order and do the derivative first. The derivative of a unit-step signal is a unit-impulse signal, so

$$y(t) = h_1(t) * \left\{\frac{d}{dt}u(t+5)\right\} = h_1(t) * \delta(t+5) = h_1(t+5) = 20e^{-3t}u(t)$$