GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Spring 2023
Quiz #3

April 10, 2023

NAME: ______________ ________________ GT username: ________________
(FIRST) (LAST) (e.g., gtbuzz2026)

Circle your recitation section:

L01 (Chen)	L07 (Davenport)	L09 (Hessler)	L11 (Hessler)
L02 (Duan)	L08 (Duan)	L10 (Chen)	

Important Notes:

- Do not unstaple the test.
- Closed book, except for one two-sided page (8.5” $\times$ 11”) of hand-written notes.
- Calculators are allowed, but no other electronics (no smartphones/watches/readers/tablets/laptops/etc).
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	36	
2	36	
3	28	
Total		

PROB. Sp23-Q3.1. An FIR filter is described by the difference equation:

$$y[n] = 2x[n-1] + \beta x[n-2] + 2x[n-3].$$

Match each input below to the value of β that results in a *zero* output ($y[n] = 0$ for all n). Indicate your answers by writing a letter (from $\{$A, B, ... I$\}$) in each answer box:

(A) $x[n] = \sqrt{3}\cos(\pi(n-1)/6 + \pi/3)$		$\beta = -4$
(B) $x[n] = \sqrt{12}\cos(\pi n/3 + 0.125\pi)$		$\beta = -\sqrt{12}$
(C) $x[n] = \cos(0.75\pi(n-1)) + \cos(1.25\pi(n+1))$		$\beta = -\sqrt{8}$
(D) $x[n] = \sqrt{3}\cos(17\pi(n-1)/6 - \pi/3)$		$\beta = -2$
(E) $x[n] = 2\sin^2(0.25\pi(n-1) + \pi/3) - 1$		$\beta = 0$
(F) $x[n] = \sqrt{12}(-1)^{n-4}$		$\beta = 2$
(G) $x[n] = \sqrt{8}(je^{j\pi(n-1)/4} - je^{-j\pi(n-1)/4})$		$\beta = \sqrt{8}$
(H) $x[n] = \sqrt{12}\cos(2\pi(n-1)/3 + 5\pi/6)$		$\beta = \sqrt{12}$
(I) $x[n] = 4\cos^2(0.25\pi n + \pi/5) + 4\sin^2(0.25\pi n + \pi/5)$		$\beta = 4$

PROB. Sp23-Q3.2. On the left below are several discrete-time signals. On the right are the corresponding DTFT, in a scrambled order. Match each signal to its corresponding DTFT by writing a letter from $\{$A, B, ... M$\}$ into each answer box.

$$x[n] = \cos(0.2\pi n)\frac{\sin(0.4\pi n)}{\pi n}$$

$$x[n] = \frac{\sin(0.4\pi n)}{\pi n}$$

$$x[n] = \delta[n] - \frac{\sin(0.8\pi n)}{\pi n} + \frac{\sin(0.2\pi n)}{\pi n}$$

$$x[n] = \sum_{k=-\infty}^{\infty} \frac{\sin(0.2\pi k)\sin(0.4\pi(n-k))}{\pi^2 k(n-k)}$$

$$x[n] = \big(\cos(0.1\pi n) + \cos(\pi n)\big)\frac{\sin(0.4\pi n)}{\pi n}$$

$$x[n] = \delta[n] - \frac{\sin(0.6\pi n)}{\pi n}$$

$$x[n] = \frac{\sin(0.2\pi n)}{2\pi n} + \frac{\sin(0.4\pi n)}{2\pi n}$$

$$x[n] = \delta[n] - \frac{\sin(0.6\pi n)}{\pi n} + \frac{\sin(0.2\pi n)}{\pi n}$$

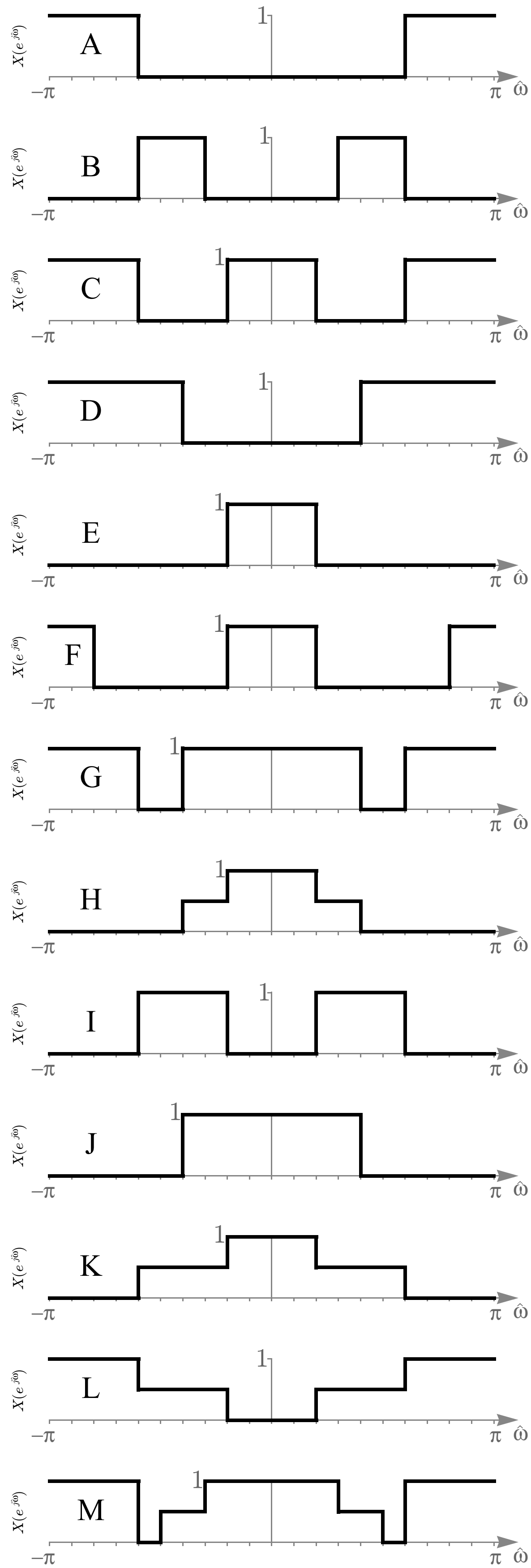
$$x[n] = 2\cos(0.4\pi n)\frac{\sin(0.2\pi n)}{\pi n}$$

$$x[n] = \delta[n] - \cos(0.2\pi n)\frac{\sin(0.4\pi n)}{\pi n}$$

$$x[n] = \cos(\pi n)\frac{\sin(0.6\pi n)}{\pi n}$$

$$x[n] = \big(1 + \cos(\pi n)\big)\frac{\sin(0.4\pi n)}{\pi n}$$

$$x[n] = \frac{\sin(0.6\pi n)}{\pi n} - \frac{\sin(0.3\pi n)}{\pi n}$$



PROB. Sp23-Q3.3.

(a) The 2-point DFT of $[x[0], x[1]] = [3, 4]$ is:

$X[0]$	$X[1]$

(b) Let $X[k]$ be the k-th coefficient after taking the 1000-point DFT of $[x[0], \ldots x[4]] = [1, 1, 1, 1, 1]$. List *all* of the value(s) of $k \in \{0, \ldots 999\}$, if any, for which $X[k] = 0$:

$X[k] = 0$ when $k \in \{$ $\}$.

(c) If the 20-point DFT of $[x[0], \ldots x[5]]$ satisfies $X[k] = e^{-jk0.3\pi}(5 + 2\cos(0.2\pi k))$ for $k \in \{0, \ldots 19\}$, then:

$x[0]$	$x[1]$	$x[2]$	$x[3]$	$x[4]$	$x[5]$

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(FIRST) (LAST) (e.g., gtbuzz2026)

Circle your recitation section:

L01 (Chen)	L07 (Davenport)	L09 (Hessler)	L11 (Hessler)
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Important Notes:

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$$|H(e^{j\hat{\omega}_0})| = |2e^{-j\hat{\omega}_0} + \beta e^{-2j\hat{\omega}_0} + 2e^{-3j\hat{\omega}_0}|$$
$$= |e^{-2j\hat{\omega}_0}(\beta + 4\cos(\hat{\omega}_0))| = 0$$
when $\beta = -4\cos(\hat{\omega}_0)$

PROB. Sp23-Q3.1. An FIR filter is described by the difference equation:

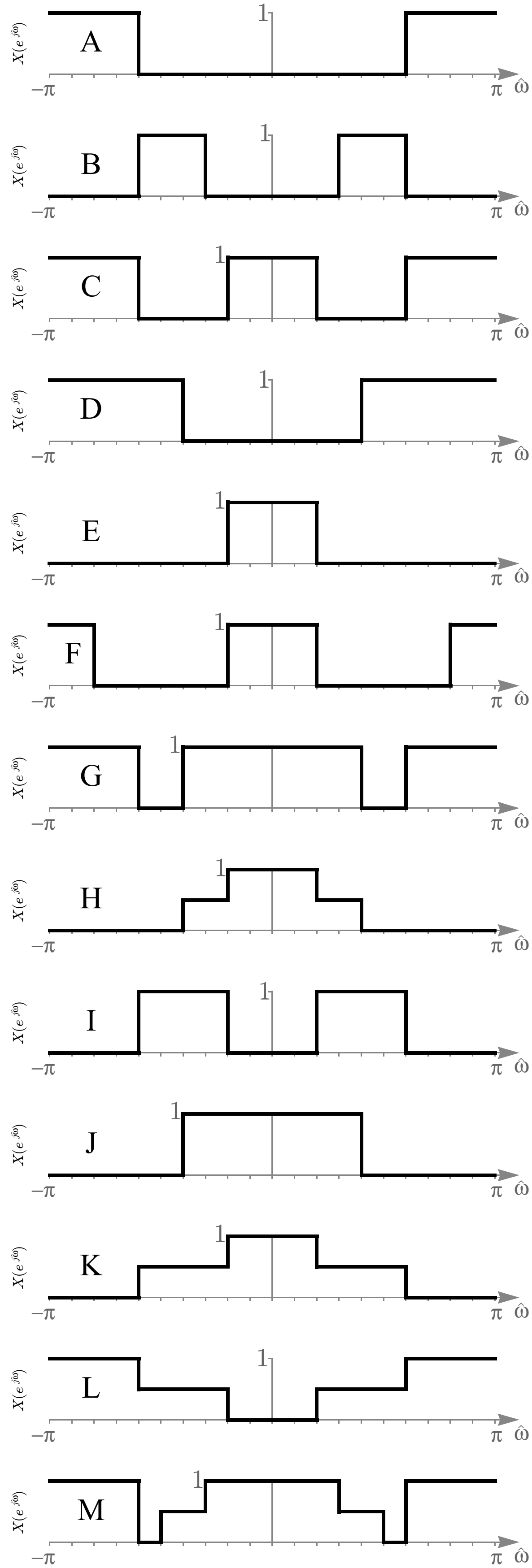
$$y[n] = 2x[n-1] + \beta x[n-2] + 2x[n-3].$$

Match each input below to the value of β that results in a *zero* output ($y[n] = 0$ for all n). Indicate your answers by writing a letter (from $\{$A, B, ... I$\}$) in each answer box:

Input	Answer	β	$\hat{\omega}_0 = \cos^{-1}(\frac{-\beta}{4})$
(A) $x[n] = \sqrt{3}\cos(\pi(n-1)/6 + \pi/3)$ $\hat{\omega}_0 = \pi/6$	I	$\beta = -4$	0
(B) $x[n] = \sqrt{12}\cos(\pi n/3 + 0.125\pi)$ $\hat{\omega}_0 = \pi/3$	A	$\beta = -\sqrt{12}$	$\pi/6$
(C) $x[n] = \cos(0.75\pi(n-1)) + \cos(1.25\pi(n+1))$ $\hat{\omega}_0 = 0.75\pi$	G	$\beta = -\sqrt{8}$	0.25π
(D) $x[n] = \sqrt{3}\cos(17\pi(n-1)/6 - \pi/3)$ $\hat{\omega}_0 = 5\pi/6$	B	$\beta = -2$	$\pi/3$
(E) $x[n] = 2\sin^2(0.25\pi(n-1) + \pi/3) - 1$ $\hat{\omega}_0 = 0.5\pi$	E	$\beta = 0$	0.5π
(F) $x[n] = \sqrt{12}(-1)^{n-4}$ $\hat{\omega}_0 = \pi$	H	$\beta = 2$	$2\pi/3$
(G) $x[n] = \sqrt{8}(je^{j\pi(n-1)/4} - je^{-j\pi(n-1)/4})$ $\hat{\omega}_0 = 0.25\pi$	C	$\beta = \sqrt{8}$	0.75π
(H) $x[n] = \sqrt{12}\cos(2\pi(n-1)/3 + 5\pi/6)$ $\hat{\omega}_0 = 2\pi/3$	D	$\beta = \sqrt{12}$	$5\pi/6$
(I) $x[n] = 4\cos^2(0.25\pi n + \pi/5) + 4\sin^2(0.25\pi n + \pi/5)$ $\hat{\omega}_0 = 0$	F	$\beta = 4$	π

PROB. Sp23-Q3.2. On the left below are several discrete-time signals. On the right are the corresponding DTFT, in a scrambled order. Match each signal to its corresponding DTFT by writing a letter from {A, B, ... M} into each answer box.

K	$x[n] = \cos(0.2\pi n)\frac{\sin(0.4\pi n)}{\pi n}$
J	$x[n] = \frac{\sin(0.4\pi n)}{\pi n}$
F	$x[n] = \delta[n] - \frac{\sin(0.8\pi n)}{\pi n} + \frac{\sin(0.2\pi n)}{\pi n}$
E	$x[n] = \sum_{k=-\infty}^{\infty}\frac{\sin(0.2\pi k)\sin(0.4\pi(n-k))}{\pi^2 k(n-k)}$
M	$x[n] = \big(\cos(0.1\pi n) + \cos(\pi n)\big)\frac{\sin(0.4\pi n)}{\pi n}$
A	$x[n] = \delta[n] - \frac{\sin(0.6\pi n)}{\pi n}$
H	$x[n] = \frac{\sin(0.2\pi n)}{2\pi n} + \frac{\sin(0.4\pi n)}{2\pi n}$
C	$x[n] = \delta[n] - \frac{\sin(0.6\pi n)}{\pi n} + \frac{\sin(0.2\pi n)}{\pi n}$
I	$x[n] = 2\cos(0.4\pi n)\frac{\sin(0.2\pi n)}{\pi n}$
L	$x[n] = \delta[n] - \cos(0.2\pi n)\frac{\sin(0.4\pi n)}{\pi n}$
D	$x[n] = \cos(\pi n)\frac{\sin(0.6\pi n)}{\pi n}$
G	$x[n] = \big(1 + \cos(\pi n)\big)\frac{\sin(0.4\pi n)}{\pi n}$
B	$x[n] = \frac{\sin(0.6\pi n)}{\pi n} - \frac{\sin(0.3\pi n)}{\pi n}$



PROB. Sp23-Q3.3.

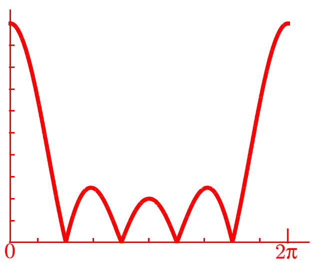
(a) The 2-point DFT of $[x[0], x[1]] = [3, 4]$ is:

$X[0]$	$X[1]$
7	–1

Sample $3 + 4e^{-j\hat{\omega}}$ at: $\hat{\omega} = 0 \Rightarrow X[0] = 7$

$\hat{\omega} = \pi \Rightarrow X[1] = -1$

(b) Let $X[k]$ be the k-th coefficient after taking the 1000-point DFT of $[x[0], \ldots x[4]] = [1, 1, 1, 1, 1]$. List *all* of the value(s) of $k \in \{0, \ldots 999\}$, if any, for which $X[k] = 0$:



$X[k] = 0$ when $k \in \{$ 200, 400, 600, 800 $\}$.

The DTFT $e^{-2j\hat{\omega}} \dfrac{\sin(2.5\hat{\omega})}{\sin(0.5\hat{\omega})}$ is zero at 4 places, when $\hat{\omega} \in \{0.4\pi, 0.8\pi, 1.2\pi, 1.6\pi\}$

$\Rightarrow$ After sampling at $\hat{\omega} = k2\pi/1000$, when $k \in \{200, 400, 600, 800\}$

(c) If the 20-point DFT of $[x[0], \ldots x[5]]$ satisfies $X[k] = e^{-jk0.3\pi}(5 + 2\cos(0.2\pi k))$ for $k \in \{0, \ldots 19\}$, then:

$x[0]$	$x[1]$	$x[2]$	$x[3]$	$x[4]$	$x[5]$
0	1	0	5	0	1

Substitute $\hat{\omega} = k2\pi/20 = k0.1\pi$

$$\Rightarrow X(e^{j\hat{\omega}}) = e^{-3j\hat{\omega}}(5 + 2\cos(2\hat{\omega})) = e^{-j\hat{\omega}} + 5e^{-3j\hat{\omega}} + e^{-5j\hat{\omega}}$$