GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 18-Apr-08 COURSE: ECE-2025

NAME: _____ GT username: _____

LAST, FIRST (ex: gpburdell3)

3 points | 3 points | 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

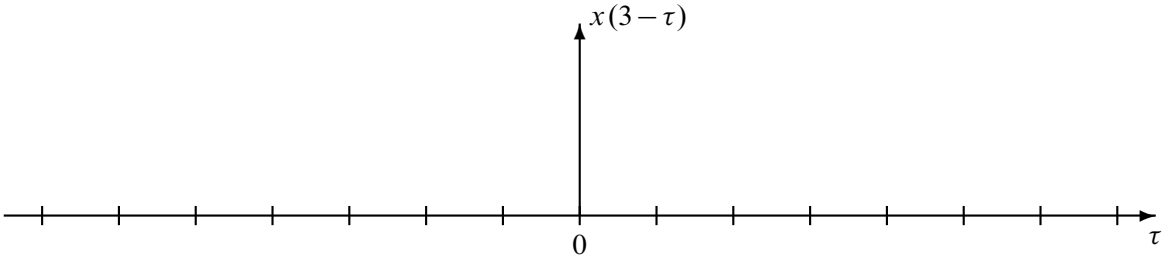
	L05:Tues-Noon (Chang)		
	L07:Tues-1:30pm (Chang)		L08:Thurs-1:30pm (Coyle)
L01:M-3pm (McClellan)	L09:Tues-3pm (Lanterman)	L02:W-3pm (Clements)	L10:Thur-3pm (Coyle)
	L11:Tues-4:30pm (Lanterman)	L04:W-4:30pm (Clements)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

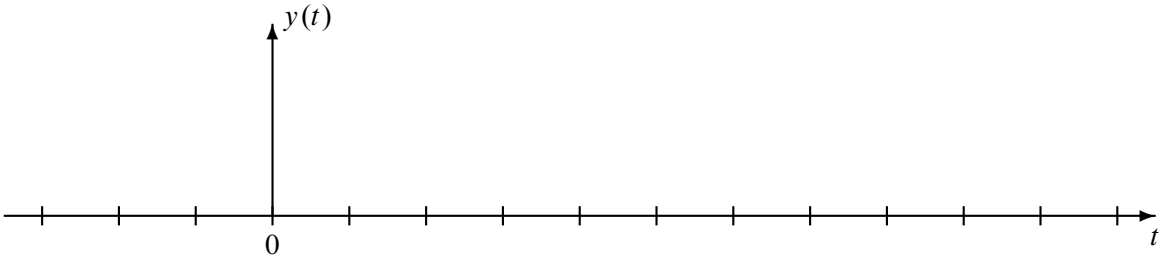
Problem	*Value*	*Score*
1	30	
2	40	
3	30	
No/Wrong Rec	−3	

PROBLEM sp-08-Q.3.1:

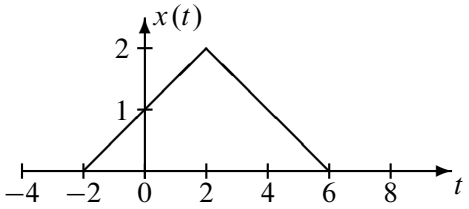
(a) Assume that $x(t) = u(t+2) - u(t-1)$. Plot $x(3-\tau)$ as a function of τ.

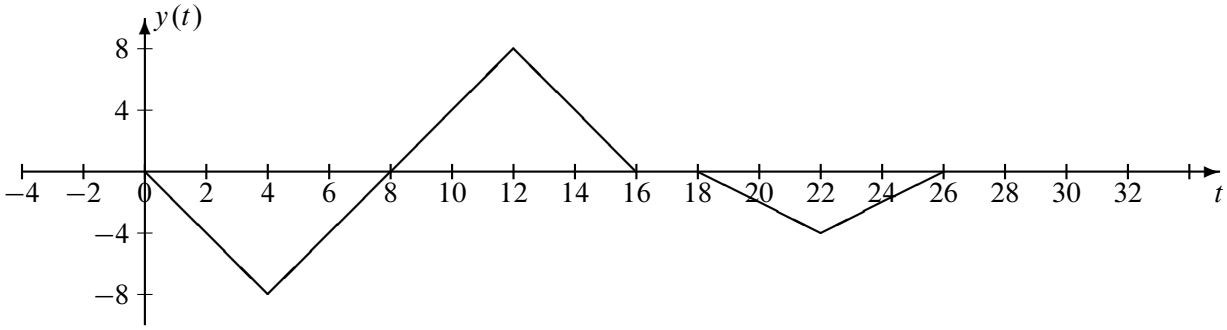


(b) If the input to an LTI system is $x(t) = u(t+2) - u(t-1)$ from part (a), determine the output signal $y(t) = x(t) * h(t)$ when $h(t) = 9\delta(t) + 9\delta(t-2)$. Give your answer as a carefully labeled sketch showing numerical values along the time axis, and also values for the amplitudes.



(c) *Deconvolution:* When the input to an LTI system is $x(t)$, the output is the signal $y(t) = x(t) * h(t)$ plotted below. Determine a formula for the impulse response $h(t)$ of the system.





$h(t) =$

PROBLEM sp-08-Q.3.2:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write your answers in the boxes provided.*** (The operator $*$ denotes convolution.)

(a) ☐ $x(t) = \delta(t) - \delta(t-8)$

(b) ☐ $x(t) = \cos(\pi t)\delta(t-4)$

(c) ☐ $x(t) = 2\cos(\pi t)\,\dfrac{\sin(\pi t)}{t}$

(d) ☐ $x(t) = \delta(t+2) * \{e^{-t+1}u(t-1)\} * \delta(t-1)$

(e) ☐ $x(t) = u(t) - u(t-8)$

(f) ☐ $x(t) = 2\cos^2(\pi t)$

(g) ☐ $x(t) = \displaystyle\int_{-\infty}^{t} e^{-t+\tau}\delta(\tau-4)\,d\tau$

(h) ☐ $x(t) = -e^{-t}u(t) + \delta(t)$

Each of the time signals above has a Fourier transform that might be in the list below.

[1] $X(j\omega) = \dfrac{j\omega}{1+j\omega}$

[2] $X(j\omega) = \dfrac{1}{1+j\omega}$

[3] $X(j\omega) = \dfrac{e^{-j4\omega}}{1+j\omega}$

[4] $X(j\omega) = 2e^{-j4\omega}\,\dfrac{\sin(4\omega)}{\omega}$

[5] $X(j\omega) = j2e^{-j4\omega}\sin(4\omega)$

[6] $X(j\omega) = \pi\{u(\omega) - u(\omega-2\pi)\}$

[7] $X(j\omega) = \pi\{2\delta(\omega) + \delta(\omega-2\pi) + \delta(\omega+2\pi)\}$

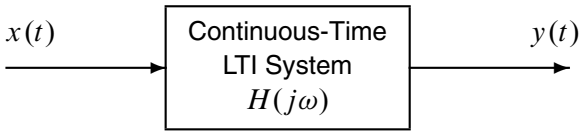
[8] $X(j\omega) = e^{-j4\omega}$

[9] $X(j\omega) = 2[\pi\delta(\omega-\pi) + \delta(\omega+\pi)]^2$

[10] $X(j\omega) = \pi\{u(\omega+2\pi) - u(\omega-2\pi)\}$

[None] $X(j\omega)$ not in the list above.

PROBLEM sp-08-Q.3.3:

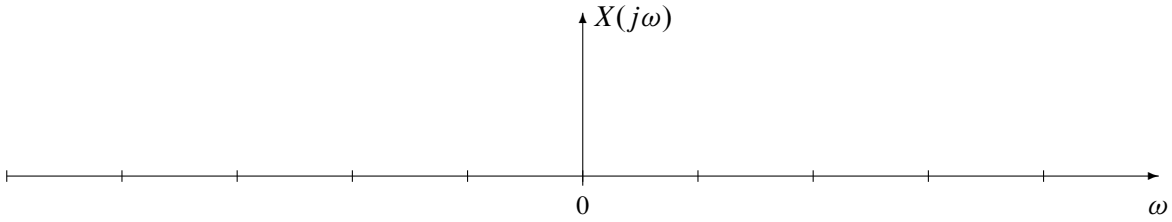


The periodic input to the above system is defined by the equation:

$$x(t) = \sum_{k=-2}^{2} a_k e^{j100kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1/\pi}{4+k^2} & k \neq 0 \\ 0.1 & k = 0 \end{cases}$$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit.

$X(j\omega) =$



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{500}{100 + j\omega}$$

Determine the ***magnitude*** of $H(j\omega)$ at the frequencies $\omega = 0, 100$, and ∞.

$|H(j0)| =$

$|H(j100)| =$

$|H(j\infty)| =$

(c) For $x(t)$ given above, the output signal can be written as $y(t) = \sum_{k=-2}^{2} b_k e^{j100kt}$

Determine the values of the parameters b_0 and b_1.

$b_0 =$

$b_1 =$

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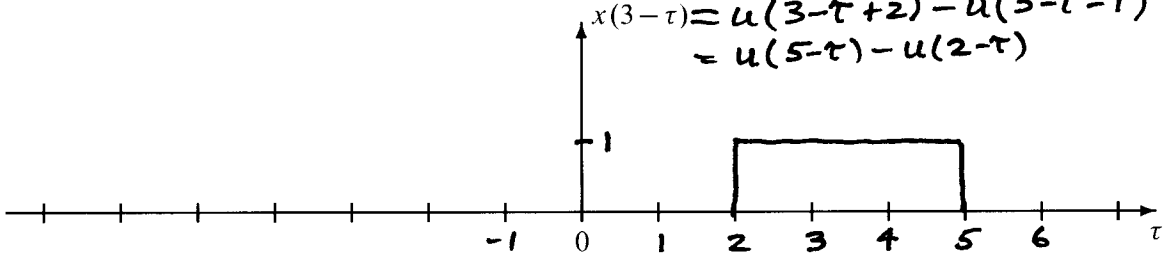
PROBLEM sp-08-Q.3.1:

(a) Assume that $x(t) = u(t+2) - u(t-1)$. Plot $x(3-\tau)$ as a function of τ.

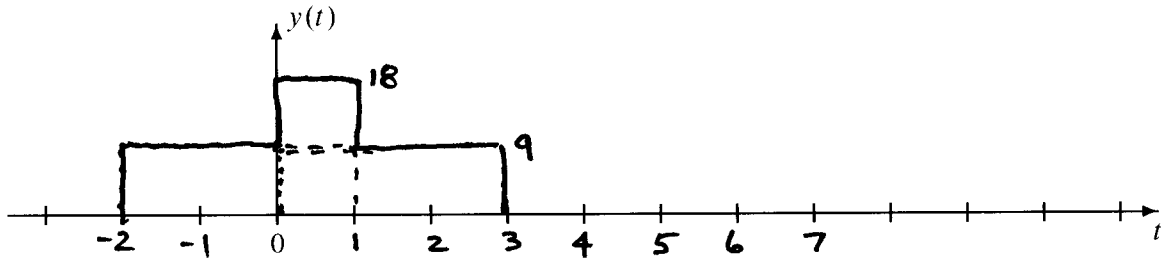
$x(3-\tau) = u(3-\tau+2) - u(3-\tau-1)$

$= u(5-\tau) - u(2-\tau)$



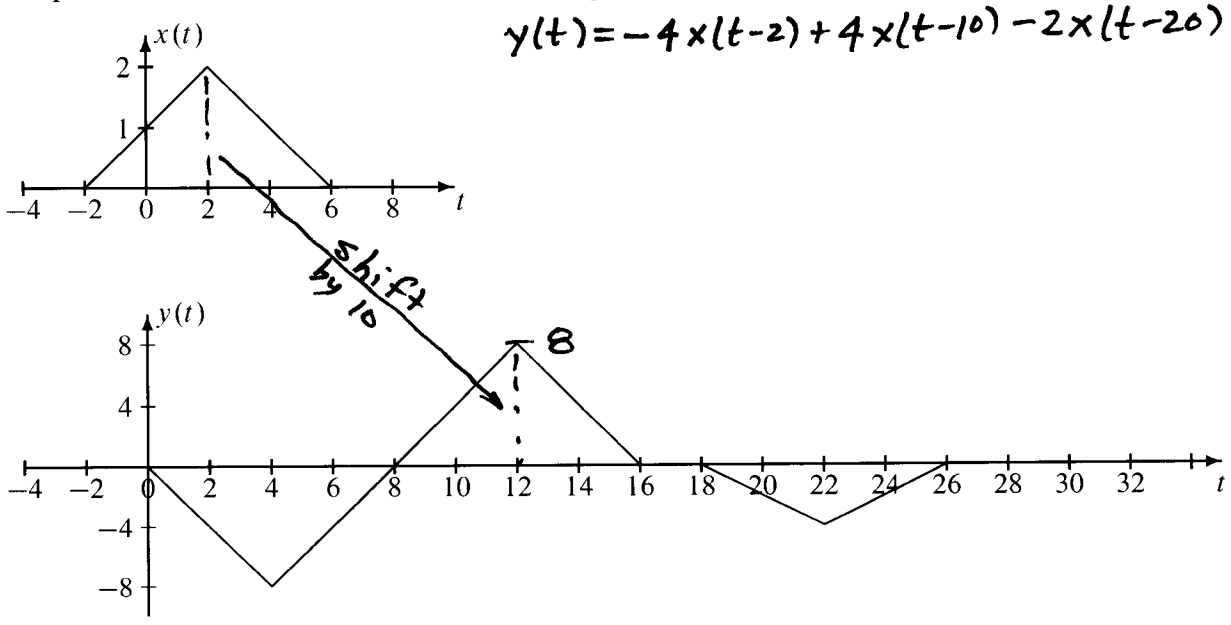
(b) If the input to an LTI system is $x(t) = u(t+2) - u(t-1)$ from part (a), determine the output signal $y(t) = x(t) * h(t)$ when $h(t) = 9\delta(t) + 9\delta(t-2)$. Give your answer as a carefully labeled sketch showing numerical values along the time axis, and also values for the amplitudes.

$y(t) = x(t) * \{9\delta(t) + 9\delta(t-2)\} = 9x(t) + 9x(t-2)$



(c) *Deconvolution:* When the input to an LTI system is $x(t)$, the output is the signal $y(t) = x(t) * h(t)$ plotted below. Determine a formula for the impulse response $h(t)$ of the system.

$y(t) = -4x(t-2) + 4x(t-10) - 2x(t-20)$



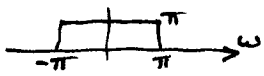
$h(t) = -4\delta(t-2) + 4\delta(t-10) - 2\delta(t-20)$

PROBLEM sp-08-Q.3.2:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write your answers in the boxes provided.*** (The operator $*$ denotes convolution.)

(a) [5] $x(t) = \delta(t) - \delta(t-8)$ $\xrightarrow{F.T.}$ $1 - e^{-j\omega 8} = e^{-j\omega 4}\underbrace{(e^{j\omega 4} - e^{-j\omega 4})}_{2j\sin(4\omega)}$

(b) [8] $x(t) = \cos(\pi t)\delta(t-4) = \cos(4\pi)\delta(t-4) = \delta(t-4) \xrightarrow{F.T.} e^{-j4\omega}$

(c) [10] $x(t) = 2\cos(\pi t)\dfrac{\sin(\pi t)}{t}$ $\xrightarrow{F.T.}$  Then shift up by π and down by π

(d) [2] $x(t) = \delta(t+2) * \{e^{-t+1}u(t-1)\} * \delta(t-1) = e^{-t}u(t)$ The time shifts cancel

(e) [4] $x(t) = u(t) - u(t-8)$ $\xrightarrow{F.T.}$ $e^{-j4\omega}\dfrac{\sin(8\omega/2)}{\omega/2}$

Rect shifted by 4

(f) [7] $x(t) = 2\cos^2(\pi t) = 1 + \cos(2\pi t) \xrightarrow{F.T.} 2\pi\delta(\omega) + \pi\delta(\omega - 2\pi) + \pi\delta(\omega + 2\pi)$

(g) [3] $x(t) = \displaystyle\int_{-\infty}^{t} e^{-t+\tau}\delta(\tau-4)\,d\tau = e^{-(t-4)}u(t-4) \xrightarrow{F.T.} e^{-j4\omega}\frac{1}{1+j\omega}$

(h) [1] $x(t) = -e^{-t}u(t) + \delta(t)$ $\xrightarrow{F.T.}$ $\dfrac{-1}{1+j\omega} + 1 = \dfrac{j\omega}{1+j\omega}$

Each of the time signals above has a Fourier transform that might be in the list below.

[1] $X(j\omega) = \dfrac{j\omega}{1+j\omega}$

[2] $X(j\omega) = \dfrac{1}{1+j\omega}$

[3] $X(j\omega) = \dfrac{e^{-j4\omega}}{1+j\omega}$

[4] $X(j\omega) = 2e^{-j4\omega}\dfrac{\sin(4\omega)}{\omega}$

[5] $X(j\omega) = j2e^{-j4\omega}\sin(4\omega)$

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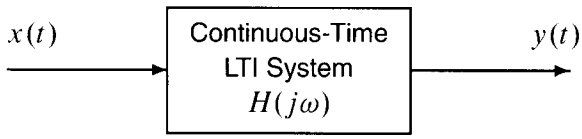
[9] $X(j\omega) = 2[\pi\delta(\omega - \pi) + \delta(\omega + \pi)]^2$

[10] $X(j\omega) = \pi\{u(\omega + 2\pi) - u(\omega - 2\pi)\}$

[None] $X(j\omega)$ not in the list above.

PROBLEM sp-08-Q.3.3:



The periodic input to the above system is defined by the equation:

$a_1 = a_{-1} = \frac{1}{5\pi}$

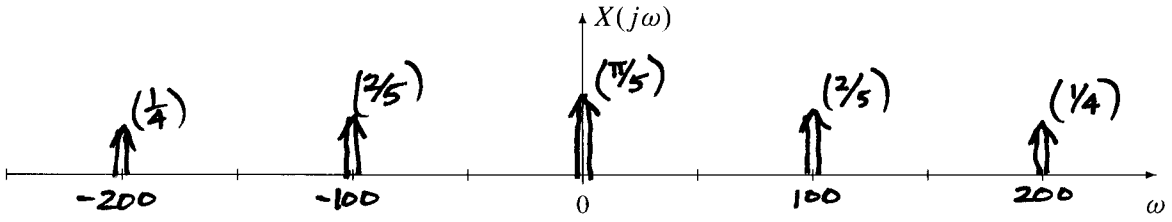
$$x(t) = \sum_{k=-2}^{2} a_k e^{j100kt}, \quad \text{where} \quad a_k = \begin{cases} \frac{1/\pi}{4+k^2} & k \neq 0 \\ 0.1 & k = 0 \end{cases}$$

$a_2 = a_{-2} = 1/8\pi$

$a_0 = 1/10$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit. $X(j\omega) = 2\pi \sum a_k \delta(\omega - 100k)$

$$X(j\omega) = \frac{2\pi}{10}\delta(\omega) + \frac{2}{5}\delta(\omega-100) + \frac{2}{5}\delta(\omega+100) + \frac{1}{4}\delta(\omega-200) + \frac{1}{4}\delta(\omega+200)$$



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{500}{100 + j\omega}$$

Determine the ***magnitude*** of $H(j\omega)$ at the frequencies $\omega = 0, 100$, and ∞.

$|H(j0)| = 5$

$H(j0) = \frac{500}{100} = 5$

$|H(j100)| = \frac{5}{\sqrt{2}} e^{-j\pi/4}$

$H(j100) = \frac{500}{100+j100} = \frac{500}{100\sqrt{2}\, e^{j\pi/4}} = \frac{5}{\sqrt{2}} e^{-j\pi/4}$

$\frac{5}{\sqrt{2}} \approx 3.5355$, $-\pi/4 \approx -0.785$

$|H(j\infty)| = 0$

$H(j\infty) \Rightarrow \frac{500}{j\omega} \rightarrow 0$

(c) For $x(t)$ given above, the output signal can be written as $y(t) = \sum_{k=-2}^{2} b_k e^{j100kt}$

Determine the values of the parameters b_0 and b_1.

$b_0 = 1/2$

$b_1 = \frac{1}{\pi\sqrt{2}} e^{-j\pi/4}$

$b_1 = 0.225 e^{-j0.785}$

$b_k = a_k H(j100k)$

$b_0 = \left(\frac{1}{10}\right)(5) = 1/2$

$b_1 = \left(\frac{1}{5\pi}\right)\left(\frac{5}{\sqrt{2}} e^{-j\pi/4}\right) = \frac{1}{\sqrt{2}\pi} e^{-j\pi/4}$

$\frac{1}{\sqrt{2}\pi} \approx 0.225$