GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
QUIZ #3

DATE: 15-Nov-13 COURSE: ECE-2026

NAME: ______________________________ GT#: ______________
LAST, FIRST ex: gtaburDEll

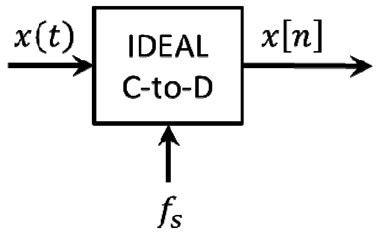
Circle your correct **recitation section** number - failing to do so will cost you 3 points

L01: Mon - (Juang)	L02: Wed - (Bloch)	L03: Mon - (Casinovi)
L04: Wed - (Bloch)	L05: Tues - (Bhatti)	L06: Thurs - (Coyle)
L07: Tues - (Bhatti)	L08: Thurs - (Coyle)	L09: Tues - (AlRegib)
L10: Thurs - (Ma)	L11: Tues - (Causey)	L12: Thurs - (Ma)
L13: Tues - (Causey)		L14: Thurs - (AlRegib)

- Write your name on the front page ONLY. **DO NOT unstaple the test**
- Closed book, but a calculator is permitted.
- One page $\left(8\frac{1}{2}'' \times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning sufficiently to receive partial credit. Explanations are also required (as applicable) to receive full credit for an answer. (i.e., ***show enough work so graders understand your approach to solving the problem***)
- You must **write your answer in the boxes provided on the exam paper itself**. **Only answers in these boxes will be graded as the final solution**. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	30	
3	30	
4	15	
Total	100	

PROBLEM fa-13-Q.2.1:
Assume the signal $x(t) = \cos(80\pi t - 0.2\pi)$ is passed through an ideal C-to-D converter as shown below:



(a) If f_s=100 Hz, find the period N_0 of the discrete time signal $x_a[n]$ (i.e., $x_a[n] = x_a[n + N_0]$).

$$X(n) = \cos\left(\frac{80\pi}{100}n - 0.2\pi\right)$$
$$= \cos\left(\frac{4\pi}{5}n - 0.2\pi\right)$$
$$N_0 = 5$$

$N_0 = 5$

(b) Find **one other value** for the sampling frequency, f_s such that the discrete time sequence $x_b[n]$ sampled from $x(t)$ will have the **same period** N_0 (i.e., $x_b[n] = x_b[n + N_0]$) as that in part (a). (NOTE: There is a simple answer. Make sure to justify our answer is correct.)

$f_s = N_0 f_0 \rightarrow$ period $\Rightarrow N_0$

$f_0 = 40 \quad \therefore \quad 5 \cdot 40 = 200$

$\boxed{f_s = 200}$

(c) Assume that

$$x_c[n] = x_a[n], \qquad 0 \le n < 30$$

(i.e., $x_c[n]$ is the same as the signal from part (a) with a length of 30 samples.)
The 30-point DFT of the signal will be *mostly zero* with the exception of two values in $X[k]$. Determine the indices *k* at which *X*[*k*] is non-zero; also give their complex values (i.e., non-zero *X*[*k*]'s)

$x_c(n) = \cos(\frac{4\pi}{5}n - 0.2\pi)$

$= \frac{1}{2}e^{j0.2\pi}e^{-j\frac{4\pi}{5}n} + \frac{1}{2}e^{-j0.2\pi}e^{j\frac{4\pi}{5}n} \leftarrow \text{DFS}$

$(\frac{2\pi}{30})\cdot -12 \qquad (\frac{2\pi}{30})\cdot 12$

DFS: $k=12, \ \frac{1}{2}e^{-j0.2\pi}$

$k=-12, \ \frac{1}{2}e^{j0.2\pi}$

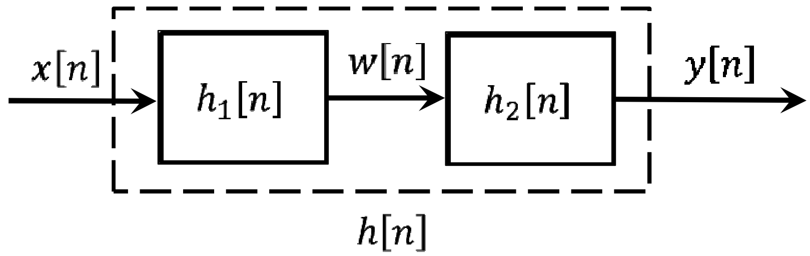
DFT = N · DFS

DFT: $k=12, \ 15e^{-j0.2\pi}$

$k=18, \ 15e^{j0.2\pi}$

k	Complex amplitude
12	$15e^{-j0.2\pi}$
18	$15e^{j0.2\pi}$

PROBLEM fa-13-Q.2.2:
Consider the following overall system:



Assume the input to the system is

$$x[n] = 3 + \cos\left(\frac{\pi}{3}n - \frac{\pi}{8}\right) + \cos\left(\frac{\pi}{6}\mathrm{n}\right) + \cos\left(\frac{2\pi}{3}n + \frac{\pi}{9}\right)$$

Also assume that $H_1(e^{j\hat{\omega}})$ and $h_2[n]$ have the form shown below:

$$H_1(e^{j\hat{\omega}}) = (2 - 2\cos(6\hat{\omega}))e^{-j6\hat{\omega}}$$

$$h_2[n] = \begin{cases} 1, & n = 0, \ldots, L-1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find $w[n]$

X(n) has frequencies @
$\hat{\omega} = \pi/3, \pi/6, 2\pi/3, 0$

$|H(0)| = 0, \; |H(\pi/3)| = 0, \; |H(\frac{2\pi}{3})| = 0, \; |H(\pi/6)| = 4$

$w(n) = 4\cos(\frac{\pi}{6}n - \pi)$

$w[n] = -4\cos(\pi/6\, n)$

(b) Find the impulse response $h_1[n]$

$$H_1(e^{j\hat{w}}) = (2 - e^{j6\hat{w}} - e^{-j6\hat{w}})(e^{-j6\hat{w}})$$

$$= 2e^{-j6\hat{w}} - 1 - e^{-j12\hat{w}}$$

$$h_1[n] = -\delta[n] + 2\delta[n-6] - \delta[n-12]$$

$h_1[n] =$ $-\delta[n] + 2\delta[n-6] - \delta[n-12]$

(c) Determine the ***smallest*** value for the parameter L in $h_2[n]$ such that $y[n] = 0$ for all n.

$$w[n] \rightarrow \hat{w} \Rightarrow \left\{ \pi/6 \right\}$$

$$H_2(e^{j\hat{w}}) = \frac{\sin(\hat{w}L/2)}{\sin(\hat{w}/2)} e^{-j\hat{w}(L-1)/2}$$

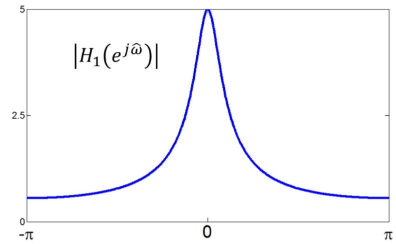
<u>Zeros</u> @: $\hat{w} = k\frac{2\pi}{L} = \pi/6 \rightarrow L = 12k$

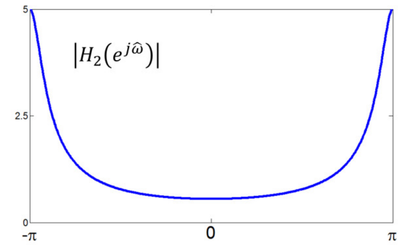
<u>Check</u>: $\hat{w} = k\pi/6$, $k=1 \rightarrow \pi/6$ ✓

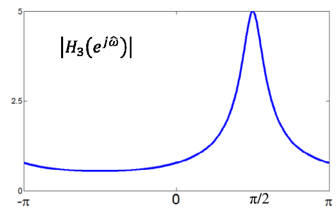
$L =$ 12

PROBLEM fa-13-Q.2.3:
The plots below are all related in some way to $h_1[n]$ with frequency response

$$H_1(e^{j\hat{\omega}}) = \frac{1}{1 - 0.8e^{-j\hat{\omega}}}$$







Determine if the statements below are True or False (Justify the answer if it is true; Provide a correction to the statement if it is false) (Place an 'X' in the box for True or False)

(a) $h_2[n] = (-1)^n h_1[n]$ TRUE FALSE

$(-1)^n = e^{-j\pi n} = \cos(\pi n)$

$\rightarrow$ Frequency shift of π which is reflected in $H_2(e^{j\hat{\omega}})$

(b) $h_1[n]$, $h_2[n]$, and $h_3[n]$ (the inverse DTFT's of the plots above) are *real-valued* signals (i.e. there are no imaginary numbers in the sequences)

TRUE FALSE

$H_3(e^{j\hat{\omega}})$ is not conjugate symmetric

(i.e., $H_3(e^{j\hat{\omega}}) \neq H_3^*(e^{j\hat{\omega}})$)

$\therefore h_3[n]$ is <u>not</u> a real sequence

(c) $h_3[n] = \delta[n - \pi/2] * h_1[n]$

TRUE FALSE

Frequency shift of $\pi/2$

$H_3(e^{j\hat{\omega}}) = H_1(e^{j(\hat{\omega} - \pi/2)})$

$\therefore h_3[n] = e^{j\pi/2 n} h_1[n]$

PROBLEM fa-13-Q.2.4:
We have seen that LTI systems can be represented in a variety of ways. For each of the LTI systems below, provide the missing representations. (Frequency response formulas can be given in any convenient form; you need not simplify them.)

(a)

Matlab Code:	`yn = conv(xn, [0, 1, 0, 4, 0 , 1]);`
Frequency Response:	$e^{-j\hat{\omega}} + 4e^{-j3\hat{\omega}} + e^{-j5\hat{\omega}}$

$$h(n) = \delta(n-1) + 4\delta(n-3) + \delta(n-5)$$

(b)

Frequency Response:	$H(e^{j\hat{\omega}}) = \dfrac{\sin(1.5\hat{\omega})}{3\sin(0.5\hat{\omega})} e^{-j\hat{\omega}3}$
Impulse Response:	$\frac{1}{3}\delta(n-2) + \frac{1}{3}\delta(n-3) + \frac{1}{3}\delta(n-4)$

$$H(e^{j\hat{\omega}}) = \underline{\frac{1}{3}} \cdot \underline{\frac{\sin(3/2\,\hat{\omega})}{\sin(\hat{\omega}/2)} e^{-j\hat{\omega}}} \cdot \underbrace{e^{-j\hat{\omega}2}}_{\text{delay}}$$

$$h(n) = \begin{cases} \frac{1}{3}, & n = 2, 3, 4 \\ 0, & \text{otherwise} \end{cases}$$

(c)

Impulse response	$h[n] = \delta[n-2] - \delta[n-3] + 2\delta[n-6]$
Difference Equation:	$y(n) = x(n-2] - x(n-3] + 2x[n-6]$

By inspection