GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 17-Apr-09 COURSE: ECE-2025

NAME: ______________________________ GT username: ____________________

LAST, FIRST (ex: gtbuzz7)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Bhatti)		L06:Thur-Noon (Barry)
	L07:Tues-1:30pm (Bhatti)		L08:Thur-1:30pm (Barry)
L01:M-3pm (Chang)	L09:Tues-3pm (Lee)	L02:W-3pm (Fekri)	
	L11:Tues-4:30pm (Lee)	L04:W-4:30pm (Fekri)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **Justify** your reasoning clearly to receive partial credit.
  Explanations are also REQUIRED to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	30	
2	30	
3	20	
4	20	
No/Wrong Rec	—3	

PROBLEM sp-09-Q.3.1:

In each of the following cases, use properties of the unit-impulse signal $\delta(t)$ to simplify the expression ***as much as possible.*** Provide some **explanation** or intermediate steps for each answer. *Note:* Star $*$ is the convolution operator.

(a) Simplify $x(t) = \dfrac{d}{dt}\left\{t^2 u(t-\tfrac{1}{2})\right\}$

(b) Simplify $y(t) = \left(\dfrac{\sin(7(t-100))}{\pi(t-100)}\,\delta(t-100)\right) * \delta(t-200)$

(c) Simplify $z(t) = \displaystyle\int_{t-4}^{t-5} \lambda^2\,\delta(\lambda-2)\,d\lambda$

PROBLEM sp-09-Q.3.2:

In each of the following cases, determine the (inverse or forward) Fourier transform. Give your answer as a plot, or a simple formula. ***Explain*** each answer (briefly) by stating which property and/or transform pair was used.

(a) Find $A(j\omega)$ when $a(t) = 7u(t-5) - 7u(t-6)$.

(b) Find $b(t)$ when $B(j\omega) = 4\pi e^{j2}\delta(\omega - 13) + 4\pi e^{-j2}\delta(\omega + 13)$. Simplify to get a real-valued answer.

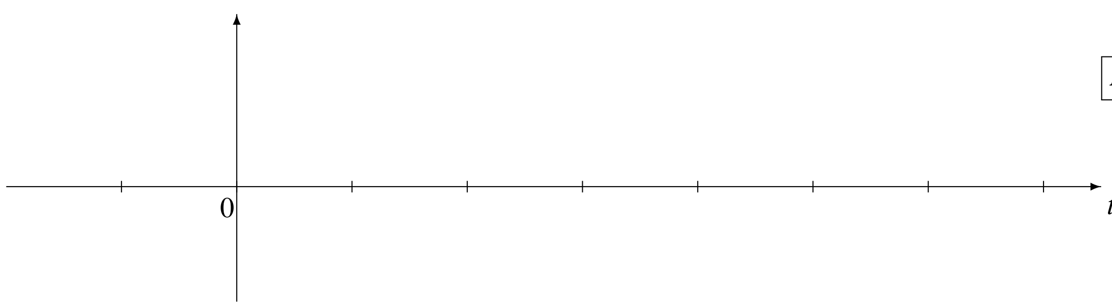
(c) Find $c(t)$ when $C(j\omega) = \dfrac{12}{8 + j2\omega}$.

PROBLEM sp-09-Q.3.3:

Two questions about convolution (denoted by the $*$ operator):

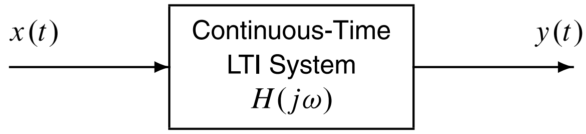
(a) Evaluate $\dfrac{\sin(50\pi t)}{50\pi t} * \dfrac{\sin(80\pi(t-2))}{\pi(t-2)}$. Give the answer as a **simple** formula.

(b) Find the output of a LTI system whose impulse response is $h(t) = \delta(t-1) - 2\delta(t-5)$ when the input is a unit-step signal, $u(t)$. Give the answer as a **plot**.



Label all points

PROBLEM sp-09-Q.3.4:



Lab #11 dealt with the case where the frequency response $H(j\omega)$ could be written in terms of a few parameters, e.g.,

$$H(j\omega) = \frac{j\omega d}{a + j\omega}$$

where a and d are parameters that can be determined from measurements.

(a) Suppose you want to generate a test case with $X(j\omega) = 2\pi\delta(\omega - 600)$ as the Fourier transform of the input to the system above with $a = 800$ and $d = 50$. Determine a *simple* formula for the output $y(t)$, which is complex-valued in this case.

$y(t) =$

(b) Now suppose that the parameters, a and d, of the system above are *unknown.* In order to identify the values of a and d, one sinusoidal measurement has been taken:

$$x(t) = \cos(500t) \qquad \longrightarrow \qquad y(t) = 0.2\cos(500t + \pi/4)$$

Explain how to set up ***two*** *linear* equations for the two unknowns, a and d. Set up the linear equations in matrix form, i.e., $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ is a 2×2 matrix. *It is not necessary to solve the equations.*

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QUIZ #2

DATE: 17-Apr-09 COURSE: ECE-2025

NAME: Answer Key
LAST, FIRST

GT username: Ver-1
(ex: gtbuzz7)

3 points 3 points 3 points

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	L05:Tues-Noon (Bhatti)		L06:Thur-Noon (Barry)
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Problem	*Value*	*Score*
1	30	
2	30	
3	20	
4	20	
No/Wrong Rec	−3	

PROBLEM sp-09-Q.2.1:

In each of the following cases, use properties of the unit-impulse signal $\delta(t)$ to simplify the expression ***as much as possible.*** Provide some **explanation** or intermediate steps for each answer. *Note:* Star $*$ is the convolution operator.

(a) Simplify $x(t) = \frac{d}{dt}\{t^2 u(t-\frac{1}{2})\}$

$$x(t) = 2t\,u(t-1/2) + \underbrace{t^2\delta(t-\tfrac{1}{2})}_{\text{eval at } t=1/2}$$

$$= 2t\,u(t-1/2) + \frac{1}{4}\delta(t-1/2)$$

(b) Simplify $y(t) = \left(\frac{\sin(7(t-100))}{\pi(t-100)}\delta(t-100)\right) * \delta(t-200)$

eval at $t=100$

at $t \to 100$, $\frac{\sin(7(t-100))}{\pi(t-100)} \longrightarrow \frac{7}{\pi}$

$$\frac{7}{\pi}\delta(t-100) * \delta(t-200) = \frac{7}{\pi}\delta(t-300)$$

(c) Simplify $z(t) = \int_{t-4}^{t-5} \lambda^2 \delta(\lambda-2)\, d\lambda$

eval at $\lambda=2$, $\lambda^2 = 4$

$$z(t) = 4\int_{t-4}^{t-5} \delta(\lambda-2)\,d\lambda = 4u(t-7) - 4u(t-6)$$

PROBLEM sp-09-Q.2.2:

In each of the following cases, determine the (inverse or forward) Fourier transform. Give your answer as a plot, or a simple formula. ***Explain*** each answer (briefly) by stating which property and/or transform pair was used.

(a) Find $A(j\omega)$ when $a(t) = 7u(t-5) - 7u(t-6)$.

Shifted Rectangle centered at $t = 5.5$
width = 1 = T

$$A(j\omega) = 7e^{-j5.5\omega}\frac{\sin(\omega/2)}{\omega/2}$$

(b) Find $b(t)$ when $B(j\omega) = 4\pi e^{j2}\delta(\omega-13) + 4\pi e^{-j2}\delta(\omega+13)$. Simplify to get a real-valued answer.

$$2\pi A e^{j\theta}\delta(\omega-\omega_1) \xrightarrow{\text{inv. F.T.}} A e^{j\theta} e^{j\omega_1 t}$$

$$b(t) = \underbrace{2e^{j2}e^{j13t} + 2e^{-j2}e^{-j13t}}_{\text{Combine w/ inverse Euler.}}$$

$$= 4\cos(13t + 2)$$

(c) Find $c(t)$ when $C(j\omega) = \dfrac{12}{8 + j2\omega}$. $= \dfrac{6}{4 + j\omega}$

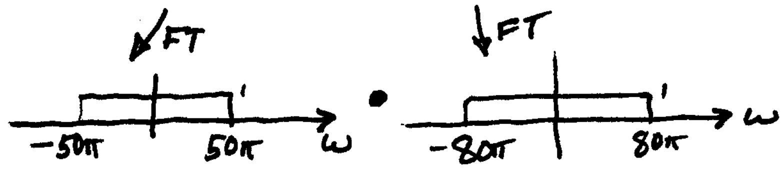
$$c(t) = 6e^{-4t}u(t)$$

PROBLEM sp-09-Q.2.3:

Two questions about convolution (denoted by the $*$ operator):

(a) Evaluate $\dfrac{\sin(50\pi t)}{50\pi t} * \dfrac{\sin(80\pi(t-2))}{\pi(t-2)}$. Give the answer as a **simple** formula.

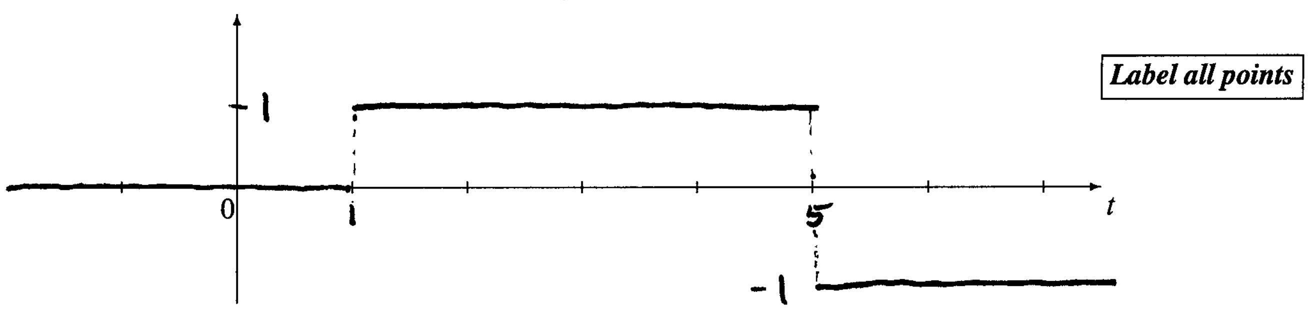
Same as $\dfrac{1}{50}\,\dfrac{\sin(50\pi t)}{\pi t} * \dfrac{\sin(80\pi t)}{\pi t} * \delta(t-2)$ ← shift by 2



Multiplying the rectangles gives the smaller one. Then inverse F.T. gives $\dfrac{\sin(50\pi t)}{\pi t}$. Then shift

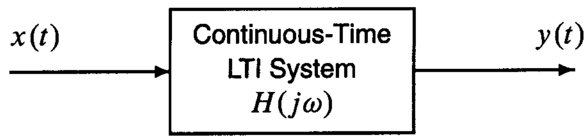
$$\frac{1}{50}\,\frac{\sin(50\pi(t-2))}{\pi(t-2)}$$

(b) Find the output of a LTI system whose impulse response is $h(t) = \delta(t-1) - 2\delta(t-5)$ when the input is a unit-step signal, $u(t)$. Give the answer as a **plot**.



$$\begin{aligned} y(t) &= u(t) * h(t) \\ &= u(t) * (\delta(t-1) - 2\delta(t-5)) \\ &= u(t-1) - 2u(t-5) \\ &= \begin{cases} 0 & t<1 \\ 1 & 1 \le t \le 5 \\ -1 & 5<t \end{cases} \end{aligned}$$

PROBLEM sp-09-Q.2.4:



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$$H(j\omega) = \frac{j\omega d}{a + j\omega}$$

where a and d are parameters that can be determined from measurements.

(a) Suppose you want to generate a test case with $X(j\omega) = 2\pi\delta(\omega - 600)$ as the Fourier transform of the input to the system above with $a = 800$ and $d = 50$. Determine a *simple* formula for the output $y(t)$, which is complex-valued in this case.

$y(t) = 30e^{j0.927}e^{j600t}$

$Y(j\omega) = X(j\omega)H(j\omega) = 2\pi\,\delta(\omega - 600)\,H(j600)$

$H(j600) = \dfrac{(j600)50}{800 + j600} = 30e^{j0.927}$ ← or 0.295π

$2\pi A e^{j\theta}\delta(\omega - 600) \longrightarrow A e^{j\theta} e^{j600t}$

(b) Now suppose that the parameters, a and d, of the system above are *unknown.* In order to identify the values of a and d, one sinusoidal measurement has been taken:

$$x(t) = \cos(500t) \qquad \longrightarrow \qquad y(t) = 0.2\cos(500t + \pi/4)$$

Explain how to set up ***two linear*** equations for the two unknowns, a and d. Set up the linear equations in matrix form, i.e., $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ is a 2×2 matrix. *It is not necessary to solve the equations.*

$H(j500)\,1 = 0.2e^{j\pi/4} = 0.1\sqrt{2} + j0.1\sqrt{2}$

$\dfrac{(j500)d}{a + j500} = 0.1\sqrt{2} + j0.1\sqrt{2}$

$j500d = 0.1\sqrt{2}\,a + j0.1\sqrt{2}\,a + j50\sqrt{2} - 50\sqrt{2}$

EQUATE Real & Imag separately to get 2 eqns

$-0.1\sqrt{2}\,a = -50\sqrt{2}$

$-0.1\sqrt{2}\,a + 500d = 50\sqrt{2}$

$$\begin{bmatrix} -0.1\sqrt{2} & 0 \\ -0.1\sqrt{2} & 500 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} -50\sqrt{2} \\ 50\sqrt{2} \end{bmatrix}$$