GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 – Fall 2023
Exam #3

NAME: ______________________________ GTemail: ____________________
FIRST LAST ex: GPburDELL@gatech.edu

- Write your name at the top of EACH PAGE.
- DO NOT unstaple the test.
- Closed book, except for one two-sided page $(8.5'' \times 11'')$ of hand-written notes permitted.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to received partial credit.
- Express all angles as a fraction of π. (i.e., write 0.4π or $\frac{2\pi}{5}$ instead of 1.257)
- All angles must be expressed in the range $(-\pi, \pi]$ for full credit.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the **boxes/spaces** provided. DO NOT write on the backs of the pages.
- All exams will be collected and uploaded to gradescope for grading.

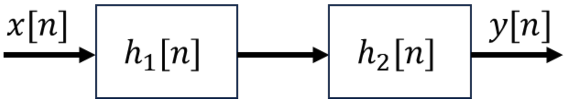
Problem	*Value*	*Score*
1	20	
2	20	
3	20	
Total		

Print Name (First Last) ______________________________

PROBLEM 1:

Parts a and b (10 points each) can be solved independently of each other.

(a) Consider the following cascade of two discrete-time LTI systems:

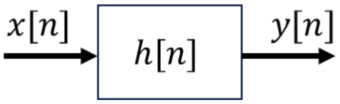


The first system has impulse response: $h_1[n] = 4\delta[n-2] + 8\delta[n-8] + 4\delta[n-14]$. The second system has frequency response: $H_2(e^{j\hat{\omega}}) = \begin{cases} 2 & |\hat{\omega}| \leq 0.35\pi \\ 0 & 0.35\pi < |\hat{\omega}| < 0.65\pi. \\ 2 & 0.65\pi \leq |\hat{\omega}| \leq \pi \end{cases}$

If $x[n] = 3 + 5\cos(0.25\pi n) + 2\cos(0.4\pi n) + 4\cos\left(\frac{11\pi}{12}n\right)$ is input to the first system, find the output $y[n]$ of the second system.

$y[n] =$

(b) Consider the following discrete-time LTI system with impulse response $h[n] = \sum_{k=0}^{8} C\delta[n-k]$.



If the input $x[n] = 5 + 8\cos\left(\frac{5\pi}{9}n + \frac{2\pi}{9}\right)$ results in the output $y[n] = 9 + M\cos(\hat{\omega}_y n + \varphi_y)$, find the values for $C, \hat{\omega}_y, M$, and φ_y.

$C =$	$\hat{\omega}_y =$	$M =$	$\varphi_y =$

Print Name (First Last) ________________________________

PROBLEM 2:

Each part (5 points each) can be solved independently of each other.

The following problems might benefit from the use of the DTFT pairs and properties tables.

(a) Evaluate the following using Parseval's Theorem: $K = \sum_{n=-\infty}^{\infty} \left| \frac{\sin(0.6\pi n)}{\pi n} \right|^2$

$K =$

(b) Find $x[n]$ if its DTFT is $X(e^{j\hat{\omega}}) = \frac{5e^{-j4\hat{\omega}}}{5+2e^{-j\hat{\omega}}}$

$x[n] =$

(c) Find $y[n] = [2\cos(0.2\pi n)\cos(0.5\pi n)] * \left(\frac{\sin(0.8\pi n)}{\pi n} - \cos(0.65\pi n)\frac{\sin(0.3\pi n)}{\pi n}\right)$ ***(where $'*'$ is convolution)***

$y[n] =$

(d) Find $y[n] = [2\cos(0.2\pi n) + \cos(0.5\pi n)] * \left(\frac{\sin(0.8\pi n)}{\pi n} - \frac{\sin(0.3\pi n)}{\pi n}\right)$ ***(where $'*'$ is convolution)***

$y[n] =$

Print Name (First Last) ________________________________

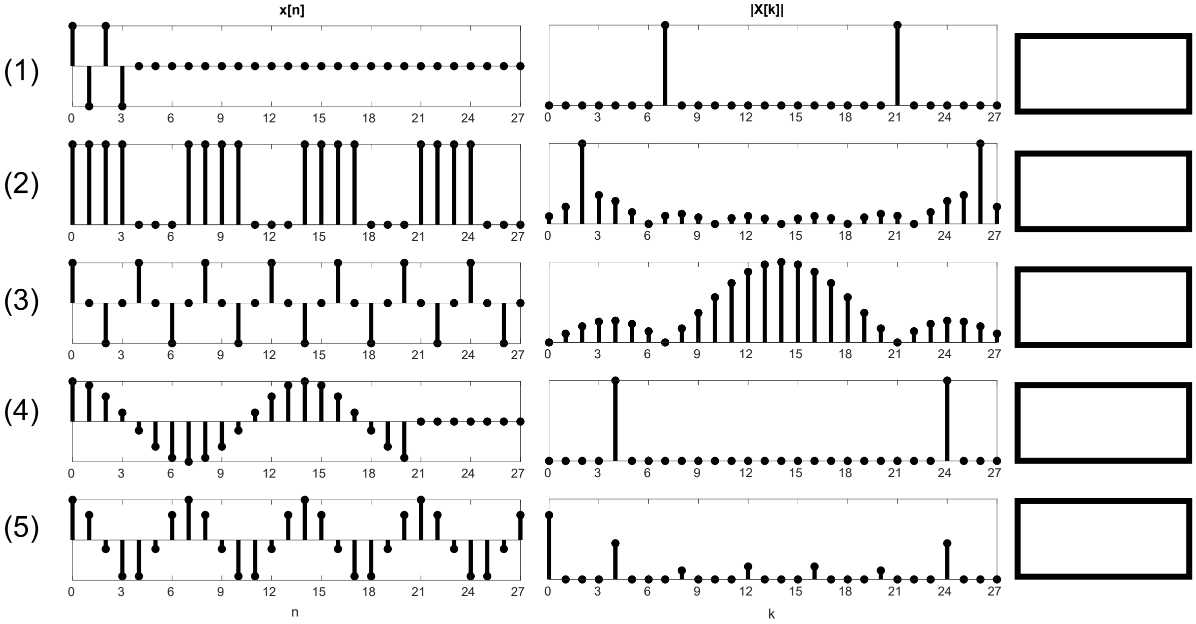
PROBLEM 3:

Parts a and b (10 points each) can be solved independently of each other.

(a) Let $x[n] = 3 + 2\cos\left(\frac{2\pi}{3}n - \frac{\pi}{6}\right) + \cos\left(\frac{\pi}{7}n + \frac{\pi}{4}\right)$. Let $\{X[0], \ldots, X[41]\}$ be the 42-point DFT of $\{x[0], \ldots, x[41]\}$. Many of these DFT coefficients are zero. Specify only the **nonzero** DFT coefficients in the table below with the corresponding index $k \in \{0, \ldots, 41\}$.

$k \geq 0$	$X[k]$

(b) Match the plots (1)-(5) for $x[n]$ below with the plots for the corresponding 28-point DFT magnitudes $|X[k]|$ by placing the appropriate number in the boxes besides the plots for $|X[k]|$.



GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 – Fall 2023
Exam #3

NAME: SOLUTIONS ______________________ GTemail: ______________________
FIRST LAST ex: GPburDELL@gatech.edu

- Write your name at the top of EACH PAGE.
- DO NOT unstaple the test.
- Closed book, except for one two-sided page $(8.5'' \times 11'')$ of hand-written notes permitted.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to received partial credit.
- Express all angles as a fraction of π. (i.e., write 0.4π or $\frac{2\pi}{5}$ instead of 1.257)
- All angles must be expressed in the range $(-\pi, \pi]$ for full credit.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the **boxes/spaces** provided. DO NOT write on the backs of the pages.
- All exams will be collected and uploaded to gradescope for grading.

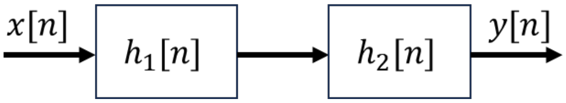
Problem	*Value*	*Score*
1	20	
2	20	
3	20	
Total		

Print Name (First Last) ______________________________

PROBLEM 1:

Parts a and b (10 points each) can be solved independently of each other.

(a) Consider the following cascade of two discrete-time LTI systems:



The first system has impulse response: $h_1[n] = 4\delta[n-2] + 8\delta[n-8] + 4\delta[n-14]$. The second system has frequency response: $H_2(e^{j\hat{\omega}}) = \begin{cases} 2 & |\hat{\omega}| \le 0.35\pi \\ 0 & 0.35\pi < |\hat{\omega}| < 0.65\pi. \\ 2 & 0.65\pi \le |\hat{\omega}| \le \pi \end{cases}$

If $x[n] = 3 + 5\cos(0.25\pi n) + 2\cos(0.4\pi n) + 4\cos\left(\frac{11\pi}{12}n\right)$ is input to the first system, find the output $y[n]$ of the second system.

$H_1(e^{j\hat{\omega}}) = e^{-j8\hat{\omega}}(8 + 8\cos(6\hat{\omega}))$

Spectrum content at $\hat{\omega} = 0, 0.25\pi, 0.4\pi, \frac{11\pi}{12}$

$\hat{\omega} = 0$: $H_1(e^{j\hat{\omega}}) = 16; H_2(e^{j\hat{\omega}}) = 2; \rightarrow DC: 3 * 16 * 2 = 96$

$\hat{\omega} = 0.25\pi$: $H_1(e^{j\hat{\omega}}) = 8; H_2(e^{j\hat{\omega}}) = 2; \rightarrow 5 * 8 * 2\cos(0.25\pi n) \rightarrow 80\cos(0.25\pi n)$

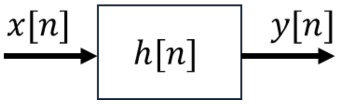
$\hat{\omega} = 0.4\pi$: $H_2(e^{j\hat{\omega}}) = 0; \rightarrow 2\cos(0.4\pi n) \rightarrow 0$

$\hat{\omega} = \frac{11\pi}{12}$: $H_1(e^{j\hat{\omega}}) = 8e^{-\frac{j88\pi}{12}} = 8e^{\frac{j2\pi}{3}}; H_2(e^{j\hat{\omega}}) = 2; \rightarrow 6\cos\left(\frac{11\pi}{12}n\right) \rightarrow 2 * 8 * 4\cos\left(\frac{11\pi}{12}n + \frac{2\pi}{3}\right) = 64\cos\left(\frac{11\pi}{12}n + \frac{2\pi}{3}\right)$

Therefore: $y[n] = 96 + 64\cos\left(\frac{11\pi}{12}n + \frac{2\pi}{3}\right)$

$y[n] =$ $96 + 80\cos(0.25\pi n) + 64\cos\left(\frac{11\pi}{12}n + \frac{2\pi}{3}\right)$

(b) Consider the following discrete-time LTI system with impulse response $h[n] = \sum_{k=0}^{8} C\delta[n-k]$.



If the input $x[n] = 5 + 8\cos\left(\frac{5\pi}{9}n + \frac{2\pi}{9}\right)$ results in the output $y[n] = 9 + M\cos(\hat{\omega}_y n + \varphi_y)$, find the values for $C, \hat{\omega}_y, M$, and φ_y.

Spectrum content at $\hat{\omega} = 0, \frac{5\pi}{9}$

$\hat{\omega}_y = \hat{\omega}_x = \frac{5\pi}{9}$

Frequency Response: $H(e^{j\hat{\omega}}) = C\frac{\sin\left(\frac{\hat{\omega}9}{2}\right)}{\sin(\hat{\omega}/2)}e^{-j\hat{\omega}4}; H(e^{j0}) = C * 9;$

Output DC: $9 = 5 * C * 9 \rightarrow C = 0.2; H\left(e^{j\left(\frac{5\pi}{9}\right)}\right) = 0.2\frac{\sin\left(\frac{\frac{5\pi}{9}9}{2}\right)}{\sin\left(\frac{5\pi}{9}/2\right)}e^{-j\frac{5\pi}{9}4} = 0.2611e^{-j0.2222\pi}$

Therefore: $\varphi_y = \frac{2\pi}{9} - \frac{2\pi}{9} = 0; M = 8 * 0.2611 = 2.0887$

$C =$ 0.2

$\hat{\omega}_y = \frac{5\pi}{9}$

$M = 2.0887$

$\varphi_y =$ 0

Print Name (First Last) ______________________________

PROBLEM 2:

Each part (5 points each) can be solved independently of each other.

The following problems might benefit from the use of the DTFT pairs and properties tables.

(a) Evaluate the following using Parseval's Theorem: $K = \sum_{n=-\infty}^{\infty}\left|\frac{\sin(0.6\pi n)}{\pi n}\right|^2$

$x[n] = \frac{\sin(0.6\pi n)}{\pi n} \rightarrow X(e^{j\hat{\omega}}) = \begin{cases} 1 & |\hat{\omega}| \leq 0.6\pi \\ 0 & 0.6\pi < |\hat{\omega}| < \pi \end{cases}$

Parseval's Theorem: $\sum_{n=-\infty}^{\infty}\left|\frac{\sin(0.6\pi n)}{\pi n}\right|^2 = \frac{1}{2\pi}\int_{-0.6\pi}^{0.6\pi} d\hat{\omega} = \left(\frac{1.2\pi}{2\pi}\right) = 0.15 = K$

$K = 0.6$

(b) Find $x[n]$ if its DTFT is $X(e^{j\hat{\omega}}) = \frac{5e^{-j4\hat{\omega}}}{5+2e^{-j\hat{\omega}}}$

$X(e^{j\hat{\omega}}) = \frac{5e^{-j4\hat{\omega}}}{5+2e^{-j\hat{\omega}}} \rightarrow mutliply\ by \frac{1/5}{1/5} \rightarrow \frac{e^{-j4\hat{\omega}}}{1+0.4e^{-j\hat{\omega}}}$

$x[n] = (-0.4)^{(n-4)}u[n-4]$

$x[n] = (-0.4)^{(n-4)}u[n-4]$

(c) Find $y[n] = [2\cos(0.2\pi n)\cos(0.5\pi n)] * \left(\frac{\sin(0.8\pi n)}{\pi n} - \cos(0.65\pi n)\frac{\sin(0.3\pi n)}{\pi n}\right)$ ***(where ' $*$ ' is convolution)***

$2\cos(0.2\pi n)\cos(0.5\pi n) \rightarrow \cos(0.3\pi n) + \cos(0.7\pi n)$

$\frac{\sin(0.8\pi n)}{\pi n} - \cos(0.65\pi n)\frac{\sin(0.3\pi n)}{\pi n} = \begin{cases} 1, & |\hat{\omega}| < 0.35\pi \\ 0.5, & 0.35\pi < |\hat{\omega}| < 0.8\pi \\ \frac{1}{2}e^{j\pi}, & 0.8\pi < |\hat{\omega}| < 0.95\pi \end{cases}$

Therefore: $y[n] = \cos(0.3\pi n) + 0.5\cos(0.7\pi n)$

$y[n] = \cos(0.3\pi n) + 0.5\cos(0.7\pi n)$

(d) Find $y[n] = [2\cos(0.2\pi n) + \cos(0.5\pi n)] * \left(\frac{\sin(0.8\pi n)}{\pi n} - \frac{\sin(0.3\pi n)}{\pi n}\right)$ ***(where ' $*$ ' is convolution)***

$2\cos(0.2\pi n) + \cos(0.5\pi n)$

$\frac{\sin(0.8\pi n)}{\pi n} - \frac{\sin(0.3\pi n)}{\pi n} = \begin{cases} 0, & |\hat{\omega}| < 0.3\pi \\ 1, & 0.3\pi < |\hat{\omega}| < 0.8\pi \\ 0, & |\hat{\omega}| > 0.8\pi \end{cases}$

Therefore: $y[n] = \cos(0.5\pi n)$

$y[n] = \cos(0.5\pi n)$

Print Name (First Last) ________________________________

PROBLEM 3:

Parts a and b (10 points each) can be solved independently of each other.

(a) Let $x[n] = 3 + 2\cos\left(\frac{2\pi}{3}n - \frac{\pi}{6}\right) + \cos\left(\frac{\pi}{7}n + \frac{\pi}{4}\right)$. Let $\{X[0], \dots, X[41]\}$ be the 42-point DFT of $\{x[0], \dots, x[41]\}$. Many of these DFT coefficients are zero. Specify only the **nonzero** DFT coefficients in the table below with the corresponding index $k \in \{0, \dots, 41\}$.

$k \geq 0$	$X[k]$
0	126
3	$21e^{\frac{j\pi}{4}}$
14	$42e^{-\frac{j\pi}{6}}$
28	$42e^{\frac{j\pi}{6}}$
39	$21e^{-\frac{j\pi}{4}}$

DFS:

$$\hat{\omega}_0 = GCD\left(\frac{2\pi}{3}, \frac{\pi}{7}\right) = \frac{2\pi}{42} \to N_0 = 42 = N$$

The DFT is the same length as the fundamental period so we will have "complete" periods in the windowed DFT and can represent each sinusoid with 2 spectral lines.

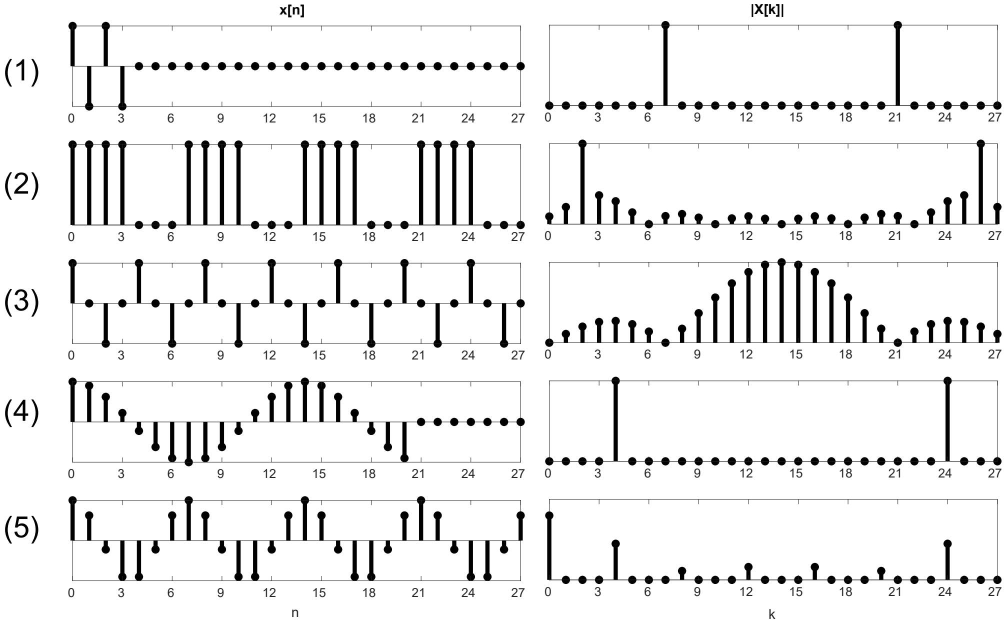
The DFS for x[n] are:

$$\hat{\omega}_0 k = 0, \pm\frac{2\pi}{3}, \pm\frac{\pi}{7} \to a_k = 3, e^{\mp\frac{j\pi}{4}}, \frac{1}{2}e^{\pm\frac{j\pi}{4}}, k = 0, \pm 14, \pm 3$$

$$X[k] = Na_k$$

$$X[k] = 42a_k \to 126, 42e^{\mp\frac{j\pi}{4}}, 21e^{\pm\frac{j\pi}{4}}, k = 0, \{14,28\}, \{3,39\}$$

(b) Match the plots (1)-(5) for $x[n]$ below with the plots for the corresponding 28-point DFT magnitudes $|X[k]|$ by placing the appropriate number in the boxes besides the plots for $|X[k]|$.



(1)	3
(2)	4
(3)	1
(4)	5
(5)	2