GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

ECE2026 – Fall 2014
Quiz #4 (Written Exam 2)
November 21, 2014

NAME: _____Solution Keys___________ **GT Username: _____________________**
LAST FIRST (e.g., gtinit0101)

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):
(Failing to circle the correct section will cost you 3 points)

	Mon	Tue	Wed	Thu
9:35-10:55				L06 (Bhatti)
12:05-13:25		L07 (Causey)		
13:35-14:55		L09 (Causey)		L10 (Bhatti)
15:05-16:25	L01 (Moore)	L11 (Davenport)	L02 (Juang)	L12 (Walkenhorst)
16:35-17:55	L03 (Aghasi)	L13 (Davenport)	L04 (Juang)	L14 (Walkenhorst)

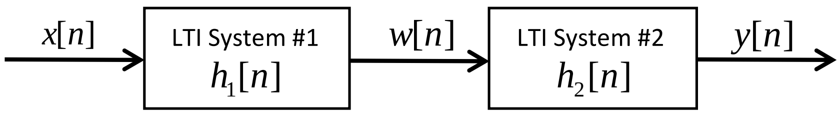
Important Notes:

- Write your name on the front page ONLY. **DO NOT** UNSTAPLE the test.
- Closed book, but a calculator is permitted.
- One page (8.5"x11") of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Partial credit for incorrect answers may be granted ONLY when you JUSTIFY your correct reasoning CLEARLY.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If more space is needed for scratch work, use the backs of previous pages.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
No/Wrong Rec	-3	
Total		

PROBLEM Fall-14-Q.4.1:

The following figure depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.



Suppose that System #1 is a 5-pt average, defined by the difference equation: $w[n] = \frac{1}{5}\sum_{k=0}^{4} x[n-k]$ and System #2 has the following frequency response:

$$H_2(e^{j\hat{\omega}}) = 2e^{-j(\frac{3\hat{\omega}-\pi}{2})}\sin(\tfrac{\hat{\omega}}{2}).$$

(a) Determine the impulse response of the 2nd system, $h_2[n]$; express your result in terms of time-shifted unit impulse functions (i.e., $h_2[n] = b_0\delta[n] + b_1\delta[n-1] + b_2\delta[n-2] + \cdots$)

$$H_2(e^{j\hat{\omega}}) = 2e^{-j(\frac{3\hat{\omega}-\pi}{2})}\sin(\tfrac{\hat{\omega}}{2}) = 2je^{-j\frac{3\hat{\omega}}{2}}\frac{e^{j\frac{\hat{\omega}}{2}} - e^{-j\frac{\hat{\omega}}{2}}}{2j} = e^{-j\hat{\omega}} - e^{-j2\hat{\omega}}$$

$$h_2[n] = \delta[n-1] - \delta[n-2]$$

$$\boxed{h_2[n] = \delta[n-1] - \delta[n-2]}$$

(b) Referring to the above figure, obtain a single difference equation that relates $y[n]$ to $x[n]$.

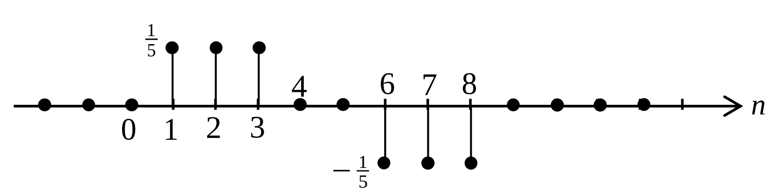
$$w[n] = \tfrac{1}{5}\sum_{k=0}^{4} x[n-k] \quad \Rightarrow h_1[n] = \tfrac{1}{5}\sum_{k=0}^{4}\delta[n-k]$$

$$h[n] = \left(\tfrac{1}{5}\sum_{k=0}^{4}\delta[n-k]\right) * (\delta[n-1] - \delta[n-2]) = \tfrac{1}{5}\left(\delta[n-1] - \delta[n-6]\right)$$

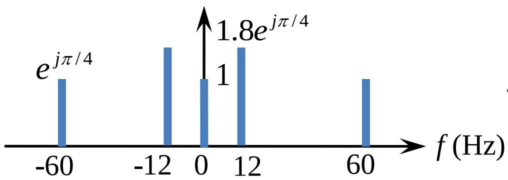
$$y[n] = \tfrac{1}{5}\left(x[n-1] - x[n-6]\right)$$

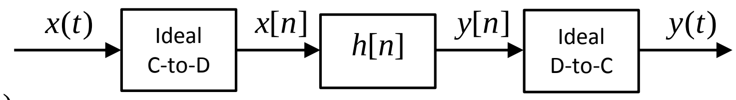
$$\boxed{y[n] = \tfrac{1}{5}\left(x[n-1] - x[n-6]\right)}$$

(c) The above cascade system is driven by an input sequence, $x[n] = u[n] - u[n-3]$, where $u[n]$ is the unit step function. Obtain and express the output $y[n]$ in a properly labeled stem plot in the box.



PROBLEM Fall-14-Q.4.2:





The spectrum of a real signal $x(t)$ is shown as above. The signal is being sampled at a rate of 72 samples/s with an ideal C-to-D, processed by an LTI system, and finally reconstructed with an ideal D-to-C operating at the same sampling rate as the C-to-D, to produce the continuous-time output $y(t)$.

(a) Write the sampled sequence $x[n]$ in mathematical expression.

$$x(t) = 1 + 3.6\cos(24\pi t + \tfrac{\pi}{4}) + 2\cos(120\pi t - \tfrac{\pi}{4}) \qquad \text{sampled at } f_s = 72$$

$$x[n] = 1 + 3.6\cos(\tfrac{24\pi}{72}n + \tfrac{\pi}{4}) + 2\cos(\tfrac{120\pi}{72}n - \tfrac{\pi}{4}) = 1 + 3.6\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4}) + 2\cos(\tfrac{5\pi}{3}n - \tfrac{\pi}{4})$$
$$= 1 + 3.6\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4}) + 2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4}) = 1 + 5.6\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4})$$

$$x[n] = 1 + 5.6\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4})$$

$x[n] = 1 + 5.6\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4})$

(b) The LTI system is specified by the difference equation: $y[n] = x[n-13] + x[n-15]$. Obtain $y(t)$.

$$H(e^{j\hat{\omega}}) = e^{-j13\hat{\omega}} + e^{-j15\hat{\omega}} = 2e^{-j14\hat{\omega}}\frac{e^{j\hat{\omega}} + e^{-j\hat{\omega}}}{2} = 2e^{-j14\hat{\omega}}\cos(\hat{\omega})$$

$$H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=0} = 2e^{-j2\bullet 0}\cos(0) = 2 \qquad H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\frac{\pi}{3}} = 2e^{-j14\bullet\pi/3}\cos(\tfrac{\pi}{3}) = e^{-j\frac{2\pi}{3}}$$

$$y[n] = 1\bullet 2 + (5.6\bullet 1)\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4} - \tfrac{2\pi}{3}) = 2 + 5.6\cos(\tfrac{\pi}{3}n - \tfrac{5\pi}{12})$$

To obtain $y(t)$, replace n with $f_s t$, i.e., $72t$

$$y(t) = 2 + 5.6\cos(\tfrac{\pi}{3}72t - \tfrac{5\pi}{12}) = 2 + 5.6\cos(24\pi t - \tfrac{5\pi}{12})$$

$y(t) = 2 + 5.6\cos(24\pi t - \tfrac{5\pi}{12})$

PROBLEM Fall-14-Q.4.3:

A sampled sequence is given: $x[n] = 2\cos(17\pi n/256 - \pi/3) + 6\cos(27\pi n/128)$ for n = 0, 1, 2, …, 2047. Now, we take the first quarter of the sequence, starting from n = 0, and compute its 512-point DFT. List all non-zero DFT coefficients, $X[k]$, both in terms of their indices k and the corresponding coefficient values.

(extend the table if necessary)

k	$X[k]$
17	$512e^{-j\frac{\pi}{3}}$
495	$512e^{j\frac{\pi}{3}}$
54	1536
458	1536

First need to make sure of the periodicity of the sequence. There is a common factor of $\frac{\pi}{256}$ among $(\frac{17\pi}{256}, \frac{27\pi}{128}, 2\pi)$. Therefore, the sequence has a period of 512 ($=\frac{2\pi\bullet 256}{\pi}$). Taking the first quarter, i.e., 512 points, would cover exactly one full period. So, we can just pursue the DFS and make equivalence to find the corresponding DFT coefficients.

$$x[n] = 2\cos(17\pi n/256 - \pi/3) + 6\cos(27\pi n/128)$$

$$= 2\cos\left(\frac{2\bullet 17\pi}{2\bullet 256}n - \frac{\pi}{3}\right) + 6\cos\left(\frac{4\bullet 27\pi}{4\bullet 128}n\right) = 2\cos\left(\frac{2\pi\bullet 17}{512}n - \frac{\pi}{3}\right) + 6\cos\left(\frac{2\pi\bullet 54}{512}n\right)$$

$$= e^{j\left(\frac{2\pi\bullet 17}{512}n - \frac{\pi}{3}\right)} + e^{-j\left(\frac{2\pi\bullet 17}{512}n - \frac{\pi}{3}\right)} + 3e^{j\left(\frac{2\pi\bullet 54}{512}n\right)} + 3e^{-j\left(\frac{2\pi\bullet 54}{512}n\right)}$$

$$= e^{j\left(\frac{2\pi\bullet 17}{512}n - \frac{\pi}{3}\right)} + e^{j\left(\frac{2\pi\bullet 495}{512}n + \frac{\pi}{3}\right)} + 3e^{j\left(\frac{2\pi\bullet 54}{512}n\right)} + 3e^{j\left(\frac{2\pi\bullet 458}{512}n\right)}$$

$$= \frac{1}{512}\left\{512\left(e^{j\left(\frac{2\pi\bullet 17}{512}n - \frac{\pi}{3}\right)} + e^{j\left(\frac{2\pi\bullet 495}{512}n + \frac{\pi}{3}\right)} + 3e^{j\left(\frac{2\pi\bullet 54}{512}n\right)} + 3e^{j\left(\frac{2\pi\bullet 458}{512}n\right)}\right)\right\}$$

The final step above accounts for the scalar factor of $\frac{1}{512}$ in the front of the summation of all complex exponentials for a 512 point IDFT. The DFS coefficients must be scaled up by 512.

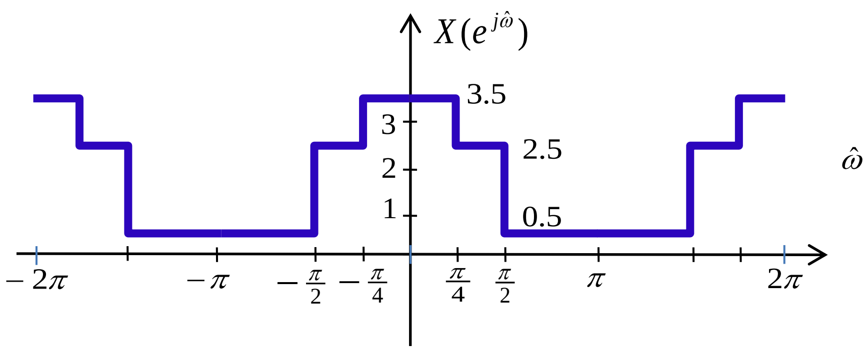
PROBLEM Fall-14-Q.4.4:

Find the Discrete-time Fourier transform (DTFT) $X(e^{j\hat{\omega}})$ of the sequence:

$$x[n] = \left(\frac{\sin(0.25\pi n)}{\pi n} + 2\delta[n]\right) * \left(\frac{\sin(0.5\pi n)}{\pi n}\right) + 0.5\delta[n]$$

You only need to provide values of the transform at three frequencies: $\hat{\omega} = 0, \hat{\omega} = \frac{\pi}{3}$, and $\hat{\omega} = \pi$.

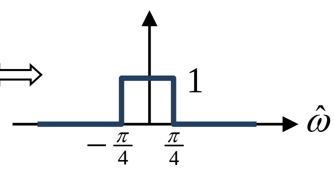
$X(e^{j\hat{\omega}})\Big\vert_{\hat{\omega}=0} = 3.5$	$X(e^{j\hat{\omega}})\Big\vert_{\hat{\omega}=\frac{\pi}{3}} = 2.5$	$X(e^{j\hat{\omega}})\Big\vert_{\hat{\omega}=\pi} = 0.5$



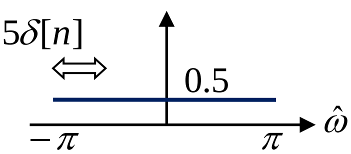
$$x[n] = \frac{\sin(0.25\pi n)}{\pi n} * \frac{\sin(0.5\pi n)}{\pi n} + 2\frac{\sin(0.5\pi n)}{\pi n} + 0.5\delta[n]$$

$$= \frac{\sin(0.25\pi n)}{\pi n} + 2\frac{\sin(0.5\pi n)}{\pi n} + 0.5\delta[n]$$

$\frac{\sin(0.25\pi n)}{\pi n} \Longleftrightarrow$



$0.5\delta[n] \Longleftrightarrow$



$2\frac{\sin(0.5\pi n)}{\pi n} \Longleftrightarrow$