GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Spring 2025
Quiz #3

April 7, 2025

NAME: ______________ ________________ GT username: ________________
(FIRST) (LAST) (e.g., gtxyz123)

Circle your recitation section:

L01 (Daniela)	L05 (Chun-Wei)	L07 (Chun-Wei)	L09 (Daniela)
L02 (Greg)	L06 (Kennedy)	L08 (Kennedy)	L10 (Greg)

Important Notes:

- Do not unstaple the test.
- Closed book, except for one two-sided page (8.5” × 11”) of hand-written notes.
- Calculators are allowed, but no other electronics (no smartphones/watches/readers/tablets/laptops/etc).
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
- Only the fronts of the each page will be scanned and graded.

Problem	Value	Score Earned
1	32	
2	36	
3	32	
Total		

PROB. Sp25-Q3.1. An FIR filter is described by the difference equation:

$$y[n] = Ax[n-1] + Bx[n-2] + Ax[n-3] + Bx[n-4] + Ax[n-5].$$

The constants A and B are unspecified and different in the two parts below.

(a) Suppose $A = 1$ and $B = 1$. What digital frequencies does this filter null?
Express your answer by listing *all* of the value(s) of $\hat{\omega} \in [0, \pi]$,
if any, for which the output in response to an input of $\cos(\hat{\omega} n)$ is zero for all time:

response to $\cos(\hat{\omega} n)$ is *zero* when $\hat{\omega} \in \{ \quad \}$.

$0 \leq \hat{\omega} \leq \pi$ only

(b) Find A and B if the output in response to:

$$x[n] = 2 + 2\cos(\pi(n-1))$$

is

$$y[n] = 24 \quad \text{(for all } n\text{, a constant)}.$$

$A =$,

$B =$.

PROB. Sp25-Q3.2. Match each discrete-time signal on the left to its corresponding DTFT plot on the right. (The y-axis scales may differ, they are not labeled, the shapes are enough.) Indicate your answer by writing a letter from {A, B, ... L} into each answer box.

$$x[n] = 8\frac{\sin(0.7\pi n)}{\pi n} + \frac{\sin(0.5\pi(n-8))}{\pi(n-8)} + \frac{\sin(0.5\pi(n+8))}{\pi(n+8)}$$

$$x[n] = 8\frac{\sin(0.7\pi n)}{\pi n} + \frac{\sin(0.5\pi(n-2))}{\pi(n-2)} + \frac{\sin(0.5\pi(n+2))}{\pi(n+2)}$$

$$x[n] = \frac{\sin(0.7\pi n)}{\pi n} + \frac{\sin(0.5\pi n)}{\pi n}$$

$$x[n] = \delta[n] - \frac{\sin(0.7\pi n)}{\pi n}$$

$$x[n] = \sum_{k=-\infty}^{\infty} \frac{\sin(0.6\pi k)\sin(0.8\pi(n-k))}{\pi^2 k(n-k)}$$

$$x[n] = \cos(0.4\pi n)\frac{\sin(0.4\pi n)}{\pi n}$$

$$x[n] = \cos(0.4\pi n)\frac{\sin(0.2\pi n)}{\pi n}$$

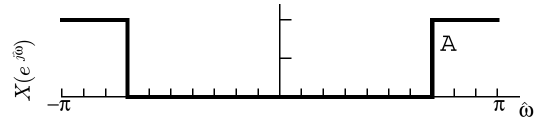
$$x[n] = \cos(0.8\pi n)\frac{\sin(0.2\pi n)}{\pi n}$$

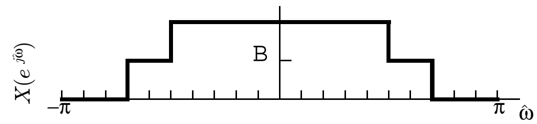
$$x[n] = (-1)^n \frac{\sin(0.2\pi n)}{\pi n}$$

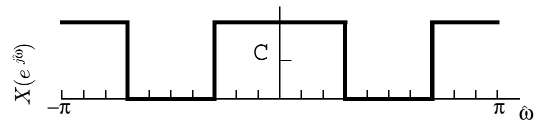
$$x[n] = \delta[n] - \frac{\sin(0.7\pi n)}{\pi n} + \frac{\sin(0.5\pi n)}{\pi n}$$

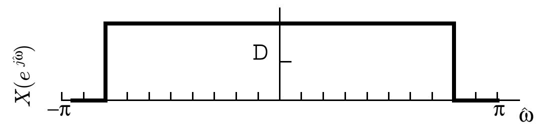
$$x[n] = \delta[n] - 2\cos(0.5\pi n)\frac{\sin(0.2\pi n)}{\pi n}$$

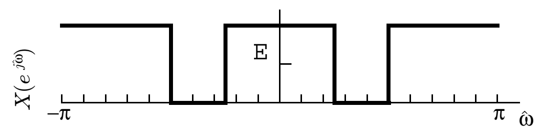
$$x[n] = \frac{\sin(0.25\pi n)}{\pi n}(1 + e^{j0.75\pi n} + e^{-j0.75\pi n})$$

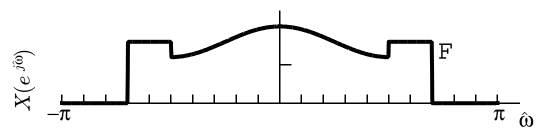


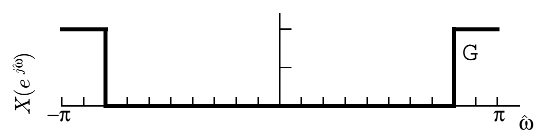


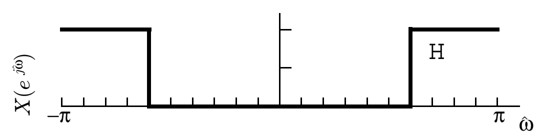


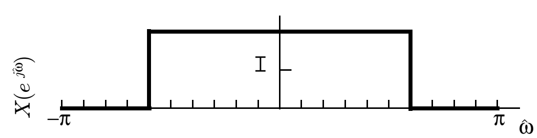


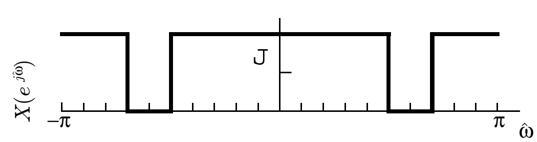


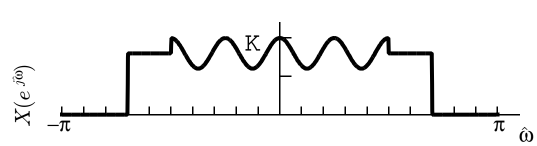


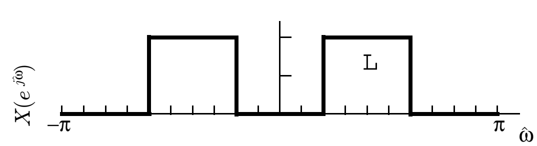












PROB. Sp25-Q3.3. (The different parts of this problem are unrelated.)

(a) The 3-point DFT of $\big[x[0],\ x[1],\ x[2]\big] = [\,5,\ -1,\ -1\,]$ is:

$X[0]$	$X[1]$	$X[2]$

(b) Suppose that the k-th coefficient after taking the 32-point DFT of $\big[x[0],\ \ldots\ x[31]\big]$ is $X[k] = (j)^k$.
List *all* of the value(s) of $n \in \{0,\ \ldots\ 31\}$, if any, for which $x[n] \neq 0$:

$x[n] \neq 0$ when $n \in \Big\{$ $\Big\}$.

(c) The 4-point DFT of $\big[x[0],\ \ldots\ x[3]\big] = [\,5,\ 7,\ 1,\ 7\,]$ is $\big[X[0],\ \ldots\ X[3]\big] = [\,20,\ 4,\ -8,\ 4\,]$.

Find the 4-point DFT of $\big[y[0],\ \ldots\ y[3]\big] = [\,7,\ 5,\ 7,\ 1\,]$.

(*Hint:* use the DFT delay property.)

$Y[0]$	$Y[1]$	$Y[2]$	$Y[3]$

Table of DTFT Pairs	
Time-Domain: $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\hat{\omega}n_0}$
$r_L[n] = u[n] - u[n-L]$	$\dfrac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}\, e^{-j\hat{\omega}(L-1)/2}$
$r_L[n]e^{j\hat{\omega}_0 n}$	$\dfrac{\sin(\frac{1}{2}L(\hat{\omega}-\hat{\omega}_0))}{\sin(\frac{1}{2}(\hat{\omega}-\hat{\omega}_0))}\, e^{-j(\hat{\omega}-\hat{\omega}_0)(L-1)/2}$
$\dfrac{\sin(\hat{\omega}_b n)}{\pi n}$	$u(\hat{\omega}+\hat{\omega}_b) - u(\hat{\omega}-\hat{\omega}_b) = \begin{cases} 1 & \lvert\hat{\omega}\rvert \le \hat{\omega}_b \\ 0 & \hat{\omega}_b < \lvert\hat{\omega}\rvert \le \pi \end{cases}$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\dfrac{1}{1 - ae^{-j\hat{\omega}}}$

Table of DTFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
Periodic in $\hat{\omega}$		$X(e^{j(\hat{\omega}+2\pi)}) = X(e^{j\hat{\omega}})$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\hat{\omega}}) + bX_2(e^{j\hat{\omega}})$
Conjugate Symmetry	$x[n]$ is real	$X(e^{-j\hat{\omega}}) = X^*(e^{j\hat{\omega}})$
Conjugation	$x^*[n]$	$X^*(e^{-j\hat{\omega}})$
Time-Reversal	$x[-n]$	$X(e^{-j\hat{\omega}})$
Delay (n_d=integer)	$x[n-n_d]$	$e^{-j\hat{\omega}n_d}X(e^{j\hat{\omega}})$
Frequency Shift	$x[n]e^{j\hat{\omega}_0 n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_0)})$
Modulation	$x[n]\cos(\hat{\omega}_0 n)$	$\frac{1}{2}X(e^{j(\hat{\omega}-\hat{\omega}_0)}) + \frac{1}{2}X(e^{j(\hat{\omega}+\hat{\omega}_0)})$
Convolution	$x[n] * h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
Autocorrelation	$x[-n] * x[n]$	$\lvert X(e^{j\hat{\omega}})\rvert^2$
Parseval's Theorem	$\displaystyle\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$	$\displaystyle\frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\hat{\omega}})\rvert^2 d\hat{\omega}$

Date: 28-Apr-2013

Table of Pairs for N-point DFT	
Time-Domain: $x[n], \quad n=0,1,2,\ldots,N-1$	*Frequency-Domain:* $X[k], \quad k=0,1,2,\ldots,N-1$
If $x[n]$ is finite length, i.e., $x[n]\neq 0$ only when $n\in[0,N-1]$ and the DTFT of $x[n]$ is $X(e^{j\hat{\omega}})$	$X[k]=X(e^{j\hat{\omega}})\Big\vert_{\hat{\omega}=2\pi k/N}$ (frequency sampling the DTFT)
$\delta[n]$	1
1	$N\delta[k]$
$\delta[n-n_0]$	$e^{-j(2\pi k/N)n_0}$
$e^{j(2\pi n/N)k_0}$	$N\delta[k-k_0]$, when $k_0\in[0,N-1]$
$r_L[n]=u[n]-u[n-L]$, when $L\leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi k/N))}{\sin(\frac{1}{2}(2\pi k/N))}\,e^{-j(2\pi k/N)(L-1)/2}$
$r_L[n]e^{j(2\pi k_0/N)n}$, when $L\leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi(k-k_0)/N))}{\sin(\frac{1}{2}(2\pi(k-k_0)/N))}\,e^{-j(2\pi(k-k_0)/N)(L-1)/2}$
$\dfrac{\sin(\frac{1}{2}L(2\pi n/N))}{\sin(\frac{1}{2}(2\pi n/N))}\,e^{j(2\pi n/N)(L-1)/2}$	$N(u[k]-u[k-L])$, when $L\leq N$

Table of DFT Properties		
Property Name	*Time-Domain:* $x[n]$ for $n\in\{0,\ldots N-1\}$	*Frequency-Domain:* $X[k]$
Periodic	periodic extension $\tilde{x}[n]=\tilde{x}[n+N]$	$X[k]=X[k+N]$
Linearity	$ax_1[n]+bx_2[n]$	$aX_1[k]+bX_2[k]$
Conjugate Symmetry	$x[n]$ is real	$X[N-k]=X^*[k]$
Conjugation	$x^*[n]$	$X^*[N-k]$
Time-Reversal	$x[N-n]$	$X[N-k]$
Delay (PERIODIC)	$\tilde{x}[n-n_d]$	$e^{-j(2\pi k/N)n_d}X[k]$
Frequency Shift	$x[n]e^{j(2\pi k_0/N)n}$	$X[k-k_0]$
Modulation	$x[n]\cos((2\pi k_0/N)n)$	$\frac{1}{2}X[k-k_0]+\frac{1}{2}X[k+k_0]$
Convolution (PERIODIC)	$\tilde{x}[n]*h[n]=\sum_{m=0}^{N-1}h[m]\tilde{x}[n-m]$	$X[k]H[k]$
Parseval's Theorem	$\displaystyle\sum_{n=0}^{N-1}\vert x[n]\vert^2=\frac{1}{N}\sum_{k=0}^{N-1}\vert X[k]\vert^2$	

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Circle your recitation section:

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L02 (Greg)	L06 (Kennedy)	L08 (Kennedy)	L10 (Greg)

Important Notes:

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Total		

PROB. Sp25-Q3.1. An FIR filter is described by the difference equation:

$$y[n] = Ax[n-1] + Bx[n-2] + Ax[n-3] + Bx[n-4] + Ax[n-5].$$

The constants A and B are unspecified and different in the two parts below.

(a) Suppose $A = 1$ and $B = 1$. What digital frequencies does this filter null?
Express your answer by listing *all* of the value(s) of $\hat{\omega} \in [0, \pi]$,
if any, for which the output in response to an input of $\cos(\hat{\omega} n)$ is zero for all time:

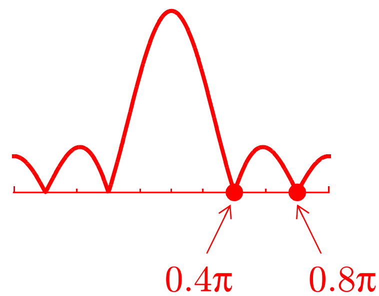
response to $\cos(\hat{\omega} n)$ is *zero* when $\hat{\omega} \in \{$ 0.4π, 0.8π $\}$.

$0 \le \hat{\omega} \le \pi$ only

From table:

$r_L[n] = u[n] - u[n-L]$	$\frac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j\hat{\omega}(L-1)/2}$

$$\Rightarrow H(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}} \frac{\sin(2.5\hat{\omega})}{\sin(0.5\hat{\omega})},$$



which is zero when the numerator is zero
and the denominator is not,
i.e. when $2.5\hat{\omega} = m\pi$ for $m = 1$ or $2 \Rightarrow \hat{\omega} = m0.4\pi$

(b) Find A and B if the output in response to:

$$x[n] = 2 + 2\cos(\pi(n-1))$$

is

$$y[n] = 24 \quad \text{(for all } n\text{, a constant)}.$$

$A =$ 2 ,

$B =$ 3 .

The constant 2 becomes 24 $\Rightarrow$ dc gain is $H(e^{j0}) = 12$.
The sinusoid at $\hat{\omega} = \pi$ is nulled $\Rightarrow$ $H(e^{j\pi}) = 0$.

Evaluating $H(e^{j\hat{\omega}}) = Ae^{-j\hat{\omega}} + Be^{-j2\hat{\omega}} + Ae^{-j3\hat{\omega}} + Be^{-j4\hat{\omega}} + Ae^{-j5\hat{\omega}}$

- at $\hat{\omega} = 0 \Rightarrow \quad 3A + 2B = 12$
- at $\hat{\omega} = \pi \Rightarrow \quad -3A + 2B = 0$

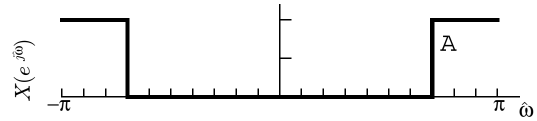
❶ $\longrightarrow A = (2/3)B$

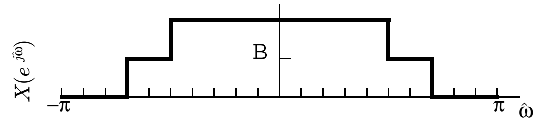
❷ $3(2/3)B + 2B = 12 \Rightarrow B = 3$

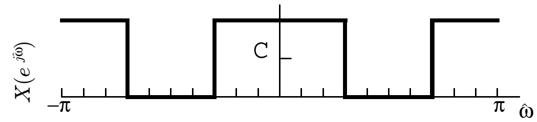
❸ $A = 2$

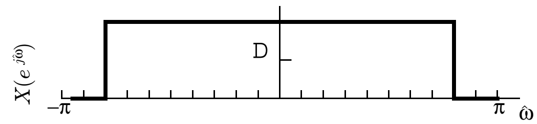
PROB. Sp25-Q3.2. Match each discrete-time signal on the left to its corresponding DTFT plot on the right. (The y-axis scales may differ, they are not labeled, the shapes are enough.) Indicate your answer by writing a letter from {A, B, ... P} into each answer box.

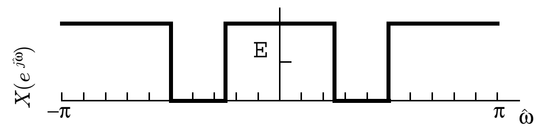
K	$x[n] = 8\dfrac{\sin(0.7\pi n)}{\pi n} + \dfrac{\sin(0.5\pi(n-8))}{\pi(n-8)} + \dfrac{\sin(0.5\pi(n+8))}{\pi(n+8)}$
F	$x[n] = 8\dfrac{\sin(0.7\pi n)}{\pi n} + \dfrac{\sin(0.5\pi(n-2))}{\pi(n-2)} + \dfrac{\sin(0.5\pi(n+2))}{\pi(n+2)}$
B	$x[n] = \dfrac{\sin(0.7\pi n)}{\pi n} + \dfrac{\sin(0.5\pi n)}{\pi n}$
A	$x[n] = \delta[n] - \dfrac{\sin(0.7\pi n)}{\pi n}$
I	$x[n] = \sum_{k=-\infty}^{\infty} \dfrac{\sin(0.6\pi k)\sin(0.8\pi(n-k))}{\pi^2 k(n-k)}$
D	$x[n] = \cos(0.4\pi n)\dfrac{\sin(0.4\pi n)}{\pi n}$
L	$x[n] = \cos(0.4\pi n)\dfrac{\sin(0.2\pi n)}{\pi n}$
H	$x[n] = \cos(0.8\pi n)\dfrac{\sin(0.2\pi n)}{\pi n}$
G	$x[n] = (-1)^n \dfrac{\sin(0.2\pi n)}{\pi n}$
J	$x[n] = \delta[n] - \dfrac{\sin(0.7\pi n)}{\pi n} + \dfrac{\sin(0.5\pi n)}{\pi n}$
C	$x[n] = \delta[n] - 2\cos(0.5\pi n)\dfrac{\sin(0.2\pi n)}{\pi n}$
E	$x[n] = \dfrac{\sin(0.25\pi n)}{\pi n}(1 + e^{j0.75\pi n} + e^{-j0.75\pi n})$

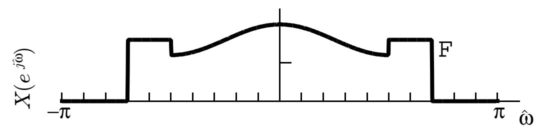


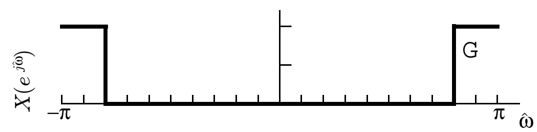


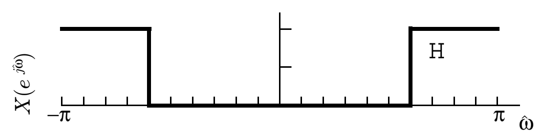


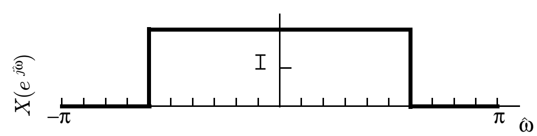


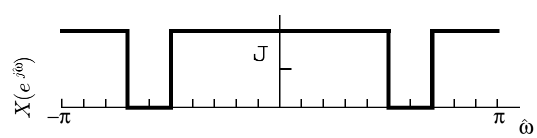


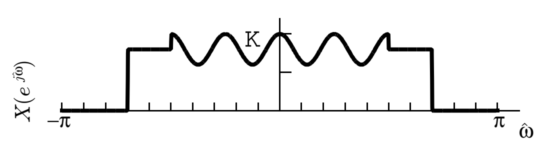


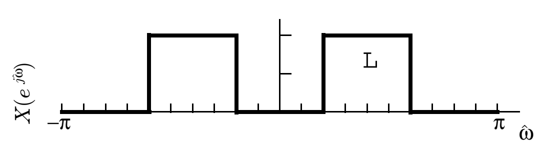












PROB. Sp25-Q3.3. (The different parts of this problem are unrelated.)

(a) The 3-point DFT of $[x[0], x[1], x[2]] = [5, -1, -1]$ is:

$X[0]$	$X[1]$	$X[2]$
3	6	6

Start with DTFT: $X(e^{j\hat{\omega}}) = 5 - e^{-j\hat{\omega}} - e^{-j2\hat{\omega}}$,
then sample:

$$X[0] = X(e^{j\hat{\omega}})\big|_{\hat{\omega}=0} = 5 - 1 - 1 = 3$$

$$X[1] = X(e^{j\hat{\omega}})\big|_{\hat{\omega}=2\pi/3} = 5 - e^{-j2\pi/3} - e^{-j4\pi/3} = 6$$

real in time domain $\Rightarrow X[2] = X^*[1] = 6$

(b) Suppose that the k-th coefficient after taking the 32-point DFT of $[x[0], \ldots x[31]]$ is $X[k] = (j)^k$.
List *all* of the value(s) of $n \in \{0, \ldots 31\}$, if any, for which $x[n] \neq 0$:

$x[n] \neq 0$ when $n \in \{$ 24 $\}$.

$$X[k] = (j)^k = e^{j0.5\pi k} = e^{-j1.5\pi k} = e^{-j2\pi(24)k/32} = \sum_{n=0}^{31} x[n]e^{-j2\pi(n)k/32}$$

equate terms
$\Rightarrow x[24] = 1$, all others zero

(c) The 4-point DFT of $[x[0], \ldots x[3]] = [5, 7, 1, 7]$ is $[X[0], \ldots X[3]] = [20, 4, -8, 4]$.

Find the 4-point DFT of $[y[0], \ldots y[3]] = [7, 5, 7, 1]$.

(*Hint:* use the DFT delay property.)

$Y[0]$	$Y[1]$	$Y[2]$	$Y[3]$
20	$-4j$	8	$4j$

From table with $n_d = 1$ and $N = 4$:

Delay (PERIODIC)	$\tilde{x}[n - n_d]$	$e^{-j(2\pi k/N)n_d} X[k]$

$$\Rightarrow Y[k] = e^{-j(2\pi k/4)} X[k] = (-j)^k X[k]:$$

$$\Rightarrow Y[0] = X[0]$$

$$Y[1] = -jX[1]$$

$$Y[2] = -X[2]$$

$$Y[3] = jX[3]$$