GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: April 6, 2018 COURSE: ECE-2026

NAME: Solutions GT ID:

LAST, FIRST (ex: buzz2b)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

			L06:Thur-Noon (Fekri)
			L08:Thurs-1:30pm (Fekri)
L01:M-3pm (Valenta)	L09:Tues-3pm (Rohling)	L02:W-3pm (Yang)	L10:Thur-3pm (Marenco)
L03:M-4:30pm (Valenta)	L11:Tues-4:30pm (Rohling)	L04:W-4:30pm (Yang)	L12:Thur-4:30pm (Marenco)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One sheet ($8\frac{1}{2}'' \times 11''$) of notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	34	
2	38	
3	28	
No/Wrong Rec	-3	

PROBLEM SPR-18-Q.3.1:

(a) (5 pts) The DTFT of a signal is

$$2e^{j\hat{\omega}} - 1 + 2e^{-j\hat{\omega}} + 2e^{-j3\hat{\omega}}$$

What is the time-domain signal?

(A) $4\cos(\hat{\omega}n) - 1 + 2e^{-j3\hat{\omega}n}$	**(B)** $2\delta[n+1] - 1 + 2\delta[n] + 2\delta[n-1]$
(C) $2\delta[n+1] - 1 + 2\delta[n-1] + 2\delta[n-3]$	**(D)** $2\delta[n+1] - \delta[n] + 2\delta[n-1] + 2\delta[n-3]$
(E) $4\delta[n-1]\cos(\hat{\omega}n) - \delta[n] + 2\delta[n-3]$	**(F)** other

(b) (5 pts) Determine the DTFT of the following sum of two exponential signals

$$(-0.5)^n u[n] + 2(0.8)^n u[n]$$

(A) $X(e^{j\hat{\omega}}) = 2\left(\frac{1}{1-0.5e^{-j\hat{\omega}}}\right)\left(\frac{1}{1+0.8e^{-j\hat{\omega}}}\right)$	**(B)** $X(e^{j\hat{\omega}}) = \frac{2}{1-0.5e^{-j\hat{\omega}}} + \frac{1}{1+0.8e^{-j\hat{\omega}}}$
(C) $X(e^{j\hat{\omega}}) = \frac{2}{1+0.5e^{-j\hat{\omega}}} - \frac{1}{1-0.8e^{-j\hat{\omega}}}$	**(D)** $X(e^{j\hat{\omega}}) = \frac{1}{1+0.5e^{-j\hat{\omega}}} + \frac{2}{1-0.8e^{-j\hat{\omega}}}$
(E) $X(e^{j\hat{\omega}}) = \frac{2}{1+0.5e^{-j\hat{\omega}}} + \frac{1}{1-0.8e^{-j\hat{\omega}}}$	**(F)** $X(e^{j\hat{\omega}}) = \frac{-1}{1+0.5e^{-j\hat{\omega}}} + \frac{2}{1-0.8e^{-j\hat{\omega}}}$
(G) $X(e^{j\hat{\omega}}) = \frac{1}{1+0.5e^{-j\hat{\omega}}} + \frac{1}{1-0.8e^{-j\hat{\omega}}}$	**(H)** $X(e^{j\hat{\omega}}) = 2\left(\frac{1}{1+0.5e^{-j\hat{\omega}}}\right)\left(\frac{1}{1-0.8e^{-j\hat{\omega}}}\right)$

(c) (14 pts) Use Parseval's theorem (see the DTFT Properties Table) and the DTFT to evaluate:

- $\displaystyle\sum_{n=-\infty}^{\infty} \left| \frac{\sin\left(\frac{\pi}{4}n\right)}{3\pi n} \right|^2$

$$\frac{\sin\left(\frac{\pi}{4}n\right)}{3\pi n} \Longleftrightarrow \begin{cases} 1/3 & |\hat{\omega}| \le \frac{\pi}{4} \\ 0 & \text{else} \end{cases}$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} \left| \frac{1}{3} \right|^2 d\omega$$

$$= \frac{1}{2\pi} \cdot \frac{1}{9} \cdot \left(\frac{\pi}{4} - \frac{-\pi}{4} \right) = \boxed{\frac{1}{36}}$$

- $\displaystyle 3 \int_{-\pi}^{\pi} \left| \frac{\sin(5\hat{\omega})}{\sin(\hat{\omega}/2)} \right|^2$

$$\frac{\sin(5\hat{\omega})}{\sin(\hat{\omega}/2)} e^{-j\hat{\omega}(10-1)/2} \Rightarrow u[n] - u[n-10]$$

$$= 3 \cdot 2\pi \cdot \sum_{n=0}^{9} |1|^2$$

$$= \boxed{60\pi}$$

(d) (10 pts) Suppose a discrete-time LTI system has the frequency response

$$H(e^{j\hat{\omega}}) = -5\cos\left(\frac{1}{2}\hat{\omega}\right)\exp\left(-j\frac{1}{2}\hat{\omega}\right)$$

Compute and simplify the convolution

$$y[n] = \left[\sqrt{2}\cos\left(\pi n + \frac{3}{16}\pi\right) - 7\right] * h[n],$$

where $h[n]$ is the impulse response associated with $H(e^{j\hat{\omega}})$. (Note: you are not required to find the impulse response to work this part.)

$H(e^{j\pi}) = 0$

$H(e^{j0}) = -5$

$\boxed{y[n] = 35}$

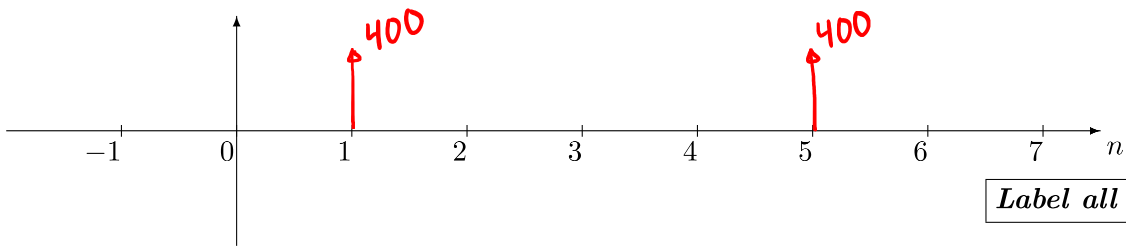
PROBLEM SPR-18-Q.3.2:

Each part of this problem is independent of the other parts.

(a) (6 pts) Determine the impulse response of the system:

$$y[n] = 400x[n-1] + 400x[n-5]$$

Give your answer as a ***stem plot.***



Label all points

(b) (12 pts) Determine the frequency response of the FIR system:

$$y[n] = 400x[n-1] + 400x[n-5]$$

Give your answer as a formula ***in the following form:*** $H(e^{j\hat{\omega}}) = e^{-j\beta\hat{\omega}} A\cos(\mu\hat{\omega})$ by finding numerical values for A, β and μ.

$A =$ 800	$\beta =$ 3	$\mu =$ 2

$$H(e^{j\hat{\omega}}) = 400e^{-j\hat{\omega}} + 400e^{-j5\hat{\omega}}$$

$$= 400e^{-j3\hat{\omega}}\left(e^{j2\hat{\omega}} + e^{-j2\hat{\omega}}\right)$$

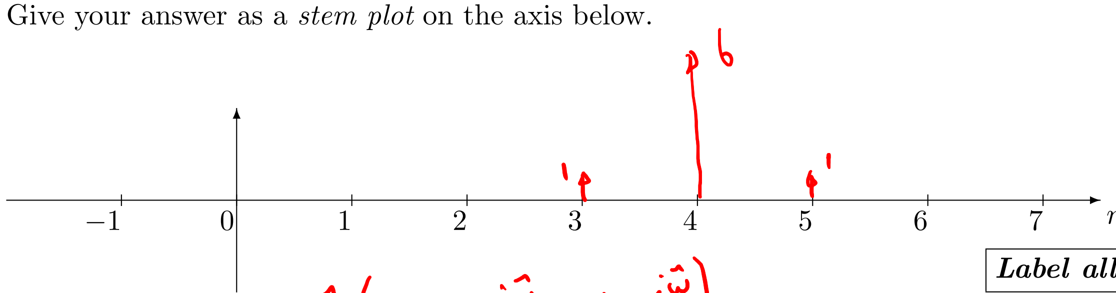
$$= 800e^{-j3\hat{\omega}}\cos(2\hat{\omega})$$

(c) (12 pts) Suppose a linear, time-invariant system is characterized by its frequency response,

$$H(e^{j\hat{\omega}}) = 2e^{-j4\hat{\omega}}(3 + \cos\hat{\omega})$$

Determine the impulse response, $h[n]$.

Give your answer as a *stem plot* on the axis below.



Label all points

$$H(e^{j\hat{\omega}}) = 2e^{-j4\hat{\omega}}\left(3 + \tfrac{1}{2}e^{j\hat{\omega}} + \tfrac{1}{2}e^{-j\hat{\omega}}\right)$$

$$= 6e^{-j4\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j5\hat{\omega}}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$6\delta[n-4] \qquad \delta[n-3] \qquad \delta[n-5]$$

(d) (8 pts) Consider a discrete-time LTI system with impulse response given by: $h[n] = 0.5^n u[n-3]$.

- Is this system *causal*? Explain why or why not.

Yes, $h[n] = 0$ for $n < 3$

- Is this system *stable*? Explain why or why not.

Yes. $\sum_n |h[n]| < 1 < \infty$

PROBLEM SPR-18-Q.3.3:

Each part of this problem is independent of the others.

(a) (14 pts) Suppose we have some unknown linear, time-invariant system, $\mathcal{S}$. When the input to the system is

$$x_1[n] = \{\ldots, 0, \overset{n=0}{1}, 1, 2, 1, 1, 0, 0, \ldots\}$$

the output is

$$y_1[n] = \{\ldots, 0, \overset{n=0}{2}, 3, 3, 2, -1, -1, -2, 0, 0, \ldots\}$$

and when the input is

$$x_2[n] = \{\ldots, 0, \overset{n=0}{1}, 1, 0, 1, 1, 0, 0, \ldots\}$$

the output is

$$y_2[n] = \{\ldots, 0, \overset{n=0}{2}, 3, -1, 0, 3, -1, -2, 0, 0, \ldots\}.$$

What is the output of the system, $\mathcal{S}$, when the input is $x_3[n]$?

$$x_3[n] = \{\ldots, 0, \overset{n=0}{-1}, -1, 2, -1, -1, 0, 0, \ldots\}$$

$$x_3[n] = x_1[n] - 2x_2[n]$$

so $$y_3[n] = y_1[n] - 2y_2[n]$$

$$= \{0, \underset{n=0}{-2}, -3, 5, 2, -7, 1, 2, 0\}$$

(b) (14 pts) The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

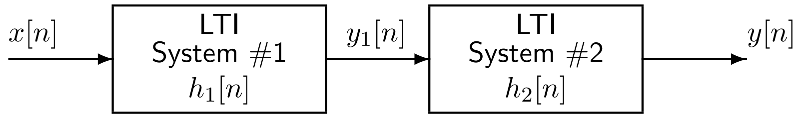


Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a filter described by the difference equation

$$y_1[n] = x[n] + \tfrac{1}{2}x[n-3] + x[n-6]$$

and System #2 is described by the FIR coefficient vector

$$b_k = \{2, \tfrac{1}{2}, \tfrac{1}{2}\} \qquad \text{for} \quad k = 0, 1, 2$$

What is the impulse response, $h[n]$, of the overall cascade system?

$$h[n] = \{\underset{n=0}{2}, \tfrac{1}{2}, \tfrac{1}{2}, 1, \tfrac{1}{4}, \tfrac{1}{4}, 2, \tfrac{1}{2}, \tfrac{1}{2}\}$$