GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
EXAM 3

DATE: 13-Nov-17 COURSE: ECE-2026

NAME: ______________________________ TSquareID: ______________
LAST, FIRST ex: gtJohnA

Circle your correct recitation section number - failing to do so will cost you 2 points

Recitation time	Mon	Tue	Wed	Thu
09:30:10:45				L06 Harper
12:00-13:15		L07 Causey		L08 Harper
13:30-14:45		L09 Yang		L10 Stuber
15:00-16:15	L01 Juang	L11 Yang	L02 Causey	L12 Stuber
16:30-17:45	L03 Marenco		L04 Causey	

- Write your name on the front page ONLY. **DO NOT unstaple the test**
- Closed book, but a calculator is permitted.
- Three pages $\left(8\frac{1}{2}'' \times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **SHOW ALL YOUR WORK ON ALL PROBLEMS TO RECEIVE CREDIT**
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Write your answers in the **boxes/spaces** provided. If more space is needed for scratch work, use the backs of previous pages.
- **WRITE ANY RADIAN ANSWERS AS A FRACTION OF PI. (i.e., write 0.4π instead of 1.257)**
- **ALL RADIAN ANSWERS SHOULD BE IN THE RANGE (-π, π].**

Problem	*Value*	*Score*
1	16	
2	24	
3	20	
No/Wrong Recitation Circled	-2	
Total		

PROBLEM 1:

(a) The continuous signal $x(t) = A\cos(2\pi f_0 t)$ has a fundamental frequency of f_0. Find the **smallest** sampling frequency, f_s (where $x[n] = x(n/f_s)$) that: (1) Avoids aliasing and (2) creates a fundamental period of $N_0 = 15$ (i.e., $x[n] = x[n + N_0] = x[n + 15]$.) (NOTE: Your answer will be expressed in terms of f_0.) (6 points)

$$f_s = \frac{N_0}{M} f_0 = \frac{15}{M} f_0 > 2f_0 \rightarrow \frac{15}{M} > 2 \rightarrow M < 7.5$$

$$f_s = \frac{15}{7} f_0$$

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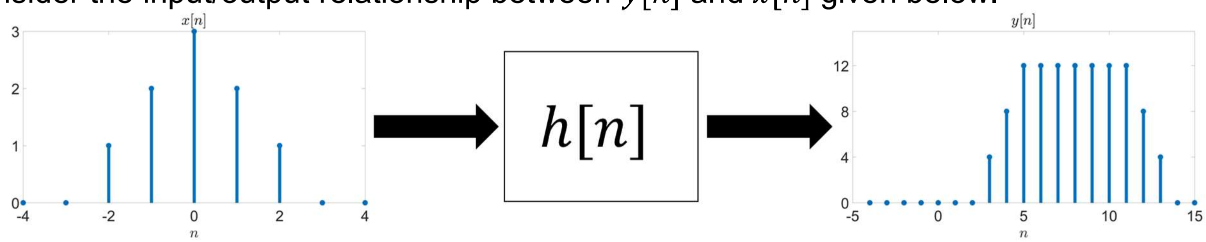
(b) Consider the input/output relationship between $y[n]$ and $x[n]$ given below.



Let $y[n] = x[n] * h[n]$ (representing convolution). If $h[n] = A(\delta[n-a] + \delta[n-b] + \delta[n-c])$ find $A, a, b,$ and c such that $y[n] = 12$ for $n = 5, \ldots, 11$ (i.e., $y[n]$=12 over the interval $5 \leq n \leq 11$). (10 points)

Need the trailing two values of 1 and 2 (at n=-1 and -2) to align with the leading values of 2 and 1 (at n=1 and 2) as the signal is scaled by 4 and shifted. We will need three "copies" of x[n] for y[n] to be constant over the interval of $5 \leq n \leq 11$.

Therefore: $h[n] = 4(\delta[n-5] + \delta[n-8] + \delta[n-11])$

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PROBLEM 2: (All parts can be solved independently)

Assume that an LTI system is defined with the following impulse response:

$$h[n] = u[n-8] - u[n-18]$$

The input to the system is $x[n] = u[n-a] - u[n-b]$ and the output is defined as $y[n] = x[n] * h[n]$ (where $y[n] = x[n] * h[n]$ denotes $x[n]$ convolved with $h[n]$).

(a) Find a and b such that $y[n]$ is non-zero over the interval $4 \leq n \leq 21$. (6 points)

y[n] has boundaries: {4 -> 21}
h[n] has boundaries: {8 -> 17}
$4 = a + 8 \rightarrow a = -4$
$21 = (b-1) + 17 \rightarrow b = 5$

$a = -4$
$b = 5$

(b) Assume that the length of $x[n]$ is greater than the length of $h[n]$ (i.e., $(b-a) > 10)$). Find the **maximum value** of $y[n]$. (i.e., $y[n] = A$ for some n and $y[n] \leq A$ for all n). (6 points) (HINT: Recall the **general formula** for convolution in an FIR filter is: $y[n] = \sum_{k=0}^{M} h[k]x[n-k]$)

$y[n] = \sum_{k=8}^{17} h[k]x[n-k]$. When $h[n]$ completely overlaps $x[n]$

$$\sum_{k=8}^{17} (1)(1) = 10$$

$A = 10$

(c) Let $p[n] = h[n] \times (\delta[n] + \delta[n-15])$ where $\times$ represents **multiplication** (NOT convolution). Write a complete expression for $p[n]$. (6 points)

$$p[n] = h[n]\delta[n] + h[n]\delta[n-15] = h[0]\delta[n] + h[15]\delta[n-15] = \delta[n-15]$$

$p[n] = \delta[n-15]$

(d) Define a new LTI system as $h_2[n] = h[n] - (u[n-9] - u[n-19])$ and $y_2[n] = x[n] * h_2[n]$. Write the **difference equation** between $y_2[n]$ and $x[n]$. (6 points) (HINT: $h_2[n]$ reduces to a very simple equation.)

$$h_2[n] = h[n] - (u[n-9] - u[n-19]) = \delta[n-8] - \delta[n-18] \rightarrow y_2[n] = x[n-8] - x[n-18]$$

$y_2[n] = x[n-8] - x[n-18]$

PROBLEM 3: (All parts can be solved independently)

(a) Suppose an LTI system is characterized by the following frequency response:

$$H(e^{j\hat{\omega}}) = 6e^{-j3\hat{\omega}}(3 - 2\cos(4\hat{\omega}))$$

(NOTE: Use this LTI system for parts (i) and (ii) below.)

(i) Find the impulse response, $h[n]$. (5 points)

$$H(e^{j\hat{\omega}}) = 6e^{-j3\hat{\omega}}(3 - e^{4j\hat{\omega}} - e^{-j4\hat{\omega}}) = 18e^{-j3\hat{\omega}} - 6e^{j\hat{\omega}} - 6e^{-j7\hat{\omega}}$$

$$h[n] = -6\delta[n+1] + 18\delta[n-3] - 6\delta[n-7]$$

$h[n] = \quad -6\delta[n+1] + 18\delta[n-3] - 6\delta[n-7]$

(ii) If $y[n] = \left\{3 + 2\cos\left(0.25\pi n - \frac{\pi}{4}\right)\right\} * h[n]$ find an equation for $y[n]$ in the form $y[n] = A + \cos(\hat{\omega}n + \phi)$. (5 points)

$$|H(e^{j(0)})| = 6 * (3-2) = 6; |H(e^{j(0.25\ \ )})| = 6e^{-\frac{j3\pi}{4}}\left(3 - 2\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) = 6e^{-\frac{j3\pi}{4}}(5) = 30e^{-\frac{j3\pi}{4}}$$

$$y[n] = 6 * 3 + 30 * 2\cos\left(0.25\pi n - \frac{\pi}{4} - \frac{3\pi}{4}\right) = 18 + 60\cos(0.25\pi n - \pi) = 18 - 60\cos(0.25\pi n)$$

$y[n] = \quad 18 + 60\cos(0.25\pi n - \pi) = 18 - 60\cos(0.25\pi n)$

(b) Let $h[n] = \sum_{k=0}^{L-1}\delta[n-k]$ and $x[n] = \sum_{k=0}^{M}\cos((\hat{\omega}_0 k)n)$ with $M\hat{\omega}_0 \le \pi$.

If $y[n] = h[n] * x[n] = 0$ and $L = 10$, find the **smallest** possible value for $\hat{\omega}_0$. (5 points)

$h[n]$ has a frequency response of $H(e^{j\hat{\omega}}) = \frac{\sin\left(\frac{10\hat{}}{2}\right)}{\sin\left(\frac{\hat{\omega}}{2}\right)}e^{-j4.5\hat{\omega}}$ with zeros at $\hat{\omega} = \frac{2\pi}{10}k = \frac{\pi}{5}k$

Therefore, $\hat{\omega}_0 = \frac{\pi}{5}$

$\hat{\omega}_0 = \frac{\pi}{5}$

(NOTE: This problem should have said $y[n] = h[n] * x[n] = constant$ since the summation started at *k*=0. We will add 1 point (20% of this problem) to every student grade)

(c) Consider the following MATLAB code: (5 points)

```
hn = [0,0,2,0,8,0,2];
yn = conv(xn, hn);
```

Find an expression for the magnitude of the frequency response.

$h[n] = 2\delta[n-2] + 8\delta[n-4] + 2\delta[n-6] \rightarrow H(e^{j\hat{\omega}}) = e^{-j4\hat{\omega}}(8 + 4\cos(2\hat{\omega}))$

$|H(e^{j\hat{\omega}})| = \quad 8 + 4\cos(2\hat{\omega})$