GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Fall 2012
Quiz #3

November 16, 2012

NAME: ______________ ________________ GT username: ________________
(FIRST) (LAST) (e.g., gtxyz123)

Circle your recitation section in the chart below (otherwise you lose 3 points!):

	Mon	Tue	Wed	Thu
9:30 – 11am				L06 (Fekri)
12 – 11:30pm		L07 (Al-Regib)		L08 (Fekri)
1:30 – 3pm		L09 (Al-Regib)		L10 (Rozell)
3 – 4:30pm	L01 (Juang)	L11 (Davenport)	L02 (Zajic)	L12 (Rozell)
4:30 – 6pm	L03 (Baxley)	L13 (Davenport)	L04 (Zajic)	
6 – 7:30pm	L05 (Baxley)			

Important Notes:

- DO NOT unstaple the test.
- One two-sided page (8.5” $\times$ 11”) of hand-written notes permitted.
- JUSTIFY your reasoning CLEARLY to receive full credit.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
No/Wrong Rec	–3	
Total		

PROB. Fall-12-Q3.1.

(a) The fundamental period N_0 of a periodic sequence $x[n]$ is the smallest positive integer such that $x[n + N_0] = x[n]$ for all n.
The sequence $x[n] = \cos(0.15\pi n)$ is periodic. Its fundamental period is $N_0 =$ ☐ .

(b) When taking the 12-point DFT of $\{x[0], x[1], \ldots x[11]\} = \{1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3\}$,
the zero-th DFT coefficient is $X[0] =$ ☐ .

(c) If a 3-point DFT of $\{x[0], x[1], x[2]\}$ yields $X[0] = 0$ and $X[1] = X^*[2] = \sqrt{27}\, e^{j\pi/6}$, then the values of $\{x[0], x[1], x[2]\}$ must be:

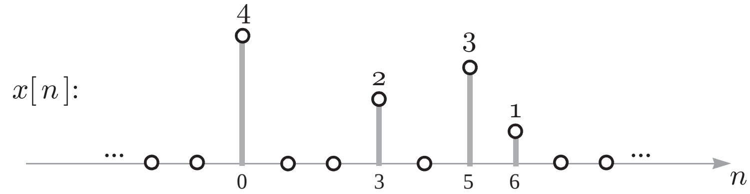
$x[0] =$ ☐

$x[1] =$ ☐

$x[2] =$ ☐ .

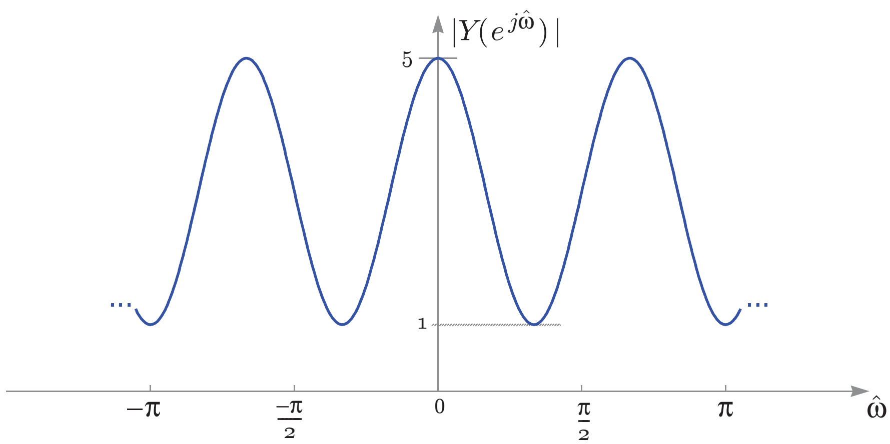
PROB. Fall-12-Q3.2. (Parts (a) and (b) are unrelated.)

(a) Write an equation for the DTFT $X(e^{j\hat{\omega}})$ of the finite sequence $x[n]$ shown in the stem plot below:



$\Rightarrow$ $X(e^{j\hat{\omega}}) =$

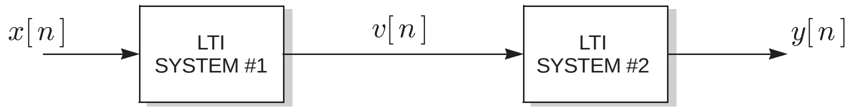
(b) Suppose that the DTFT of a sequence $y[n]$ can be written as $Y(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}}|Y(e^{j\hat{\omega}})|$, where $|Y(e^{j\hat{\omega}})|$ oscillates sinusoidally between 1 and 5, with a period of $2\pi/3$, as shown below:



Find numerical values for the sequence $y[n]$ in the time domain at times $n \in \{0, 1, 2, 3, 4, 5, 6\}$:

$y[0]$	$y[1]$	$y[2]$	$y[3]$	$y[4]$	$y[5]$	$y[6]$

PROB. Fall-12-Q3.3. The figure below depicts a *cascade* connection of two LTI systems, where the output of system#1 is the input to system#2, and the output of system#2 is the overall output:



The first system is defined by the frequency response $H_1(e^{j\hat{\omega}}) = 3\,e^{-j\hat{\omega}} - 3\,e^{-3j\hat{\omega}}$.

The second system is defined by the difference equation $y[n] = \frac{2}{3}\sum_{k=0}^{3} v[n-k]$.

(a) In response to $x[n] = \cos(\hat{\omega}_1 n)$, the output of system#1 will be $v[n] = A\cos(\hat{\omega}_1 n + \theta)$ for some amplitude A and phase θ.

The value of $\hat{\omega}_1$ in the range $0 \leq \hat{\omega}_1 \leq \pi$ that *maximizes* the output amplitude A is: $\hat{\omega}_1 =$ radians.

(b) Find the values of the impulse response $h[n]$ of the *overall* cascade system for $n \in \{0, 1, 2, \ldots 6\}$:

$h[0]$	$h[1]$	$h[2]$	$h[3]$	$h[4]$	$h[5]$	$h[6]$

PROB. Fall-12-Q3.4. We have seen that LTI systems can be represented in a variety of ways. For each of the LTI systems below, provide the missing representations. (Frequency response formulas can be given in any convenient form; you need not simplify them.)

(a)

difference equation:	$y[n] = \sum_{k=0}^{2} (1+k)x[n-k]$
impulse response:	
frequency response:	

(b)

MATLAB code:	`yy = conv(xx, [0,0,-2,0,2,0]);`
impulse response:	
frequency response:	

(c)

frequency response:	$H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}(5\cos(2\hat{\omega}) + 15\cos(\hat{\omega}))$
difference equation:	

(d)

impulse response:	$h[n] = 3\delta[n-2] - 5\delta[n-3] + 3\delta[n-4]$
difference equation:	
frequency response:	

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NAME: ANSWER KEY (FIRST) (LAST) GT username: VERSION #1 (e.g., gtxyz123)

Circle your recitation section in the chart below (otherwise you lose 3 points!):

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Total		

PROB. Fall-12-Q3.1.

(a) The fundamental period N_0 of a periodic sequence $x[n]$ is the smallest positive integer such that $x[n+N_0] = x[n]$ for all n.
The sequence $x[n] = \cos(0.15\pi n)$ is periodic. Its fundamental period is $N_0 = \boxed{40}$.

$$= \cos(\hat{\omega} n + \hat{\omega} N_0)$$

$\Rightarrow$ N_0 = smallest integer satisfying $\hat{\omega} N_0 = \ell 2\pi$

$$\Rightarrow N_0 = \ell \frac{2\pi}{\hat{\omega}} = \ell \frac{2\pi}{0.15\pi} = \ell \frac{40}{3} = 40$$

$\ell = 3$ is smallest

(b) When taking the 12-point DFT of $\{x[0], x[1], \ldots x[11]\} = \{1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3\}$, the zero-th DFT coefficient is $X[0] = \boxed{24}$.

$$X[0] = \sum_n x[n] e^{-jn(0)} = \sum_n x[n] = 1+2+3+1+2+3+\ldots+3$$
$$= 24$$

(c) If a 3-point DFT of $\{x[0], x[1], x[2]\}$ yields $X[0] = 0$ and $X[1] = X^*[2] = \sqrt{27}\, e^{j\pi/6}$, then the values of $\{x[0], x[1], x[2]\}$ must be:

$x[0] = \boxed{3}$

$x[1] = \boxed{-3}$

$x[2] = \boxed{0}$.

Inverse DFT:

$$x[n] = \frac{1}{N} \sum_k X[k] e^{jk2\pi n/N}$$

$$= \frac{1}{3}\left(0 + \sqrt{27} e^{j\pi/6} e^{j2\pi n/3} + \sqrt{27} e^{-j\pi/6} e^{-j2\pi n/3}\right)$$

$$= 2\sqrt{3} \cos\left(\frac{2\pi n}{3} + \frac{\pi}{6}\right)$$

$$= 2\sqrt{3} \cos\left((4n+1)\frac{\pi}{6}\right)$$

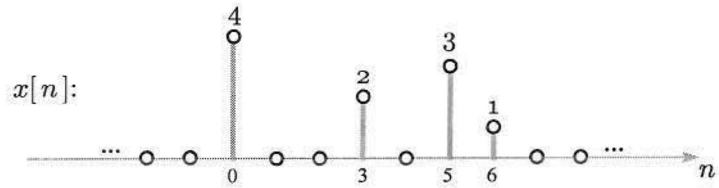
$$\Rightarrow x[0] = 2\sqrt{3}\cos\left(\frac{\pi}{6}\right) = 2\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = 3$$

$$x[1] = 2\sqrt{3}\cos\left(\frac{5\pi}{6}\right) = 2\sqrt{3}\left(\frac{-\sqrt{3}}{2}\right) = -3$$

$$x[2] = 2\sqrt{3}\cos(9\pi/6) = 0$$

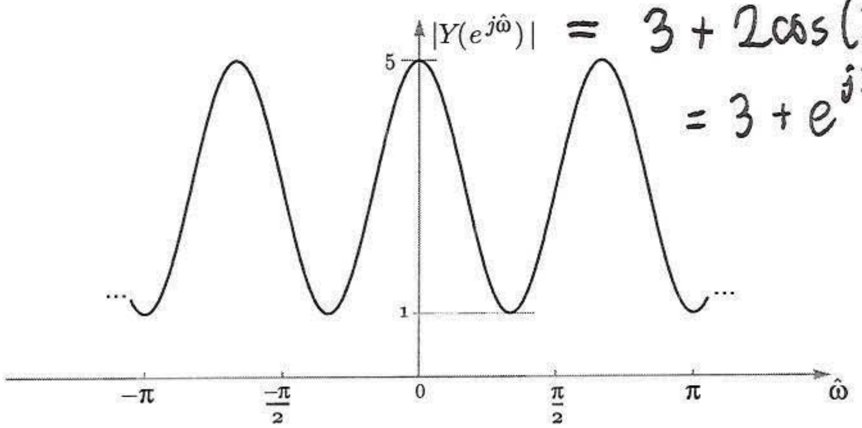
PROB. Fall-12-Q3.2. (Parts (a) and (b) are unrelated.)

(a) Write an equation for the DTFT $X(e^{j\hat{\omega}})$ of the finite sequence $x[n]$ shown in the stem plot below:



$\Rightarrow$ $X(e^{j\hat{\omega}}) = 4 + 2e^{-j3\hat{\omega}} + 3e^{-j5\hat{\omega}} + e^{-j6\hat{\omega}}$

(b) Suppose that the DTFT of a sequence $y[n]$ can be written as $Y(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}}|Y(e^{j\hat{\omega}})|$, where $|Y(e^{j\hat{\omega}})|$ oscillates sinusoidally between 1 and 5, with a period of $2\pi/3$, as shown below:



$|Y(e^{j\hat{\omega}})| = 3 + 2\cos(3\hat{\omega})$

$= 3 + e^{j3\hat{\omega}} + e^{-j3\hat{\omega}}$

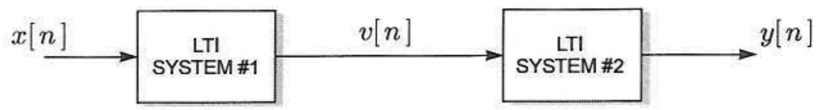
Find numerical values for the sequence $y[n]$ in the time domain at times $n \in \{0, 1, 2, 3, 4, 5, 6\}$:

$\Rightarrow Y(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}}|\cdot| = e^{-j3\hat{\omega}}(3 + e^{j3\hat{\omega}} + e^{-j3\hat{\omega}})$

$= 1 + 3e^{-j3\hat{\omega}} + e^{-j6\hat{\omega}}$

$y[0]$	$y[1]$	$y[2]$	$y[3]$	$y[4]$	$y[5]$	$y[6]$
1	0	0	3	0	0	1

PROB. Fall-12-Q3.3. The figure below depicts a *cascade* connection of two LTI systems, where the output of system#1 is the input to system#2, and the output of system#2 is the overall output:



The first system is defined by the frequency response $H_1(e^{j\hat{\omega}}) = 3e^{-j\hat{\omega}} - 3e^{-3j\hat{\omega}}$. $= 6je^{-j2\hat{\omega}} \sin\hat{\omega}$

The second system is defined by the difference equation $y[n] = \frac{2}{3}\sum_{k=0}^{3} v[n-k]$.

(a) In response to $x[n] = \cos(\hat{\omega}_1 n)$, the output of system#1 will be $v[n] = A\cos(\hat{\omega}_1 n + \theta)$ for some amplitude A and phase θ.

The value of $\hat{\omega}_1$ in the range $0 \le \hat{\omega}_1 \le \pi$ that *maximizes* the output amplitude A is: $\hat{\omega}_1 = \boxed{\frac{\pi}{2}}$ radians.

$\Rightarrow |H_1(e^{j\hat{\omega}})| = 6|\sin\hat{\omega}|$

is maximum when $\hat{\omega} = \frac{\pi}{2}$

(b) Find the values of the impulse response $h[n]$ of the *overall* cascade system for $n \in \{0, 1, 2, \ldots 6\}$:

$h[0]$	$h[1]$	$h[2]$	$h[3]$	$h[4]$	$h[5]$	$h[6]$
0	2	2	0	0	-2	-2

$h_1[n]$:	0	3	0	-3			
$h_2[n]$:	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$			
	0	0	0	0			
		2	2	2	2		
			0	0	0	0	
				-2	-2	-2	-2
	0	2	2	0	0	-2	-2

PROB. Fall-12-Q3.4. We have seen that LTI systems can be represented in a variety of ways. For each of the LTI systems below, provide the missing representations. (Frequency response formulas can be given in any convenient form; you need not simplify them.)

(a) difference equation: $y[n] = \sum_{k=0}^{2}(1+k)x[n-k] = x[n] + 2x[n-1] + 3x[n-2]$

impulse response: $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$

frequency response: $H(e^{j\hat{\omega}}) = 1 + 2e^{-j\hat{\omega}} + 3e^{-j2\hat{\omega}}$

(b) MATLAB code:

```
yy = conv(xx, [0,0,-2,0,2,0]);
```

impulse response: $h[n] = -2\delta[n-2] + 2\delta[n-4]$

frequency response: $H(e^{j\hat{\omega}}) = -2e^{-j\hat{\omega}2} + 2e^{-j4\hat{\omega}}$

$= 2.5 + 7.5e^{-j\hat{\omega}} + 7.5e^{-j3\hat{\omega}} + 2.5e^{-j4\hat{\omega}}$

(c) frequency response: $H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}(5\cos(2\hat{\omega}) + 15\cos(\hat{\omega}))$

difference equation: $y[n] = 2.5x[n] + 7.5x[n-1] + 7.5x[n-3] + 2.5x[n-4]$

(d) impulse response: $h[n] = 3\delta[n-2] - 5\delta[n-3] + 3\delta[n-4]$

difference equation: $y[n] = 3x[n-2] - 5x[n-3] + 3x[n-4]$

frequency response: $H(e^{j\hat{\omega}}) = 3e^{-j2\hat{\omega}} - 5e^{-j3\hat{\omega}} + 3e^{-j4\hat{\omega}}$