GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 13-MAR-15 COURSE: ECE 2026A,B

NAME: **Solutions** **version 1** STUDENT #: ________________

LAST, FIRST

2 points 2 points 2 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L00:Tue-9:30am (Zhang)	L01:Mon-3:00pm (Casinovi)
L03:Mon-4:30pm (Casinovi)	L05:Tue-12:00pm (Zhang)
L06:Thu-12:00pm (Walkenhorst)	L07:Tue-1:30pm (Zajic)
L08:Thu-1:30pm (Walkenhorst)	L09:Tue-3:00pm (Zajic)
L10:Thu-3:00pm (Fekri)	L12:Thu-4:30pm (Fekri)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Unless stated otherwise, JUSTIFY your reasoning clearly to receive any partial credit. Showing your work is required to receive any partial credit.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	12	
3	16	
Rec	2	
Total	50	

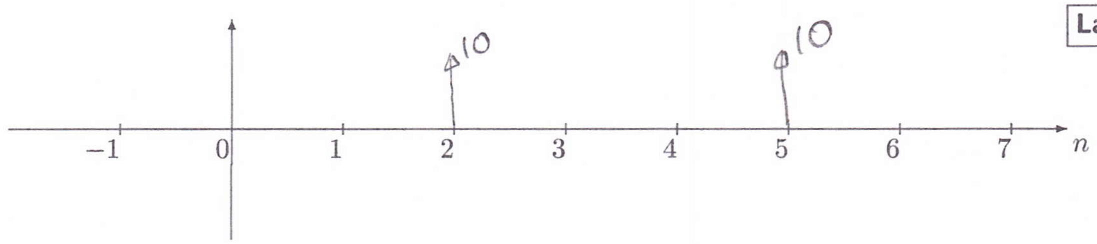
Problem Q1.1:
Frequency Response

(a) (6 pts) Determine the impulse response of the system:

$$y[n] = 10x[n-2] + 10x[n-5]$$

Give your answer as a **stem plot.**

Label all points



When $x[n] = \delta[n]$, you get the impulse response, $h[n]$. Thus,

$$h[n] = 10\delta[n-2] + 10\delta[n-5]$$

(b) (8 pts) Determine the frequency response of the FIR system:

$$y[n] = 10x[n-2] + 10x[n-5]$$

Give your answer as a formula **in the following form:** $H(e^{j\hat{\omega}}) = e^{-j\beta\hat{\omega}} A\cos(\mu\hat{\omega})$ by finding numerical values for A, β and μ.

$A = 20$	$\beta = 3.5$	$\mu = 1.5$

The frequency response is $H(e^{j\hat{\omega}}) = \sum_{n=0}^{M} h[n]e^{-jk\hat{\omega}}$. Thus,

$$H(e^{j\hat{\omega}}) = 10e^{-j2\hat{\omega}} + 10e^{-j5\hat{\omega}} = 10e^{-j3.5\hat{\omega}}\left(e^{j1.5\hat{\omega}} + e^{-j1.5\hat{\omega}}\right)$$

$$H(e^{j\hat{\omega}}) = 20e^{-j3.5\hat{\omega}}\cos(1.5\hat{\omega})$$

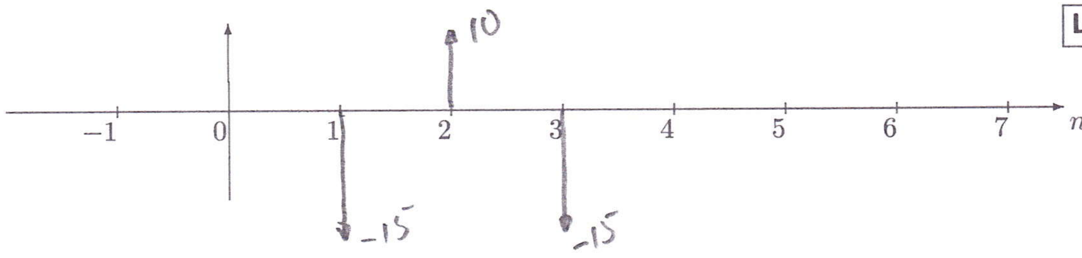
(c) (6 pts) Suppose a linear, time-invariant system is characterized by its frequency response,

$$H(e^{j\hat{\omega}}) = 5e^{-j2\hat{\omega}}(2 - 6\cos\hat{\omega})$$

Determine the impulse response, $h[n]$.

Give your answer as a *stem plot* on the axis below.

Label all points



$$H(e^{j\hat{\omega}}) = 5e^{-j2\hat{\omega}}\left(2 - 3e^{j\hat{\omega}} - 3e^{-j\hat{\omega}}\right)$$

$$= 10e^{-j2\hat{\omega}} - 15e^{-j\hat{\omega}} - 15e^{-j3\hat{\omega}}$$

$$\Updownarrow \qquad \Updownarrow \qquad \Updownarrow$$

$$10\,\delta[n-2] \qquad -15\,\delta[n-1] \qquad -15\,\delta[n-3]$$

Problem Q1.2:
LTI Systems

(12 pts) Suppose a linear, time-invariant system is given by the following difference equation.

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] + x[n-5]$$

If the input to this system is

$$x[n] = 3\cos(0.5\pi n - 0.25\pi) + 2\cos(\frac{2\pi}{3}n - 0.5\pi)$$

find the output, $y[n]$, and express it as a sum of sinusoids. *Hint:* you may want to find the frequency response first. *Note: there should be no more than two sinusoids in your answer.*

$$\boxed{y[n] = 3\sqrt{2}\cos\left(\frac{\pi}{2}n - \pi/2\right)}$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{5} e^{-j\hat{\omega}k} = \frac{1-e^{-j6\hat{\omega}}}{1-e^{-j\hat{\omega}}}$$

$$= \frac{e^{-j3\hat{\omega}}}{e^{-j\hat{\omega}/2}} \frac{\sin(3\hat{\omega})}{\sin(\hat{\omega}/2)} = e^{-j\frac{5\hat{\omega}}{2}} \frac{\sin(3\hat{\omega})}{\sin(\hat{\omega}/2)}$$

$$H(e^{j\frac{\pi}{2}}) = e^{-j\frac{5}{4}\pi} \frac{\overbrace{\sin(\tfrac{3\pi}{2})}^{-1}}{\underbrace{\sin(\pi/4)}_{1/\sqrt{2}}} = -\sqrt{2}\,e^{+j3\pi/4} = \sqrt{2}\,e^{-j\frac{\pi}{4}}$$

$$H(e^{j\frac{2\pi}{3}}) = e^{-j\frac{5\pi}{3}} \frac{\overbrace{\sin(2\pi)}^{0}}{\sin(\pi/3)} = 0$$

Problem Q1.3:
LTI Systems

Each part of this problem is independent of the others.

(a) (8 pts) Suppose we have some unknown linear, time-invariant system, $\mathcal{S}$. When the input to the system is

$$x_1[n] = \{\ldots, 0, \overset{n=0}{2}, 1, 2, 1, 2, 0, 0, \ldots\}$$

the output is

$$y_1[n] = \{\ldots, 0, \overset{n=0}{-2}, 5, -3, 3, -3, 4, -4, 0, 0, \ldots\}$$

and when the input is

$$x_2[n] = \{\ldots, 0, \overset{n=0}{0}, 1, 0, 1, 0, 0, 0, \ldots\}$$

the output is

$$y_2[n] = \{\ldots, 0, \overset{n=0}{0}, -1, 3, -3, 3, -2, 0, 0, 0, \ldots\}.$$

What is the output of the system, $\mathcal{S}$, when the input is $x_3[n]$?

$$x_3[n] = \{\ldots, 0, \overset{n=0}{6}, 1, 6, 1, 6, 0, 0, \ldots\}$$

$x_3[n] = 3x_1[n] - 2x_2[n]$

$y_3[n] = \underset{n=0}{-6}, 17, -15, 15, -15, 16, -12$

(b) (8 pts) The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

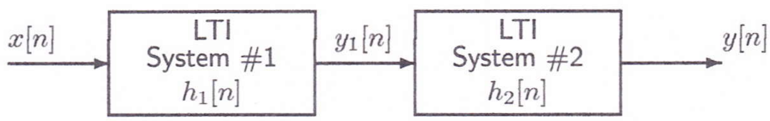


Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a filter described by the difference equation

$$y_1[n] = x[n] + \tfrac{1}{2}x[n-3] + x[n-6]$$

and System #2 is described by the FIR coefficient vector

$$b_k = \left\{-1, \tfrac{1}{2}, -\tfrac{1}{4}\right\} \qquad \text{for} \quad k = 0, 1, 2$$

What is the impulse response, $h[n]$, of the overall cascade system?

$-1, \frac{1}{2}, \frac{-1}{4}, \frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, -1, \frac{1}{2}, \frac{-1}{4}$

$n=0$