GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 04-APR-14 COURSE: ECE 2026A,B

NAME: ______________________________ STUDENT #: ______________

LAST, FIRST

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L01:Mon-3:00pm (Causey)	L02:Wed-3:00pm (Romberg)
L03:Mon-4:30pm (Causey)	L04:Wed-4:30pm (Romberg)
L05:Tues-Noon (Fekri)	L06:Thur-Noon (Chang)
L07:Tues-1:30pm (Fekri)	L08:Thur-1:30pm (Stüber)
L10:Thur-3:00pm (Stüber)	L12:Thur-4:30pm (Chang)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Unless stated otherwise, JUSTIFY your reasoning clearly to receive any partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.
- None of these problems requires tedious math. Reconsider your approach before wasting too much time.

Problem	*Value*	*Score*
1	32	
2	30	
3	35	
Rec	3	
Total	100	

Problem Q3.1:

In parts (a)-(d), when the input to an LTI filter is $\delta[n]$, the output is $\delta[n-1] + 3\delta[n-4]$.

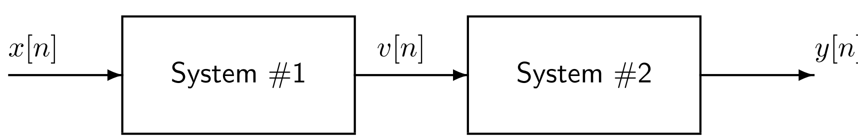
(a) (6 pts) Draw a stem plot of $h[n]$, the impulse response.

(b) (6 pts) Write the difference equation for the filter relating the output $y[n]$ to the input $x[n]$.

(c) (6 pts) Suppose we want to filter a signal `xx` in MATLAB with the FIR filter described above. What should `hh` be in the command `conv(xx,hh)`?

(d) (7 pts) Suppose a constant signal $x[n] = -3$ is input to the FIR filter. Give a formula for the output $y[n]$ which is valid for all n.

(e) (7 pts) Consider a cascade connection of two systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.



Suppose System 1 is described by the input/output relation $v[n] = x[n] - 3x[n-1]$, and System 2 is described by the input/output relation:

$$y[n] = \sqrt{v[n]}$$

In ECE2026, we have learned that under certain conditions, the order of systems in a cascade can be changed without affecting the output, i.e., putting System #2 in front of System #1 would not change $y[n]$. *Do these conditions apply in this case? If so, state the conditions and demonstrate their presence; if not describe their absence.*

Problem Q3.2:

Suppose a discrete-time LTI system has the frequency response

$$H(e^{j\hat{\omega}}) = -3\cos\left(\frac{3}{2}\hat{\omega}\right)\exp\left(-j\frac{3}{2}\hat{\omega}\right)$$

(a) (8 pts) Find the impulse response $h[n]$ of this system.

(b) (8 pts) Compute and simplify the convolution

$$y[n] = \sqrt{2}\cos\left(\frac{\pi}{3}n + \frac{3}{16}\pi\right) * h[n],$$

where $h[n]$ is the impulse response found in part (a). (Note: you do not actually need the impulse response from part (a) to work this part.)

(c) (8 pts) Consider a discrete-time LTI system with impulse response given by: $h_1[n] = 0.5^n u[n+3]$. Is this system *causal*? Explain why or why not. Is this system *stable*? Explain why or why not.

(d) (6 pts) Consider the same LTI system with impulse response given by: $h_1[n] = 0.5^n u[n+3]$. Find $H_1(e^{j\hat{\omega}})$, its DTFT.

Problem Q3.3:

Let $X[k]$ be the 4-point DFT of the signal $x[n]$. $X[0] = 0$, $X[1] = 1 + j$, $X[2] = 5$, $X[3] = 1 - j$. (Or, `X=[0,1+j,5,1-j]`). This description applies to all parts below. All parts can be worked independently. Computing $x[n]$ for all n is not required for any part.

(7 pts) (a) $X[1] = ax[0] + bx[1] + cx[2] + dx[3]$. Find a, b, c, and d and express them in terms of real and imaginary parts. (Repeat: do not find $x[n]$.)

(7 pts) (b) Find $x[1]$ using what should be simple arithmetic.

(7 pts) (c) Find $\sum_{n=0}^{3} x[n]x^*[n] = \sum_{n=0}^{3} |x[n]|^2$ (again without expicitly finding $x[n]$).

(7 pts) (d) Let $y[n] = j^n x[n]$. Find $Y[k]$, the 4-point DFT of $y[n]$.

(7 pts) (e) Let $w[n] = x[n]$ with 4 appended zeros. In other words, $w[n]$ is an 8-point sequence: $\{ x[0], x[1], x[2], x[3], 0, 0, 0, 0 \}$. Find $W[2]$, where $W[k]$ is the 8-point DFT of $w[n]$.

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QUIZ #3 SOLUTION Version A

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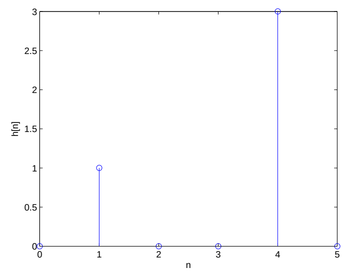
Problem Q3.1:

In parts (a)-(d), when the input to an LTI filter is $\delta[n]$, the output is $\delta[n-1] + 3\delta[n-4]$.

(a) (6 pts) Draw a stem plot of $h[n]$, the impulse response.



(b) (6 pts) Write the difference equation for the filter relating the output $y[n]$ to the input $x[n]$.

$y[n] = x[n-1] + 3x[n-4]$

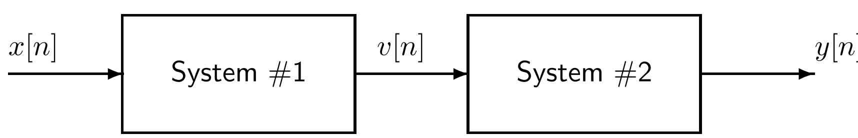
(c) (6 pts) Suppose we want to filter a signal `xx` in MATLAB with the FIR filter described above. What should `hh` be in the command `conv(xx,hh)`?

`hh = [0,1,0,0,3]`

(d) (7 pts) Suppose a constant signal $x[n] = -3$ is input to the FIR filter. Give a formula for the output $y[n]$ which is valid for all n.

$H(e^{j0}) = 4 \rightarrow y[n] = -12 \ \forall n.$

(e) (7 pts) Consider a cascade connection of two systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.



Suppose System 1 is described by the input/output relation $v[n] = x[n] - 3x[n-1]$, and System 2 is described by the input/output relation:

$$y[n] = \sqrt{v[n]}$$

In ECE2026, we have learned that under certain conditions, the order of systems in a cascade can be changed without affecting the output, i.e., putting System #2 in front of System #1 would not change $y[n]$. *Do these conditions apply in this case? If so, state the conditions and demonstrate their presence; if not describe their absence.*

No. Both systems must be linear and time invariant. System 2 is non-linear.

Problem Q3.2:

Suppose a discrete-time LTI system has the frequency response

$$H(e^{j\hat{\omega}}) = -3\cos\left(\frac{3}{2}\hat{\omega}\right)\exp\left(-j\frac{3}{2}\hat{\omega}\right)$$

(a) (8 pts) Find the impulse response $h[n]$ of this system.

$$H(e^{j\hat{\omega}}) = -\frac{3}{2}(e^{j\frac{3}{2}\hat{\omega}} + e^{-j\frac{3}{2}\hat{\omega}})(e^{-j\frac{3}{2}\hat{\omega}}) = -\frac{3}{2}(1 + e^{-j3\hat{\omega}}) \rightarrow h[n] = -\frac{3}{2}(\delta[n] + \delta[n-3])$$

(b) (8 pts) Compute and simplify the convolution

$$y[n] = \sqrt{2}\cos\left(\frac{\pi}{3}n + \frac{3}{16}\pi\right) * h[n],$$

where $h[n]$ is the impulse response found in part (a). (Note: you do not actually need the impulse response from part (a) to work this part.)

The input frequency is $\frac{\pi}{3}$. At $\hat{\omega} = \frac{\pi}{3}$, the $\cos\left(\frac{3}{2}\hat{\omega}\right)$ factor becomes $\cos(\frac{\pi}{2}) = 0$. Therefore, $y[n] = 0 \ \forall n$.

(c) (8 pts) Consider a discrete-time LTI system with impulse response given by: $h_1[n] = (0.5)^n u[n+3]$. Is this system *causal*? Explain why or why not. Is this system *stable*? Explain why or why not.

Causal? No. The impulse response starts before $n = 0$.
Stable? Yes. $\sum_{n=-3}^{\infty} |h_1[n]| < \infty$.

(d) (6 pts) Consider the same LTI system with impulse response given by: $h_1[n] = (0.5)^n u[n+3]$. Find $H_1(e^{j\hat{\omega}})$, its DTFT.

$h_1[n] = 0.5^{n+3}u[n+3](0.5)^{-3}$

Using the table: $(0.5)^n u[n] \rightarrow \frac{1}{1-0.5e^{-j\hat{\omega}}}$

$h_1[n]$ is a scaled and shifted version of this table entry.

$H_1(e^{j\hat{\omega}}) = \frac{0.5^{-3}e^{j3\hat{\omega}}}{1-0.5e^{-j\hat{\omega}}}$

Problem Q3.3:

Let $X[k]$ be the 4-point DFT of the signal $x[n]$. $X[0] = 0$, $X[1] = 1 + j$, $X[2] = 5$, $X[3] = 1 - j$. (Or, `X=[0,1+j,5,1-j])`. This description applies to all parts below. All parts can be worked independently. Computing $x[n]$ for all n is not required for any part.

(7 pts) (a) $X[1] = ax[0] + bx[1] + cx[2] + dx[3]$. Find a, b, c, and d and express them in terms of real and imaginary parts. (Repeat: do not find $x[n]$.)

$X[k] = \sum_{n=0}^{3} x[n]e^{-jk\frac{\pi}{2}n}$

$X[1] = \sum_{n=0}^{3} x[n]e^{-j\frac{\pi}{2}n}$

$a = 1$, $b = -j$, $c = -1$, $d = j$.

(7 pts) (b) Find $x[1]$ using what should be simple arithmetic.

$x[n] = \frac{1}{4}\sum_{k=0}^{3} X[k]e^{jk\frac{\pi}{2}n}$

$x[1] = \frac{1}{4}\sum_{k=0}^{3} X[k]e^{jk\frac{\pi}{2}}$

$x[1] = \frac{1}{4}(X[0] + jX[1] - X[2] - jX[3])$

$x[1] = \frac{1}{4}(0 + (j - 1) - 5 + (-j - 1)) = -7/4$

(7 pts) (c) Find $\sum_{n=0}^{3} x[n]x^*[n] = \sum_{n=0}^{3} |x[n]|^2$ (again without expicitly finding $x[n]$).

From Table: $\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X[k]|^2$

$\sum_{n=0}^{3} |x[n]|^2 = \frac{1}{4}(0 + 2 + 25 + 2) = 29/4$

(7 pts) (d) Let $y[n] = j^n x[n]$. Find $Y[k]$, the 4-point DFT of $y[n]$.

$j^n = e^{j\frac{\pi}{2}n} = e^{j\frac{2\pi}{4}n}$

This gives a circular shift in k by 1 to the right.

$Y[k] = X[((k-1))_4] = [1-j, 0, 1+j, 5]$

(7 pts) (e) Let $w[n] = x[n]$ with 4 appended zeros. In other words, $w[n]$ is an 8-point sequence: $\{ x[0], x[1], x[2], x[3], 0, 0, 0, 0 \}$. Find $W[2]$, where $W[k]$ is the 8-point DFT of $w[n]$.

$W[2] = X[1] = 1 - j$
