GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
EXAM 3

DATE:11-Nov-19 COURSE: ECE-2026

NAME: Solutions ______________________ CanvasID: ______________
LAST, FIRST ex: gtAlpha

Circle your correct recitation section number - failing to do so will cost you 2 points

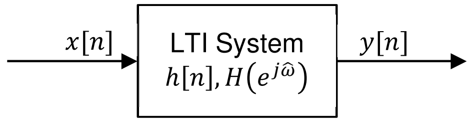
Recitation time	Mon	Tue	Wed	Thu
09:30:10:45		L12 Farahmand		L06 Causey
12:00-13:15		L07 Farahmand		L08 Barry
13:30-14:45		L09 Farahmand		L10 Barry
15:00-16:15	L01 Juang	L11 Farahmand	L02 Casinovi	
16:30-17:45			L04 Casinovi	

- Write your name on the front page ONLY. **DO NOT unstaple the test**
- Closed book, but a calculator is permitted.
- One page $\left(8\frac{1}{2}'' \times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **SHOW ALL YOUR WORK ON ALL PROBLEMS TO RECEIVE CREDIT. PROBLEMS WITH NO WORK AND JUST ANSWERS MAY RECEIVE 0 CREDIT, EVEN IF THE ANSWER IS CORRECT. YOU MUST SHOW SOME NUMERICAL WORK, REASONING, OR EXPLANATION FOR YOUR ANSWER. (I.E., DON'T JUST PUT AN ANSWER AND LEAVE THE WORK AREA BLANK)**
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the **boxes/spaces** provided. If more space is needed for scratch work, use the backs of previous pages.
- **WRITE ANY RADIAN ANSWERS AS A FRACTION OF PI. (i.e., write 0.4π or $\frac{2\pi}{5}$ instead of 1.257)**
- **ALL RADIAN ANSWERS MUST BE IN THE RANGE $(-\pi, \pi]$ FOR CREDIT.**

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
No/Wrong Recitation Circled	-2	
Total		

PROBLEM 1: Parts a and b can be solved independently of each other

(a) Consider the LTI system shown below with input $x[n]$ and output $y[n]$, whose frequency response is $H\left(e^{j\hat{\omega}}\right) = 10e^{-j26\hat{\omega}}(10 + 10\cos(10\hat{\omega}))$:



(i.) Find the **<u>difference equation</u>** that relates $y[n]$ to $x[n]$. (5 points)

$$H\left(e^{j\hat{\omega}}\right) = 10e^{-j26\hat{\omega}}(10 + 10\cos(10\hat{\omega})) = 100e^{-j26\hat{\omega}} + 50e^{-j16\hat{\omega}} + 50e^{-j36\hat{\omega}}$$

$$\rightarrow h[n] = 50\delta[n-16] + 100\delta[n-26] + 50\delta[n-36]$$

$$y[n] = 50x[n-16] + 100x[n-26] + 50x[n-36]$$

(ii.) If $x[n] = 2 + 2\cos\left(\frac{\pi}{5}n\right)$ find and equation for the output $y[n]$. (Hint: It has the form $y[n] = A + B\cos(\hat{\omega}_0 n + \phi)$). (5 points)

$$H\left(e^{j0}\right) = 200; H\left(e^{\frac{j\pi}{5}}\right) = 200e^{-\frac{j26\pi}{5}} \rightarrow y[n] = 400 + 400\cos\left(\frac{\pi}{5}n - \frac{26\pi}{5}\right) = 400 + 400\cos\left(\frac{\pi}{5}n + \frac{4\pi}{5}\right)$$

$$y[n] = 400 + 400\cos\left(\frac{\pi}{5}n + \frac{4\pi}{5}\right)$$

(b) Consider the impulse response $h[n] = \frac{1}{5}(u[n-20] - u[n-33])$. **<u>Use the provided tables of DTFT pairs and properties</u>** to find the frequency response $H\left(e^{j\hat{\omega}}\right)$ and determine the magnitude of the frequency response at $\hat{\omega} = 0$ (i.e., $H\left(e^{j0}\right)$). (NOTE: You must express the frequency response in the form $H\left(e^{j\hat{\omega}}\right) = A(\hat{\omega})e^{j\varphi(\hat{\omega})}$, where $A(\hat{\omega})$ presents the overall amplitude function and $\varphi(\hat{\omega})$ represents the overall phase function of the frequency response) (10 points)

This is a "box" in discrete time of length 13 samples shifted by 20 samples.

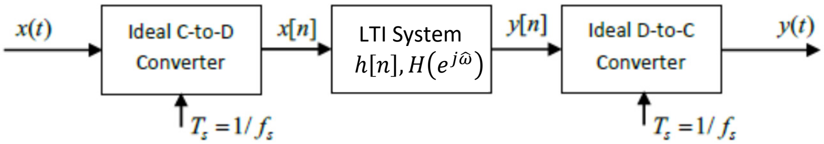
$$H\left(e^{j\hat{\omega}}\right) = \frac{\sin\left(\frac{13\hat{\omega}}{2}\right)}{5\sin\left(\frac{\hat{\omega}}{2}\right)}e^{-j\hat{\omega}26} \rightarrow H\left(e^{j0}\right) = \frac{13}{5}$$

$$H\left(e^{j\hat{\omega}}\right) = \frac{\sin\left(\frac{13\hat{\omega}}{2}\right)}{5\sin\left(\frac{\hat{\omega}}{2}\right)}e^{-j\hat{\omega}26}$$

$$H\left(e^{j0}\right) = \frac{13}{5}$$

PROBLEM 2: All parts can be solved independently of each other

Use the following ideal C-to-D / D-to-C system with the defined LTI System $H(e^{j\hat{\omega}})$ for all parts below.



$$H(e^{j\hat{\omega}}) = \begin{cases} 0 & |\hat{\omega}| < 0.3\pi \\ 2e^{-j\hat{\omega}9} & 0.3\pi \leq |\hat{\omega}| \leq 0.5\pi \\ 0 & 0.5\pi < |\hat{\omega}| \leq \pi \end{cases}$$

Let $x(t) = 9 + 3\cos(900\pi t) + 6\cos(1100\pi t)$

(a) If $f_s = 2000\ Hz$, find the output $y(t)$. (12 points)

$$x[n] = 9 + 2\cos(0.45n) + 6\cos(0.55\pi n)$$

$$y[n] = 0 * 9 + 2 * 3\cos(0.45\pi n - 4.05\pi) + 0 * 6\cos(0.55\pi n - 4.95\pi) \rightarrow 4\cos(0.45\pi n - 0.05\pi)$$

$$y(t) = 6\cos(900\pi t - 0.05\pi)$$

$y(t) = 6\cos(900\pi t - 0.05\pi)$

(b) For what range of sampling rates is there both **no aliasing** and a **nonzero output** $y(t)$. Your answer should be expressed as $A \leq f_s \leq B$ where you must determine A and B. (Hint: a nonzero output for $y(t)$ means that a minimum of one of the components of $x(t) = 9 + 3\cos(900\pi t) + 6\cos(1100\pi t)$ is passed by the LTI system) (8 points)

For $x(t)$ has frequencies of 900π and 1100π. This problem requires (1) No aliasing (2) At least one digital frequency in the range $0.3\pi \leq |\hat{\omega}| \leq 0.5\pi$.

(1) No aliasing: $f_s > 1100\ Hz$

(2) At least one component in the range $0.3\pi \leq |\hat{\omega}| \leq 0.5\pi$

$$\rightarrow \frac{900\pi}{fs} \leq 0.5\pi \rightarrow f_s \geq 1800;\ \frac{900\pi}{fs} \geq 0.3\pi \rightarrow f_s \leq 3000$$

$$\rightarrow \frac{1100\pi}{fs} \leq 0.5\pi \rightarrow f_s \geq 2200;\ \frac{1100\pi}{fs} \geq 0.3\pi \rightarrow f_s \leq \frac{1100}{0.3} \approx 3666.67$$

Combining the ranges yields: $1800 \leq f_s \leq \frac{1100}{0.3} \approx 3666.67$

$1800 \leq f_s \leq \frac{1100}{0.3} \approx 3666.67$

PROBLEM 3: All parts can be solved independently of each other

(a) An LTI system with output $y[n]$ has impulse response $h[n] = 4\delta[n-1] + 3\delta\ [n-3] - 4\delta[n-7]$. If the input to the system is the unit step function $x[n] = u[n]$, find $y[4]$ and $y[9]$.
(10 points) (NOTE: You don't need to make a table)

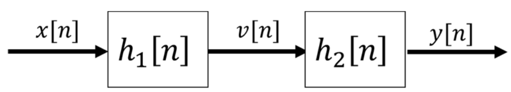
$$y[n] = 4u[n-1] + 3u[n-3] - 4u[n-7] \rightarrow y[4] = 4u[3] + 3u[1] - 4u[-3] = 7$$

$$y[9] = 4u[8] + 3u[6] - 4u[2] = 3$$

$y[4] = 7$

$y[9] = 3$

(b) Consider the cascaded LTI system below



where $h_1[n] = \sum_{k=3}^{8} \delta[n-k]$ and $h_2[n] = \delta[n] - \delta[n-2]$. Find and equation for $y[n]$ that is valid for all n, when $x[n] = 2\delta[n-3]$. (10 points)

Overall impulse response is: $h[n] = \delta[n-3] + \delta[n-4] - \delta[n-9] - \delta[n-10]$

$$y[n] = 2\delta[n-6] + 2\delta[n-7] - 2\delta[n-12] - 2\delta[n-13]$$

$y[n] = 2\delta[n-6] + 2\delta[n-7] - 2\delta[n-12] - 2\delta[n-13]$

Table of DTFT Pairs	
Time-Domain: $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
$\delta[n]$	1
$\delta[n-n_d]$	$e^{-j\hat{\omega}n_d}$
$r_L[n] = u[n] - u[n-L]$	$\dfrac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j\hat{\omega}(L-1)/2}$
$r_L[n]e^{j\hat{\omega}_0 n}$	$\dfrac{\sin(\frac{1}{2}L(\hat{\omega}-\hat{\omega}_o))}{\sin(\frac{1}{2}(\hat{\omega}-\hat{\omega}_o))} e^{-j(\hat{\omega}-\hat{\omega}_o)(L-1)/2}$
$\dfrac{\sin(\hat{\omega}_b n)}{\pi n}$	$u(\hat{\omega}+\hat{\omega}_b) - u(\hat{\omega}-\hat{\omega}_b) = \begin{cases} 1 & \lvert\hat{\omega}\rvert \le \hat{\omega}_b \\ 0 & \hat{\omega}_b < \lvert\hat{\omega}\rvert \le \pi \end{cases}$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\dfrac{1}{1-ae^{-j\hat{\omega}}}$

Table of DTFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
Periodic in $\hat{\omega}$		$X(e^{j(\hat{\omega}+2\pi)}) = X(e^{j\hat{\omega}})$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\hat{\omega}}) + bX_2(e^{j\hat{\omega}})$
Conjugate Symmetry	$x[n]$ is real	$X(e^{-j\hat{\omega}}) = X^*(e^{j\hat{\omega}})$
Conjugation	$x^*[n]$	$X^*(e^{-j\hat{\omega}})$
Time-Reversal	$x[-n]$	$X(e^{-j\hat{\omega}})$
Delay (n_d=integer)	$x[n-n_d]$	$e^{-j\hat{\omega}n_d}X(e^{j\hat{\omega}})$
Frequency Shift	$x[n]e^{j\hat{\omega}_0 n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_0)})$
Modulation	$x[n]\cos(\hat{\omega}_0 n)$	$\frac{1}{2}X(e^{j(\hat{\omega}-\hat{\omega}_0)}) + \frac{1}{2}X(e^{j(\hat{\omega}+\hat{\omega}_0)})$
Convolution	$x[n] * h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
Autocorrelation	$x^*[-n] * x[n]$	$\lvert X(e^{j\hat{\omega}})\rvert^2$
Parseval's Theorem	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\hat{\omega}})\rvert^2 d\hat{\omega}$