GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 Spring 2022
Quiz #3

April 11, 2022

NAME: ____________ (FIRST) ______________ (LAST) GT username: ______________ (e.g., gtxyz123)

To earn 2 points, circle your recitation section:

L01 (Tai)	L07 (Tai)	L09 (Hessler)	L11 (Hessler)
L02 (Duan)	L08 (Sadiq)	L10 (Sadiq)	L12 (Duan)

Important Notes:

- Do not unstaple the test.
- One two-sided page (8.5” × 11”) of hand-written notes permitted.
- Calculators are allowed, but no smartphones/tablets/readers/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score
1	35	
2	33	
3	30	
RECITATION	2	
Total		

PROB. Sp22-Q3.1. (35points, 5 pts each answer)

An FIR filter is described by the difference equation:

$$y[n] = 4x[n] + 4x[n-1] + \beta x[n-2].$$

The unspecified parameter β is different in each part below.

(a) When $\beta = 0$, the frequency response can be written as $H(e^{j\hat{\omega}}) = Ae^{-jB\hat{\omega}}\cos(C\hat{\omega})$, where:

$A =$ ☐ , $B =$ ☐ , $C =$ ☐ .

(b) When $\beta = 4$, the *number* of distinct *nulling* frequencies (*i.e.*, frequencies in the range $\hat{\omega} \in (-\pi, \pi]$ for which $H(e^{j\hat{\omega}}) = 0$) is ☐ .

(c) If a constant input $x[n] = 1$ (for all n) results in a constant output $y[n] = 0.2$ (for all n), then:

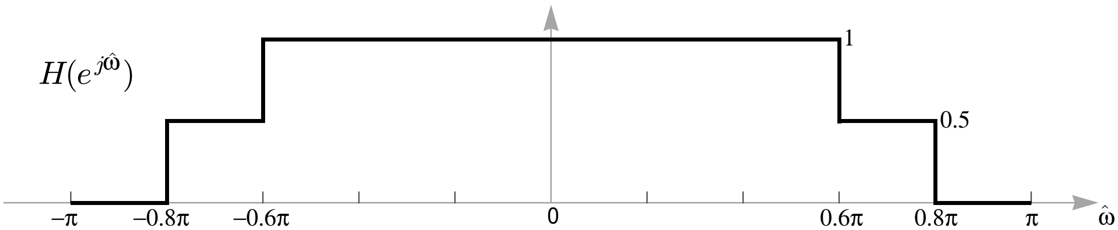
$\beta =$ ☐ .

(d) If the sinusoidal input $x[n] = \cos(0.5\pi n)$ results in an output of the form $y[n] = 5\cos(0.5\pi n + \theta)$, for some θ, then β must be one of two possible values. Specify them both: Either

$\beta =$ ☐ , or $\beta =$ ☐ .

PROB. Sp22-Q3.2. (33 points, 3pts each answer)

Consider an LTI filter whose frequency response is the real-valued function of $\hat{\omega}$ shown below:



TRUE FALSE

(a) ☐ ☐ The filter is FIR.

(b) When the input is $x[n] = \cos(0.7\pi n) + \cos(0.9\pi n)$, the output is $y[n] =$ ☐.

(c) The impulse response can be written as $h[n] = A\dfrac{\sin(B\pi n)}{\pi n} + C\dfrac{\sin(0.8\pi n)}{\pi n}$, where:

$A =$ ☐, $B =$ ☐, $C =$ ☐.

(d) The impulse response can be written as $h[n] = \dfrac{\sin(0.6\pi n)}{\pi n} + \cos(D\pi n)\dfrac{\sin(E\pi n)}{\pi n}$, where:

$D =$ ☐, $E =$ ☐.

(e) The impulse response can be written as $h[n] = \dfrac{\sin(0.8\pi n)}{\pi n} - \cos(F\pi n)\dfrac{\sin(G\pi n)}{\pi n}$, where:

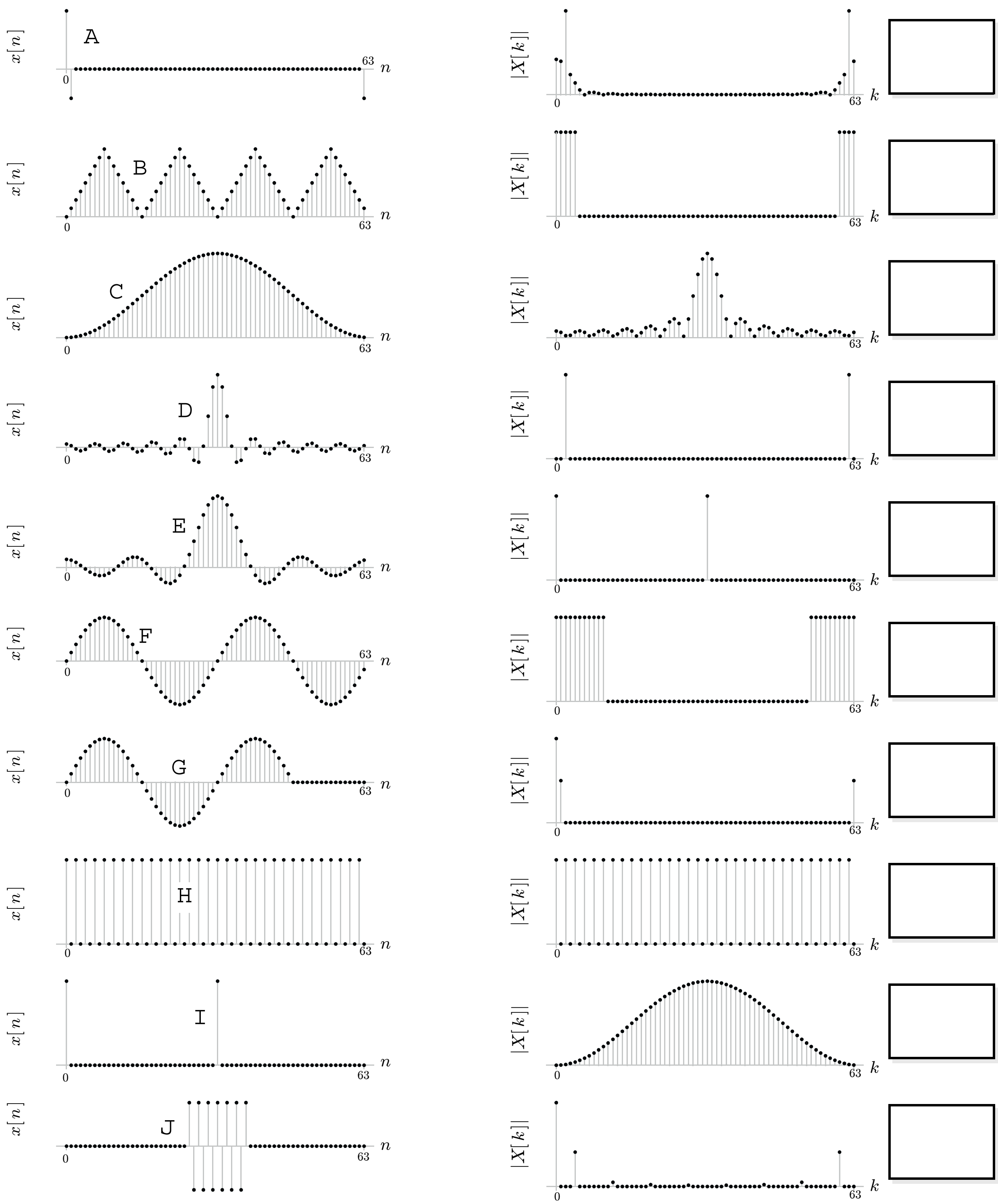
$F =$ ☐, $G =$ ☐.

(f) The impulse response can be written as $h[n] = K\cos(0.1\pi n)\dfrac{\sin(L\pi n)}{\pi n}$, where:

$K =$ ☐, $L =$ ☐.

PROB. Sp22-Q3.3. (30 points, 3 pts each answer)

Shown below on the left are the plots of ten different signal segments $[x[0], \ldots x[63]]$, labeled A through J, where each $x[n]$ is plotted versus $n \in \{0, 1, \ldots 63\}$. Let $[X[0], \ldots X[63]]$ be the $N = 64$-point DFT of $[x[0], \ldots x[63]]$. Shown on the right are the corresponding plots of the DFT magnitudes $|X[k]|$ versus $k \in \{0, 1, \ldots 63\}$, but in a scrambled order. Match each DFT magnitude plot to its corresponding signal segment by writing a letter (from A through J) into each of the ten answer boxes. (None of the y-axis scales are specified, they are not needed to solve the problem.)



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An FIR filter is described by the difference equation:

$$y[n] = 4x[n] + 4x[n-1] + \beta x[n-2].$$

The unspecified parameter β is different in each part below.

(a) When $\beta = 0$, the frequency response can be written as $H(e^{j\hat{\omega}}) = Ae^{-jB\hat{\omega}}\cos(C\hat{\omega})$, where:

$A = $ 8 , $B = $ 0.5 , $C = $ 0.5 .

$$H(e^{j\hat{\omega}}) = 4 + 4e^{-j\hat{\omega}} = 4e^{-j\hat{\omega}/2}(e^{j\hat{\omega}/2} + e^{-j\hat{\omega}/2})$$
$$= 4e^{-j\hat{\omega}/2}(2\cos(\hat{\omega}/2))$$

(b) When $\beta = 4$, the *number* of distinct *nulling* frequencies (*i.e.*, frequencies in the range $\hat{\omega} \in (-\pi, \pi]$ for which $H(e^{j\hat{\omega}}) = 0$) is 2 .

From table 1 with $L = 3$: $\frac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}e^{-j\hat{\omega}(L-1)/2}$ $\Rightarrow$ zero twice (when $\hat{\omega} = \pm 2\pi/3$)

(c) If a constant input $x[n] = 1$ (for all n) results in a constant output $y[n] = 0.2$ (for all n), then:

$\beta = $ −7.8 .

dc gain $= 4 + 4 + \beta = 0.2$

(d) If the sinusoidal input $x[n] = \cos(0.5\pi n)$ results in an output of the form $y[n] = 5\cos(0.5\pi n + \theta)$, for some θ, then β must be one of two possible values. Specify them both: Either

$\beta = $ 1 , or $\beta = $ 7 .

At frequency 0.5π, the frequency response evaluates to

$$H(e^{j0.5\pi}) = 4 + 4e^{-j0.5\pi} + \beta e^{-j\pi}$$
$$= 4 - \beta - 4j.$$

Set $|H(e^{j0.5\pi})| = 5$ and solve for β

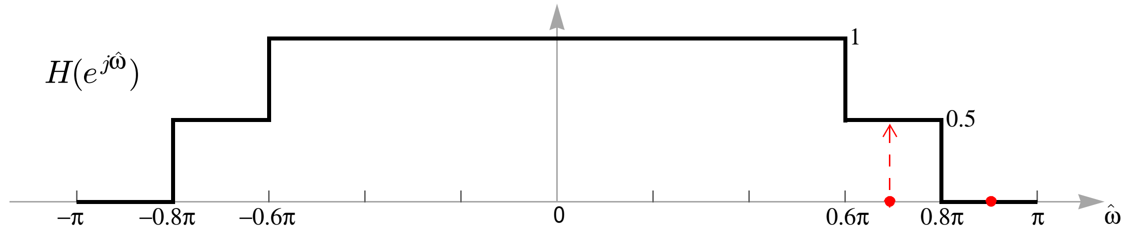
$$\Rightarrow 25 = |H(e^{j0.5\pi})|^2 = (4-\beta)^2 + (4)^2$$

$$\Rightarrow (4-\beta)^2 = 25 - 16 = 9$$

$$\Rightarrow 4 - \beta = \pm 3$$

PROB. Sp22-Q3.2. (33 points, 3pts each answer)

Consider an LTI filter whose frequency response is the real-valued function of $\hat{\omega}$ shown below:



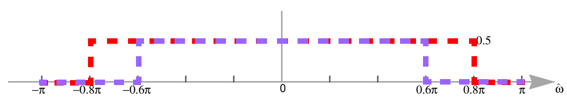
(a) TRUE ☐ FALSE ☒ The filter is FIR.

(b) When the input is $x[n] = \cos(0.7\pi n) + \cos(0.9\pi n)$, the output is $y[n] =$ **0.5cos(0.7πn)**.

(c) The impulse response can be written as $h[n] = A\dfrac{\sin(B\pi n)}{\pi n} + C\dfrac{\sin(0.8\pi n)}{\pi n}$, where:

$A =$ **0.5**, $B =$ **0.6**, $C =$ **0.5**.

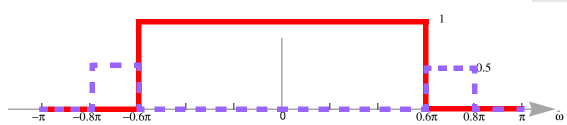
Add purple to red:



(d) The impulse response can be written as $h[n] = \dfrac{\sin(0.6\pi n)}{\pi n} + \cos(D\pi n)\dfrac{\sin(E\pi n)}{\pi n}$, where:

$D =$ **0.7**, $E =$ **0.1**.

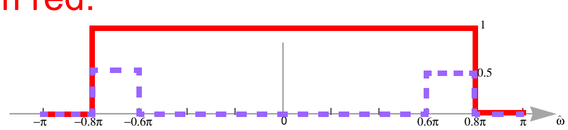
Add purple to red:



(e) The impulse response can be written as $h[n] = \dfrac{\sin(0.8\pi n)}{\pi n} - \cos(F\pi n)\dfrac{\sin(G\pi n)}{\pi n}$, where:

$F =$ **0.7**, $G =$ **0.1**.

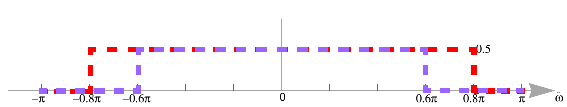
Subtract purple from red:



(f) The impulse response can be written as $h[n] = K\cos(0.1\pi n)\dfrac{\sin(L\pi n)}{\pi n}$, where:

$K =$ **1**, $L =$ **0.7**.

Add purple to red:



PROB. Sp22-Q3.3. (30 points, 3 pts each answer)

Shown below on the left are the plots of ten different signal segments $[x[0], \ldots x[63]]$, labeled A through J, where each $x[n]$ is plotted versus $n \in \{0, 1, \ldots 63\}$. Let $[X[0], \ldots X[63]]$ be the $N = 64$-point DFT of $[x[0], \ldots x[63]]$. Shown on the right are the corresponding plots of the DFT magnitudes $|X[k]|$ versus $k \in \{0, 1, \ldots 63\}$, but in a scrambled order. Match each DFT magnitude plot to its corresponding signal segment by writing a letter (from A through J) into each of the ten answer boxes. (None of the y-axis scales are specified, they are not needed to solve the problem.)