

GEORGIA INSTITUTE OF TECHNOLOGY
 SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #3

DATE: 1-Apr-05

COURSE: ECE-2025

NAME: _____
 LAST, FIRST

GT #: _____
 (ex: gtz123q)

3 points

3 points

3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Chang)		L06:Thur-Noon (Ingram)
	L07:Tues-1:30pm (Chang)		L08:Thurs-1:30pm (Zhou)
L01:M-3pm (Williams)	L09:Tues-3pm (Casinovi)	L02:W-3pm (Juang)	L10:Thur-3pm (Zhou)
L03:M-4:30pm (Casinovi)	L11:Tues-4:30pm (Casinovi)	L04:W-4:30pm (Juang)	GTSav: (Moore)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
 Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
 Only these answers will be graded. Circle your answers, or write them in the boxes provided.
 If space is needed for scratch work, use the backs of previous pages.

<i>Problem</i>	<i>Value</i>	<i>Score</i>
1	25	
2	25	
3	25	
4	25	
No Rec	-3	

PROBLEM sp-05-Q.3.1:

- (a) Determine the frequency response of the FIR system:

$$y[n] = 10x[n - 2] - 10x[n - 5]$$

Give your answer as a formula **in the following form:** $H(e^{j\hat{\omega}}) = j e^{-j\alpha\hat{\omega}} \beta \sin(\lambda\hat{\omega})$ by finding numerical values for α , β and λ .

$\alpha =$	$\beta =$	$\lambda =$
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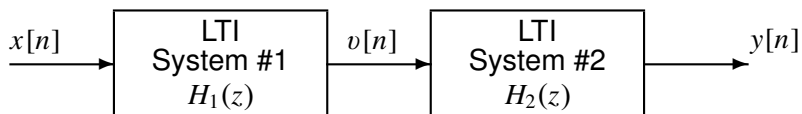
- (b) For the system in part (a), determine the output signal $y[n]$ when the input signal is

$$x[n] = \sqrt{7} + 100 \cos(0.1\pi n)$$

- (c) Write a few lines of MATLAB code that would compute the specific values of the frequency response needed in part (b).

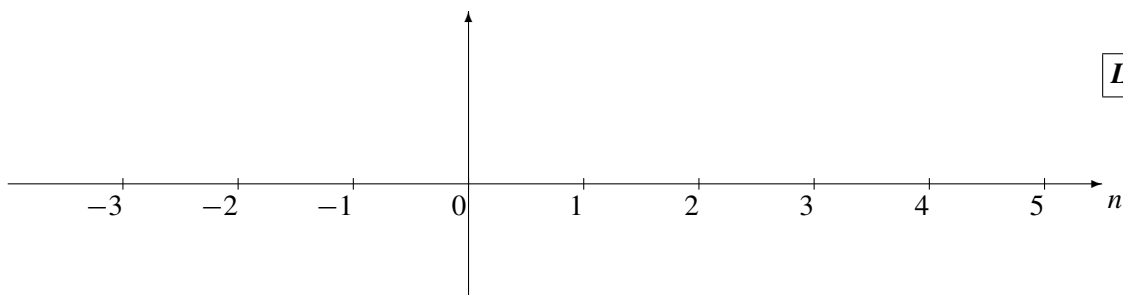
PROBLEM sp-05-Q.3.2:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.



System #1 is an IIR filter described by the system function: $H_1(z) = \frac{z^{-2} - 10z^{-3}}{1 - 0.9z^{-1}}$

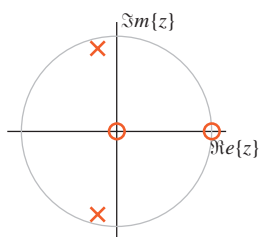
- (a) Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a **plot**.



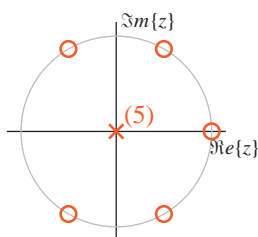
- (b) Determine the **difference equation** of the second system so that the following condition is true: When the input is a *shifted impulse*, $x[n] = \delta[n - 1]$, the output is another shifted impulse, $y[n] = \delta[n - 3]$. The difference equation must be given in the following form: $y[n] = a_1y[n - 1] + b_0v[n] + b_1v[n - 1]$, so find the numerical values for a_1 , b_0 and b_1 .

$a_1 =$	$b_0 =$	$b_1 =$
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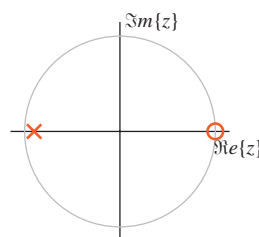
PROBLEM sp-05-Q.3.3:



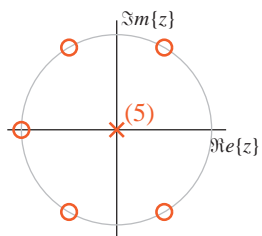
Pole-Zero Plot #1



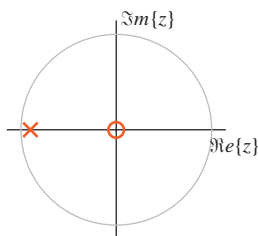
Pole-Zero Plot #2



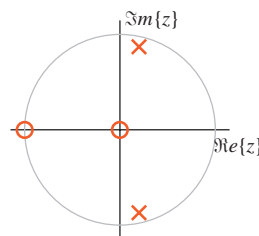
Pole-Zero Plot #3



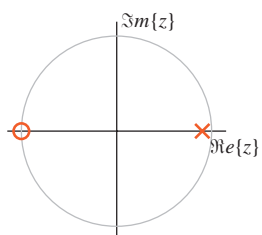
Pole-Zero Plot #4



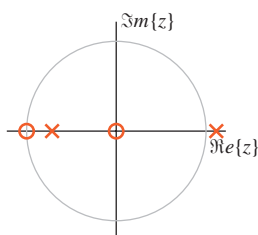
Pole-Zero Plot #5



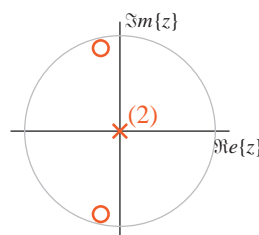
Pole-Zero Plot #6



Pole-Zero Plot #7



Pole-Zero Plot #8



Pole-Zero Plot #9

For each of systems below determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. *Note:* the unit circle is shown for reference.

Enter your answers here:

$S_1 : y[n] = -0.4y[n - 1] - 0.8y[n - 2] + 1.6x[n] - 1.6x[n - 1]$

$S_2 : H(z) = (45 + 18z^{-1} + 36z^{-2})$

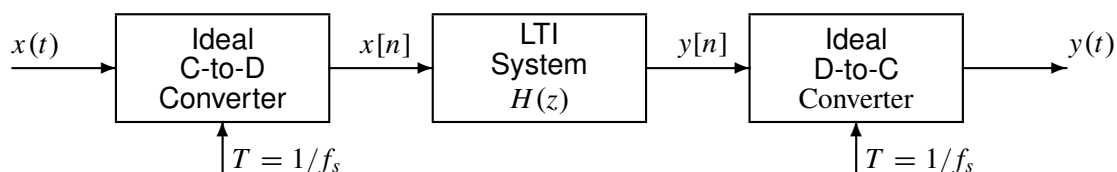
$S_3 : y[n] = \frac{5}{3}x[n] - \frac{5}{3}x[n - 1] + \frac{5}{3}x[n - 2] - \frac{5}{3}x[n - 3] + \frac{5}{3}x[n - 4] - \frac{5}{3}x[n - 5]$

$S_4 : h[n] = \frac{1}{2}\delta[n] + 0.95(0.9)^{n-1}u[n - 1]$

$S_5 : y[n] = -0.9y[n - 1] + x[n]$

$S_6 : H(z) = \frac{5}{3}(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5})$

PROBLEM sp-05-Q.3.4:



In all parts below, the sampling rates of the C-to-D and D-to-C converters **are equal to** $f_s = 80$ **samples/sec,**

and the LTI system is a 20-point running-sum filter whose system function is $H(z) = \frac{1 - z^{-20}}{1 - z^{-1}}$.

- (a) Determine the DC response of the digital filter, i.e., the output $y[n]$ when the input is $x[n] = 1$.
- (b) If the input signal is $x(t) = 10 \cos(22\pi t)$, and the sampling rate of the C-to-D and D-to-C converters is $f_s = 80$ samples/sec, determine the output signal, $y(t)$.
- (c) If the input signal is a sinusoid of the form $x(t) = \cos(2\pi f_0 t + \phi)$, and the sampling rates are $f_s = 80$ samples/sec, determine a value for the input frequency f_0 so that the output signal is zero. **Explain.**

$f_0 =$ Hz

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 LAST, FIRST

GT #: **VERSION #1**
 (ex: gtz123q)

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PROBLEM sp-05-Q.3.1:

- (a) Determine the frequency response of the FIR system:

$$y[n] = 10x[n - 2] - 10x[n - 5]$$

Give your answer as a formula *in the following form*: $H(e^{j\hat{\omega}}) = je^{-j\alpha\hat{\omega}} \beta \sin(\lambda\hat{\omega})$ by finding numerical values for α , β and λ .

$\alpha = 3.5$	$\beta = 20$	$\lambda = 1.5$
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$$H(e^{j\hat{\omega}}) = 10e^{-j2\hat{\omega}} - 10e^{-j5\hat{\omega}} = 10e^{-j3.5\hat{\omega}} (e^{j1.5\hat{\omega}} - e^{-j1.5\hat{\omega}})$$

$$H(e^{j\hat{\omega}}) = j20e^{-j3.5\hat{\omega}} \sin(1.5\hat{\omega})$$

- (b) For the system in part (a), determine the output signal
- $y[n]$
- when the input signal is

$$x[n] = \sqrt{7} + 100 \cos(0.1\pi n)$$

$$H(e^{j\hat{\omega}}) = j20e^{-j3.5\hat{\omega}} \sin(1.5\hat{\omega}) \text{ evaluated at } \hat{\omega} = 0, 0.1\pi$$

$$\text{Gives } H(e^{j0}) = 0 \text{ and } H(e^{j0.1\pi}) = 9.08e^{j0.15\pi} = 9.08e^{j0.471}$$

$$\text{Thus, } y[n] = (\sqrt{7})(0) + (100)(9.08) \cos(0.1\pi n + 0.15\pi) = 907.98 \cos(0.1\pi n + 0.471)$$

- (c) Write a few lines of MATLAB code that would compute the specific values of the frequency response needed in part (b).

```

bb = [ 0, 0, 10, 0, 0, -10 ];
HH = freqz( bb, 1, [0,0.1*pi] )
abs(HH), angle(HH)

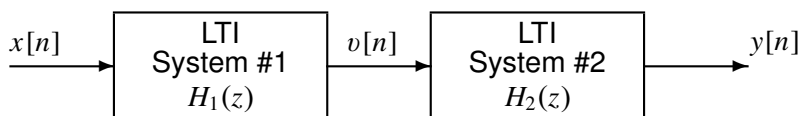
```

Note: you should include the change to polar form with `abs` and `angle`, because `HH` will be displayed in rectangular form.

Note: it would also be OK to write code for the expression in part (a), and then evaluate at the two frequencies present in $x[n]$ of part (b).

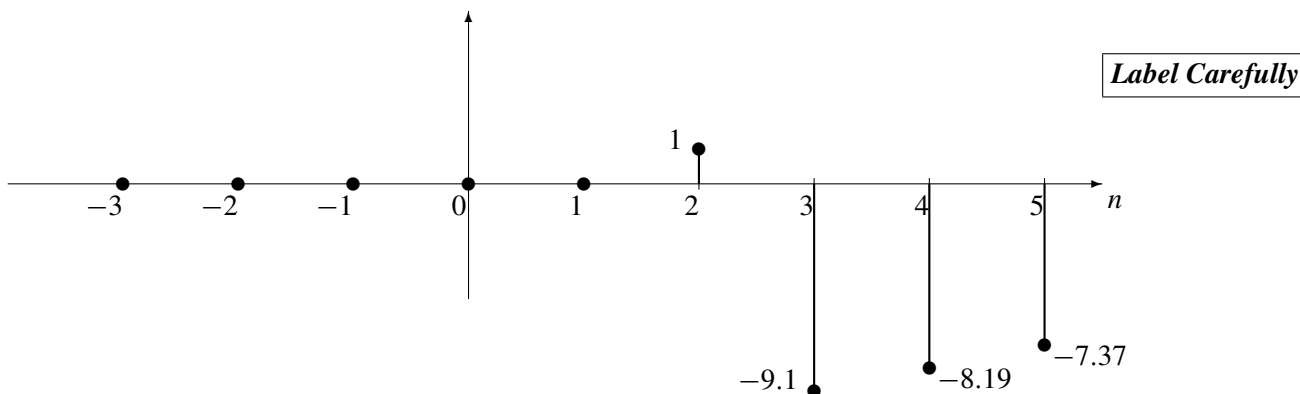
PROBLEM sp-05-Q.3.2:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.



System #1 is an IIR filter described by the system function: $H_1(z) = \frac{z^{-2} - 10z^{-3}}{1 - 0.9z^{-1}}$

- (a) Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a *plot*.



Interpret the numerator as two delays, and write via the inverse z -transform:

$$h_1[n] = (0.9)^{n-2}u[n-2] - 10(0.9)^{n-3}u[n-3]$$

Then evaluate over the range needed for the plot:

$$h_1[n] = \delta[n-2] - 9.1\delta[n-3] - 8.19\delta[n-4] - 7.3710\delta[n-5] - 6.6339\delta[n-6] + \dots$$

- (b) Determine the *difference equation* of the second system so that the following condition is true: When the input is a *shifted impulse*, $x[n] = \delta[n-1]$, the output is another shifted impulse, $y[n] = \delta[n-3]$. The difference equation must be given in the following form: $y[n] = a_1y[n-1] + b_0v[n] + b_1v[n-1]$, so find the numerical values for a_1 , b_0 and b_1 .

$$Y(z) = H_2(z)H_1(z)X(z) \text{ which can be solved for } H_2(z) = \frac{Y(z)}{H_1(z)X(z)}$$

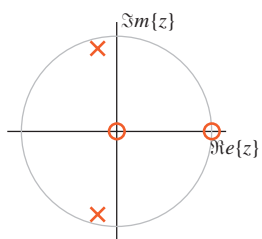
Then $X(z) = z^{-1}$ and $Y(z) = z^{-3}$, so we get

$$H_2(z) = \frac{z^{-3}}{\left(\frac{z^{-2} - 10z^{-3}}{1 - 0.9z^{-1}}\right)(z^{-1})} = \frac{1 - 0.9z^{-1}}{1 - 10z^{-1}} = \frac{b_0 + b_1z^{-1}}{1 - a_1z^{-1}}$$

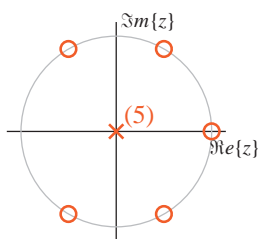
Then we identify coefficients to write the difference equation:

$$y[n] = 10y[n-1] + v[n] - 0.9v[n-1]$$

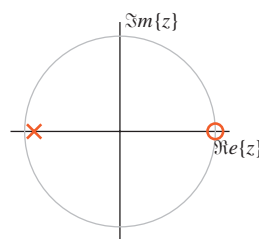
PROBLEM sp-05-Q.3.3:



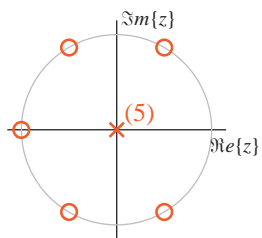
Pole-Zero Plot #1



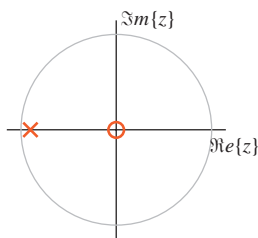
Pole-Zero Plot #2



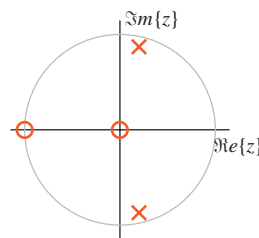
Pole-Zero Plot #3



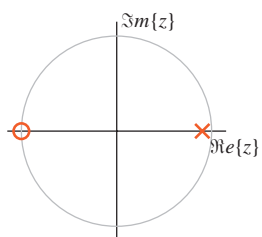
Pole-Zero Plot #4



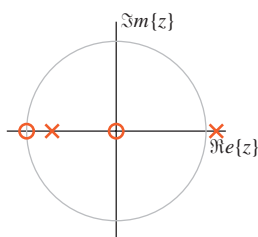
Pole-Zero Plot #5



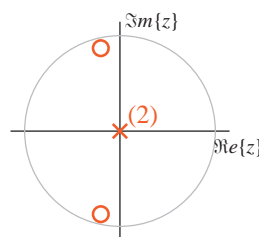
Pole-Zero Plot #6



Pole-Zero Plot #7



Pole-Zero Plot #8



Pole-Zero Plot #9

For each of systems below determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. *Note:* the unit circle is shown for reference.

Enter your answers here:

\mathcal{S}_1 : #1 \mathcal{S}_2 : #9 \mathcal{S}_3 : #2 \mathcal{S}_4 : #7 \mathcal{S}_5 : #5 \mathcal{S}_6 : #4

$$\mathcal{S}_1 : y[n] = -0.4y[n-1] - 0.8y[n-2] + 1.6x[n] - 1.6x[n-1]$$

$$\mathcal{S}_2 : H(z) = (45 + 18z^{-1} + 36z^{-2})$$

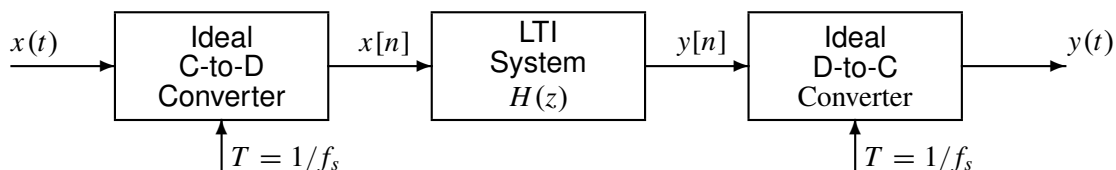
$$\mathcal{S}_3 : y[n] = \frac{5}{3}x[n] - \frac{5}{3}x[n-1] + \frac{5}{3}x[n-2] - \frac{5}{3}x[n-3] + \frac{5}{3}x[n-4] - \frac{5}{3}x[n-5]$$

$$\mathcal{S}_4 : h[n] = \frac{1}{2}\delta[n] + 0.95(0.9)^{n-1}u[n-1]$$

$$\mathcal{S}_5 : y[n] = -0.9y[n-1] + x[n]$$

$$\mathcal{S}_6 : H(z) = \frac{5}{3}(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5})$$

PROBLEM sp-05-Q.3.4:



In all parts below, the sampling rates of the C-to-D and D-to-C converters **are equal to** $f_s = 80$ **samples/sec**, and the LTI system is a 20-point running-sum filter whose system function is $H(z) = \frac{1 - z^{-20}}{1 - z^{-1}}$.

- (a) Determine the DC response of the digital filter, i.e., the output $y[n]$ when the input is $x[n] = 1$.

$$y[n] = \sum_{k=0}^{19} x[n - k] = \sum_{k=0}^{19} 1 = 20$$

- (b) If the input signal is $x(t) = 10 \cos(22\pi t)$, and the sampling rate of the C-to-D and D-to-C converters is $f_s = 80$ samples/sec, determine the output signal, $y(t)$.

Need to evaluate the frequency response at $\hat{\omega} = 22\pi/80$, or $H(z)$ at $z = e^{j22\pi/80}$

which gives $H(e^{j\hat{\omega}}) \Big|_{\hat{\omega}=22\pi/80} = \frac{1 - e^{-j22\pi/4}}{1 - e^{-j22\pi/80}} = 1.689e^{-j0.612\pi}$.

Equivalent values for the phase are -1.924 rads, or -110.25° .

Then $y(t) = (10)(1.689) \cos(22\pi t - 0.612\pi) = 16.89 \cos(22\pi t - 1.924)$.

- (c) If the input signal is a sinusoid of the form $x(t) = \cos(2\pi f_0 t + \phi)$, and the sampling rates are $f_s = 80$ samples/sec, determine a value for the input frequency f_0 so that the output signal is zero. **Explain.**

$f_0 = \boxed{4 \text{ Hz}}$

One number would be sufficient, but here is a general formula:

$f_0 = \boxed{(80/20)\ell = 4\ell \text{ Hz}}$ where $\ell = 1, 2, \dots, 19$

$H(e^{j\hat{\omega}}) = \frac{1 - e^{-j20\hat{\omega}}}{1 - e^{-j\hat{\omega}}}$ is zero when the numerator is zero.

Thus, $1 - e^{-j20\hat{\omega}} = 0$ which leads to the condition that $20\hat{\omega} = 2\pi\ell$, or $\hat{\omega} = 2\pi\ell/20$.

Converting to analog frequency via the relationship for a D-C converter: $f = f_s\hat{\omega}/(2\pi)$

gives $f = f_s(2\pi\ell/20)/(2\pi) = (80/20)\ell = 4\ell \text{ Hz}$.

However, we must exclude $\ell = 0$ and ℓ equal to a multiple of 20.