GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 – Fall 2024
Exam #3

NAME: Solutions ______________________ GTemail: ______________________

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- Write your name at the top of EACH PAGE.
- DO NOT unstaple the test.
- Closed book, except for one two-sided page $(8.5'' \times 11'')$ of hand-written notes permitted.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. (i.e., write 0.4π or $\frac{2\pi}{5}$ instead of 1.257)
- All angles/phase must be expressed in the range $(-\pi, \pi]$ for full credit.
- You must show your work in the space provided on the exam paper itself. Only these answers with shown work can receive credit. Write your answers in the **boxes/spaces** provided. DO NOT write on the backs of the pages.
- All exams will be collected and uploaded to gradescope for grading.
- **NOTE: CAREFULLY REMOVE THE REFERENCE TABLES AT THE END WITHOUT REMOVING THE STAPLE BINDING THE REST OF YOUR EXAM PAGES**

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
Total		

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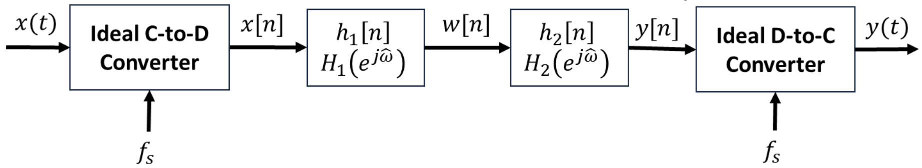
PROBLEM 1:

Consider the system below of C-to-D/D-to-C converters and two ideal LTI systems connected for all parts below.



- $H_1(e^{j\hat{\omega}}) = (4 + 4\cos(9\hat{\omega}))e^{-j12\hat{\omega}}$
- $H_2(e^{j\hat{\omega}}) = \begin{cases} 0 & 0 < |\hat{\omega}| < 0.6\pi \\ 3e^{-j5\hat{\omega}} & 0.6\pi \leq |\hat{\omega}| \leq \pi \end{cases}$

(a) Find the difference equation relating $x[n]$ to $w[n]$ through system $H_1(e^{j\hat{\omega}})$. (5 points)

$$H_1(e^{j\hat{\omega}}) = (4 + 4\cos(9\hat{\omega}))e^{-j12\hat{\omega}} \rightarrow 2e^{-j3\hat{\omega}} + 4e^{-j12\hat{\omega}} + 2e^{-j21\hat{\omega}}$$
$$h[n] = 2\delta[n-3] + 4\delta[n-12] + 2\delta[n-21]$$
$$w[n] = 2x[n-3] + 4x[n-12] + 2x[n-21]$$

Difference equation relating $x[n]$ to $w[n]$:

(b) Considering only system $H_1(e^{j\hat{\omega}})$, assume that an input of $x[n] = 2 + 4\cos(\hat{\omega}_0 n)$ resulted in an output of $w[n] = 2A$. Find the **largest value** for $\hat{\omega}_0$ in the range $\mathbf{0 < \hat{\omega}_0 \leq \frac{\pi}{2}}$ that would make this true and find the value for $A > 0$. (5 points)

The input/output suggests that $H_1(e^{j\hat{\omega}_0}) = 0$ and $H_1(e^{j0}) = A$

$$H_1(e^{j\hat{\omega}_0}) = 4 + 4\cos(9\hat{\omega}_0) = 0 \ @\hat{\omega}_0 = \frac{\pi}{9}k, k = 1,3,5,7,9$$

For the condition that the largest value be in the range $0 < \hat{\omega}_0 \leq \frac{\pi}{2} \rightarrow \hat{\omega}_0 = \frac{\pi}{3}$

$$H_1(e^{j0}) = (4 + 4\cos(9(0))) = 4 + 4 = 8 = A$$

$\hat{\omega}_0 =$

$A =$

(c) Let $x(t) = 5 + 6\cos(300\pi t) + 2\cos(700\pi t) + 9\cos(1800\pi t)$ and f_s =1000 Hz, find $y(t)$. (10 points)

$$x[n] = 5 + 6\cos(0.3\pi n) + 2\cos(0.7\pi n) + 9\cos(1.8\pi n) \rightarrow 5 + 6\cos(0.3\pi n) + 2\cos(0.7\pi n) + 9\cos(0.2\pi n)$$

$H_2(e^{j\hat{\omega}}) = 0$ for all $\hat{\omega} < 0.6\pi$ so the only remaining component for consideration will be:

$$x[n] = 2\cos(0.7\pi n)$$
$$H_1(e^{j(0.7\pi)}) = (4 + 4\cos(9 * 0.7\pi))e^{-j12(0.7\pi)} = 6.3511e^{-j0.4\pi}$$
$$H_2(e^{j(0.7\pi)}) = 3e^{-j5(0.7\pi)} = 3e^{j0.5\pi}$$
$$y[n] = (6.3511) * (3) * 2\cos(0.7\pi n - 0.4\pi + 0.5\pi) = 38.11\cos(0.7\pi n + 0.1\pi)$$
$$y(t) = 38.11\cos(700\pi t + 0.1\pi)$$

$y(t) =$

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PROBLEM 2:

Each part (5 points each) can be solved independently of each other.

The following problems might benefit from the use of the DTFT pairs and properties tables.

(a) Find $X(e^{j\hat{\omega}})$ if $x[n] = 100\frac{\sin(0.6\pi n)}{\pi n} \times \delta[n-8]$ (where × represents multiplication)

$$x[n] = 100\frac{\sin(0.6\pi n)}{\pi n} \times \delta[n-8] = 100\frac{\sin(0.6\pi(8))}{\pi(8)}\delta[n-8]$$

$$100\frac{\sin(0.6\pi(8))}{\pi(8)}\delta[n-8] = 2.34\delta[n-8]$$

$$X(e^{j\hat{\omega}}) = 2.34e^{-j8\hat{\omega}}$$

$X(e^{j\hat{\omega}}) =$

(b) Find $X(e^{j\hat{\omega}})$ if is $x[n] = 2(u[n-2] - u[n-18])$

Rectangular function (L=16) w/delay of 2 samples:

$$2(u[n-2] - u[n-18]) \rightarrow \frac{2\sin(8\hat{\omega})}{\sin\left(\frac{\hat{\omega}}{2}\right)}e^{-j\frac{15}{2}\hat{\omega}}e^{-j2\hat{\omega}} \rightarrow \frac{2\sin(8\hat{\omega})}{\sin\left(\frac{\hat{\omega}}{2}\right)}e^{-j\frac{19}{2}\hat{\omega}}$$

$X\left(e^{j\hat{\omega}}\right) =$

(c) Find the most compact (reduced) form of $x[n]$ if $X(e^{j\hat{\omega}}) = \left(\frac{4}{1-0.9e^{-j\left(\hat{\omega}-\frac{\pi}{3}\right)}} + \frac{4}{1-0.9e^{-j\left(\hat{\omega}+\frac{\pi}{3}\right)}}\right)e^{-j4\hat{\omega}}$

Right sided exponential: $\frac{8}{1-0.9e^{-j\hat{\omega}}} \rightarrow 8(0.9)^n u[n]$

Modulation by cos $\left(\frac{\pi}{3}n\right)$

Delay by 4 samples: $e^{-j4\hat{\omega}}$

$$x[n] = 8(0.9)^{(n-4)}u[n-4]\cos\left(\frac{\pi}{3}(n-4)\right)$$

$x[n] =$

(d) Find $x[n] = [2\cos(0.4\pi n) + 5\cos(0.1\pi n) + 2\cos(0.8\pi n)] * \left(6\frac{\sin(0.6\pi n)}{\pi n} - 3\frac{\sin(0.3\pi n)}{\pi n}\right)$ ***(' $*$ ' is convolution)***

$$\left(6\frac{\sin(0.6\pi n)}{\pi n} - 3\frac{\sin(0.3\pi n)}{\pi n}\right) \rightarrow \begin{cases} 0 & 0.6\pi < |\hat{\omega}| \leq \pi \\ 6 & 0.3\pi < |\hat{\omega}| \leq 0.6\pi \\ 3 & 0 \leq |\hat{\omega}| \leq 0.3\pi \end{cases}$$

Therefore:

$$x[n] = (2*6)\cos(0.4\pi n) + (3*5)\cos(0.1\pi n)$$

$$x[n] = 12\cos(0.4\pi n) + 15\cos(0.1\pi n)$$

$x[n] =$

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PROBLEM 3:

Parts a and b (10 points each) can be solved independently of each other.

(a) Let $x[n] = -2 + 5\cos\left(\frac{3\pi}{8}n + \frac{\pi}{6}\right) + 9\cos\left(\frac{7\pi}{8}n - \frac{7\pi}{9}\right)$. Let $\{X[0], \ldots, X[63]\}$ be the 64-point DFT of $\{x[0], \ldots, x[63]\}$. Many of these DFT coefficients are zero. Specify only the **nonzero** DFT coefficients in the table below with the corresponding index $k \in \{0, \ldots, 63\}$.

DFS:

$$\hat{\omega}_0 = GCD\left(\frac{3\pi}{8}, \frac{7\pi}{8}\right) = \frac{\pi}{8} \rightarrow N_0 = 16 \rightarrow N = 4 * N_0 = 64 \rightarrow \hat{\omega}_k = \frac{2\pi}{64}k = \frac{\pi}{32}k$$

The DFT is an integer multiple of the fundamental period so we will have "complete" periods in the windowed DFT and can represent each sinusoid with 2 spectral lines.

The DFS for x[n] are:

$$\hat{\omega}_0 k = 0, \pm\frac{3\pi}{8}, \pm\frac{7\pi}{8} \rightarrow a_k = 2e^{j\pi}, 2.5e^{\pm\frac{j\pi}{6}}, 4.5e^{\mp\frac{j7\pi}{9}}, k = 0, \pm 3, \pm 7$$

$$X[k] = Na_k$$

$$X[k] = 64a_k \rightarrow 128e^{j\pi}, 160e^{\pm\frac{j\pi}{6}}, 288e^{\mp\frac{j7\pi}{9}}, k = 0, \{12,52\}, \{28,36\}$$

$k \geq 0$	0	12	28	36	52		
$X[k]$	$128e^{j\pi}$	$160e^{\frac{j\pi}{6}}$	$288e^{-\frac{j7\pi}{9}}$	$288e^{\frac{j7\pi}{9}}$	$160e^{-\frac{j\pi}{6}}$		

(b) Consider the 16-point sequence defined by $x[n] = 5\delta[n-3] + 5\delta[n-13]$ for $n = 0, \ldots, 15$. Let $X[k]$ represent the 16-point DFT of $x[n]$ in the form $X[k] = A\cos(Bk)\, e^{-jCk}, k = 0, \ldots, 15$. Find $A > 0$, $0 < B \leq \pi$, $0 < C \leq \pi$ and the value of $X[0]$ (i.e., the value of the 16-point DFT at index $k = 0$).

Note: The value of $X[0]$ can be found by the sum of the values of $x[n]$:

$$X[0] = \sum_{k=0}^{15}(5\delta[n-3] + 5\delta[n-13]) = 5 + 5 = 10$$

$$x[n] = 5\delta[n-3] + 5\delta[n-13]$$

$$X(e^{j\hat{\omega}}) = 5e^{-j3\hat{\omega}} + 5e^{-j13\hat{\omega}} \rightarrow 10\cos(5\hat{\omega})\, e^{-j8\hat{\omega}}$$

$$\hat{\omega}_k = \frac{2\pi}{16}k$$

$$X[k] = 10\cos\left(\frac{5\pi}{8}k\right)e^{-j\pi k}$$

$$A = 10, B = \frac{5\pi}{8}, C = \pi$$

$A =$	$B =$	$C =$	$X[0] =$