GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

ECE2026 – Fall 2025
Quiz #3
November 14, 2025

NAME: ___________________________ **GT Username: _____________________**
FIRST **LAST** **(e.g., gtinit0101)**

Important Notes:

- Write your name on each page.
- Closed book, but a calculator is permitted.
- One page (8.5”x11”) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Partial credit for incorrect answers may be granted ONLY when you JUSTIFY your correct reasoning CLEARLY.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If more space is needed for scratch work, use the backs of previous pages.

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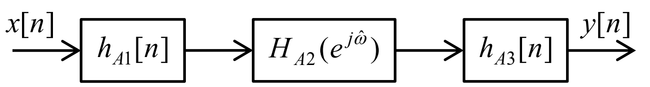
PROBLEM Fall-25-Q.3.1:

This problem involves discrete-time LTI systems. For Part a) and b) below, find the impulse response of the overall system depicted in the respective figures (System A) and (System B).

a) System A



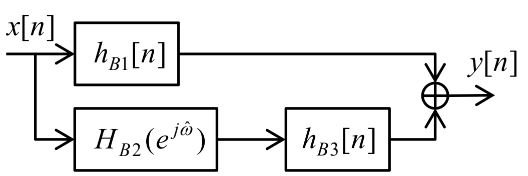
$h_{A1}[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

$H_{A2}(e^{j\hat{\omega}}) = e^{-j20\hat{\omega}}$; $h_{A3}[n] = \delta[n] - \delta[n-1]$

Solution:

$$h_A[n] = \delta[n-20] - \delta[n-23]$$

b) System B



$h_{B1}[n] = -\delta[n-11] - \delta[n-12]$

$H_{B2}(e^{j\hat{\omega}}) = e^{-j10\hat{\omega}}$; $h_{B3}[n] = \delta[n] + \delta[n-1]$

Solution:

$$h_B[n] = \delta[n-10] - \delta[n-12]$$

c) An input given as $x[n] = 3 + 2\,cos(n\pi/2)$ is being processed by System B. Find the output $y[n]$.

Solution:

$$h_B[n] = \delta[n-10] - \delta[n-12], \qquad H_B(e^{j\hat{\omega}}) = e^{-j10\hat{\omega}} - e^{-j12\hat{\omega}}$$

$$H_B(e^{j\hat{\omega}})\Big|_{\hat{\omega}=0} = 1 - 1 = 0$$

$$H_B(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\frac{\pi}{2}} = e^{-j10\frac{\pi}{2}} - e^{-j12\frac{\pi}{2}} = e^{-j5\pi} - e^{-j6\pi} = -1 - 1 = -2 = 2e^{j\pi}$$

$$y[n] = 3 \times 0 + 2 \times 2\,cos(n\pi/2 + \pi) = 4\,cos(\tfrac{n\pi}{2} + \pi)$$

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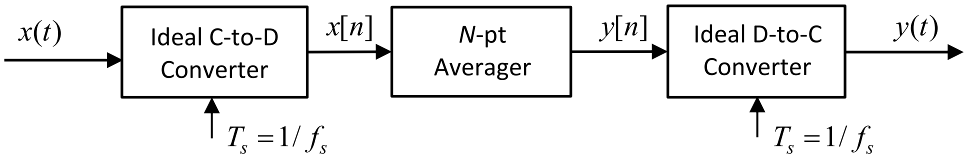
PROBLEM Fall-25-Q.3.2:
Consider the following system for discrete-time filtering of a continuous-time signal:



The discrete-time system is an N-point averager, defined by the difference equation
$y[n] = \frac{1}{N}\sum_{i=0}^{N-1} x[n-i]$.

(a) Suppose the continuous-time input is given as $x(t) = 1 + 2\,cos(8\pi t) + 3\,cos(20\pi t - \pi/3)$ for $-\infty < t < \infty$ and N is chosen to be 7. Determine the lowest sampling rate f_s in samples/s such that the continuous-time output takes the form, $y(t) = A + B\,cos(20\pi t + \varphi)$ for $-\infty < t < \infty$ and where both A and $B \neq 0$; i.e., only the 4Hz sinusoid is eliminated. The sampling rates at the C-to-D and D-to-C are assumed to be identical and you are not asked to find $A, B,$ and φ.

Answer: An N-point averager has zeros at digital frequencies, $\hat{\omega} = k\frac{2\pi}{N}$, where k is non-zero integer. With $N = 7$, the digital frequencies at which the frequency response is 0 are thus $\hat{\omega} = \pm\frac{2\pi}{7}, \pm\frac{4\pi}{7}, \pm\frac{6\pi}{7}$ for $-\pi < \hat{\omega} \leq \pi$. Since the 10 Hz sinusoid remains in the output, the sampling rate must be greater than 20 and the digital frequency that corresponds to 4Hz cannot be greater than $2\pi\frac{4}{20} = \frac{2\pi}{5}$. This eliminates the null frequencies at $\pm\frac{4\pi}{7}, \pm\frac{6\pi}{7}\pi$. We therefore conclude

$$\hat{\omega} = \frac{2\pi}{7} = \frac{8\pi}{f_s} \Rightarrow f_s = 28 \text{ Samples/s}$$

(b) Suppose now the sampling rate is chosen to be 24 samples/s and the continuous time input is given as $x(t) = 3 + cos(48\pi t + \frac{\pi}{3})$ for $-\infty < t < \infty$. The system is a 12-pt averager, i.e., $N = 12$. Obtain the output $y(t)$. Again, assume the sampling rates at C-to-D and D-to-C are identical.

Answer:

$$x(t) = 3 + cos(48\pi t + \frac{\pi}{3}) \text{ for } -\infty < t < \infty$$
$$x[n] = 3 + cos(48\pi n/24 + \pi/3) = 3.5$$

$$H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=0} = e^{-j\frac{11}{2}\hat{\omega}}\frac{sin(\frac{12}{2}\hat{\omega})}{12\,sin(\frac{1}{2}\hat{\omega})} = 1$$

$$y[n] = 3.5 \times 1 = 3.5$$

$$y(t) = 3.5 \text{ for } -\infty < t < \infty$$

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PROBLEM Fall-25-Q.3.3:

(a) Find the discrete-time sequence $x[n]$, the Discrete Time Fourier Transform (DTFT) of which is given by $X(e^{j\hat{\omega}}) = e^{-j5\hat{\omega}}(2 + 4\,cos^2(\,2\hat{\omega}))$. Express your result as sum of delayed unit impulses.

Answer:

$$X(e^{j\hat{\omega}}) = e^{-j5\hat{\omega}}(2 + 4\,cos^2(\,2\hat{\omega})) = e^{-j5\hat{\omega}}(2 + e^{j4\hat{\omega}} + 2 + e^{-j4\hat{\omega}})$$
$$= e^{-j\hat{\omega}} + 4e^{-j5\hat{\omega}} + e^{-j9\hat{\omega}}$$

$$x[n] = \delta[n-1] + 4\delta[n-5] + \delta[n-9]$$

(b) Based on the code:

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n=1:1000; yn = conv( [1 0 0 -1], cos(pi*n) )
```

give the value of `yn(6)`.

Answer:

$cos(\,n\pi) = (-1)^n = [-1,\ 1, -1,\ 1, -1,\ 1, -1, \cdots]$

After $n>3$, the output alternates between $1 \times 1 + (-1) \times 0 + 1 \times 0 + (-1) \times (-1) = 2$ for $n = 4, 6, 8, \ldots$ and $-1 \times 1 + 1 \times 0 + (-1) \times 0 + 1 \times (-1) = -2$ for $n = 5, 7, 9, \ldots$

`yn(6)=2`

(c) Given

$x[n] = \frac{sin(\pi n/6)}{\pi n} * \left(\delta[n-1] + \frac{sin(\pi n/4)}{\pi n}\right)$ where * denotes convolution, evaluate $x[n]$ at $n = 1$.

Answer:

$$x[n] = \frac{sin(\,\pi n/6)}{\pi n} * \left(\delta[n-1] + \frac{sin(\,\pi n/4)}{\pi n}\right) = \frac{sin(\,\pi(n-1)/6)}{\pi(n-1)} + \frac{sin(\,\pi n/6)}{\pi n}$$

$$x[n]\Big|_{n\,=\,1} = \lim_{n\to 1} \frac{sin(\,\pi(n-1)/6)}{\pi(n-1)} + \frac{sin(\,\pi/6)}{\pi} = \frac{1}{6} + \frac{1}{2\pi} = \frac{\pi+3}{6\pi} = 0.3258$$