GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 19-Nov-04 COURSE: ECE-2025

NAME: ______________________________ GT # ____________________

LAST, FIRST

(e.g. gtg123a)

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L02:Thurs-9:30 (Anderson)	L03:Tues-Noon (Williams)	L05:Tues-1:30 (Williams)
L06:Thurs-1:30 (Anderson)	L07:Tues-3:00 (Durey)	L08:Thurs-3:00 (Smith)
L09:Tues-4:30 (Durey)	L10:Thurs-4:30 (Smith)	L13:Mon-3:00 (McClellan)
L14:Wed-3:00 (Taylor)	L15:Mon-4:30 (Hayes)	L16:Wed-4:30 (Taylor)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- JUSTIFY your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

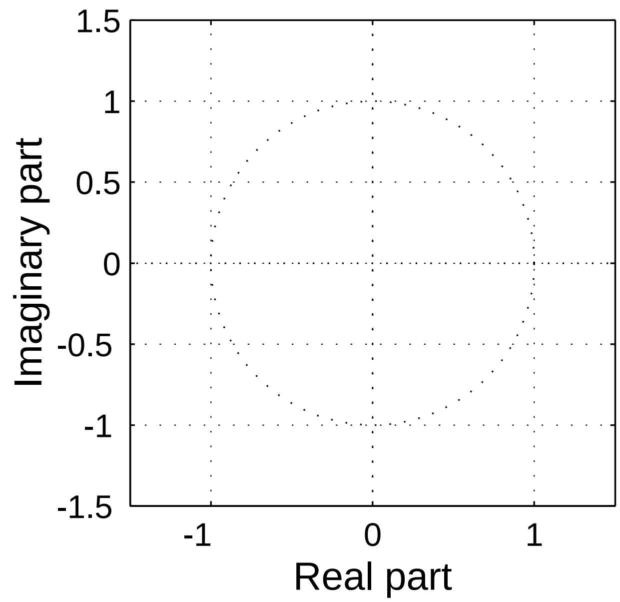
PROBLEM fa-04-Q.3.1:

A discrete-time system (FIR filter) is defined by the following z-transform system function:

$$H(z) = (1 + 0.6z^{-1})(1 - e^{j2\pi/3}z^{-1})(1 - e^{-j2\pi/3}z^{-1})$$

(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system. Give the numerical values of all filter coefficients.

(b) Determine *all* the zeros of $H(z)$ and plot them in the z-plane.



(c) If the input is of the form $x[n] = s[n] + A\cos(\omega_0 n + \phi)$, where $s[n]$ is a speech signal, for what value of frequency ω_0 (in the range $0 < \omega_0 < \pi$) will the filter completely remove the sinusoidal component? **EXPLAIN your answer.**

PROBLEM fa-04-Q.3.2:

For each of the following expressions, select the correct match from the second list below. (The operator $*$ denotes convolution.)

(a) $\square$ $\displaystyle\int_{-\infty}^{0} \delta(t-4)dt$

(b) $\square$ $u(4)$

(c) $\square$ $e^{-t}u(t) * \delta(t-4)$

(d) $\square$ $u(t-1) * u(t-3)$

(e) $\square$ $e^{-t}u(t)\delta(t-4)$

(f) $\square$ $\delta(t-1) * \delta(t-3)$

(g) $\square$ $e^{-t}u(t) * u(t-4)$

(h) $\square$ $\displaystyle\frac{d}{dt}\left\{e^{-t}u(t-4)\right\}$

Each of the expressions above is equivalent to one (and only one) of the expressions below:

[1] $-e^{-t}u(t-4) + e^{-4}\delta(t-4)$

[2] $(t-4)u(t-4)$

[3] $\left(1 - e^{-t+4}\right) u(t-4)$

[4] $e^{-(t-4)}u(t-4)$

[5] $e^{-4}\delta(t-4)$

[6] 0

[7] $\delta(t-4)$

[8] 1

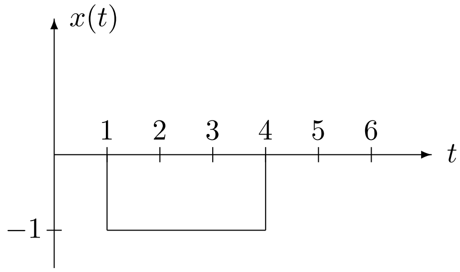
[9] e^{-4}

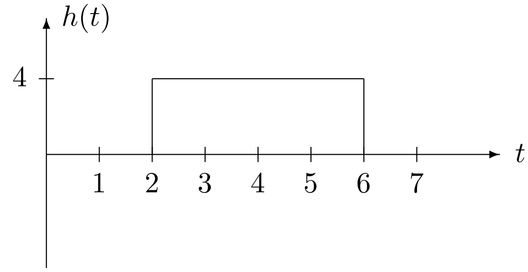
[10] $-e^{-t}u(t-4)$

[11] $u(t-4)$

PROBLEM fa-04-Q.3.3:

The following figure shows the signal $x(t) = -\,u(t-1) + u(t-4)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right) is $h(t) = 4u(t-2) - 4u(t-6)$.





(a) Sketch $h(6-\tau)$ as a function of τ in the space below.

(b) Determine the value of the output of the LTI system, $y(t)$, at $t = 6$; that is, determine $y(6)$. It is not necessary to evaluate $y(t)$ for all t, only for $t = 6$. Note: This problem may be answered without performing any integration.

(c) $y(t)$ reaches its minimum value for $T_1 \le t \le T_2$. Find the minimum value, y_{min} and also the values for T_1 and T_2.

$y_{min} =$ ____________
$T_1 =$ ____________ sec
$T_2 =$ ____________ sec

PROBLEM fa-04-Q.3.4:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = -\frac{4}{3}e^{-2t/3}u(t) + 2\delta(t)$

$\boxed{}$

(b) $x(t) = \dfrac{4}{3}e^{(-2+j3)t}u(t)$

$\boxed{}$

(c) $x(t) = \delta(t-3)\,\sin(\pi t)$

$\boxed{}$

(d) $x(t) = u(t-2) - u(t-4)$

$\boxed{}$

(e) $x(t) = \Im m\{\delta(t-4) * e^{j\pi t}\}$

$\boxed{}$

Each of the time signals above has a Fourier transform that can be found in the list below.

[0] $X(j\omega) = \dfrac{j6\omega}{2+j3\omega}$

[1] $X(j\omega) = 2e^{-j3\omega}\,\dfrac{\sin(3\omega)}{\omega}$

[2] $X(j\omega) = \dfrac{-9}{2+j3\omega}$

[3] $X(j\omega) = j2e^{-j3\omega}\,\sin(3\omega)$

[4] $X(j\omega) = 0$

[5] $X(j\omega) = 2e^{-j3\omega}\,\dfrac{\sin(\omega)}{\omega}$

[6] $X(j\omega) = \dfrac{\sin(\omega)}{\omega/2}$

[7] $X(j\omega) = j\pi\delta(\omega+\pi) - j\pi\delta(\omega-\pi)$

[8] $X(j\omega) = \frac{1}{2\pi}e^{-j3\omega} * [j\pi u(\omega+\pi) - j\pi u(\omega-\pi)]$

[9] $X(j\omega) = \dfrac{4/3}{2+j(\omega-3)}$

PROBLEM fa-04-Q.3.1:
A discrete-time system (FIR filter) is defined by the following z-transform system function:

$$H(z) = (1 + 0.6z^{-1})(1 - e^{j2\pi/3}z^{-1})(1 - e^{-j2\pi/3}z^{-1})$$

(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system. Give the numerical values of all filter coefficients.

$$H(z) = (1+0.6z^{-1})(1+z^{-1}+z^{-2})$$
$$= 1+1.6z^{-1}+1.6z^{-2}+0.6z^{-3}$$

$$\boxed{y[n] = x[n] + 1.6x[n-1] + 1.6x[n-2] + 0.6x[n-3]}$$

(b) Determine *all* the zeros of $H(z)$ and plot them in the z-plane.

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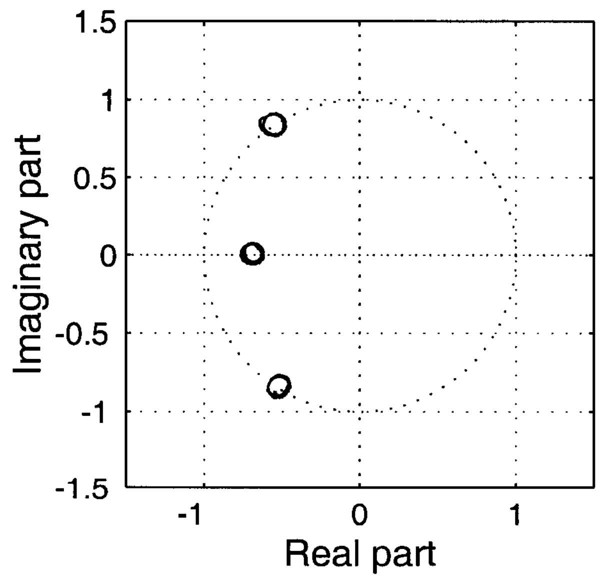
$z = -0.6$
$z = e^{\pm j2\pi/3}$



(c) If the input is of the form $x[n] = s[n] + A\cos(\omega_0 n + \phi)$, where $s[n]$ is a speech signal, for what value of frequency ω_0 (in the range $0 < \omega_0 < \pi$) will the filter completely remove the sinusoidal component? **EXPLAIN your answer.**

$$\boxed{\omega_0 = \frac{2\pi}{3}}$$

$$H(j\tfrac{2\pi}{3}) = H(-j\tfrac{2\pi}{3}) = 0$$

PROBLEM fa-04-Q.3.2:

For each of the following expressions, select the correct match from the second list below. (The operator $*$ denotes convolution.)

(a) $\boxed{[6]}$ $\displaystyle\int_{-\infty}^{0} \delta(t-4)dt$

(b) $\boxed{[8]}$ $u(4)$

(c) $\boxed{[4]}$ $e^{-t}u(t) * \delta(t-4)$

(d) $\boxed{[2]}$ $u(t-1) * u(t-3)$

(e) $\boxed{[5]}$ $e^{-t}u(t)\delta(t-4)$

(f) $\boxed{[7]}$ $\delta(t-1) * \delta(t-3)$

(g) $\boxed{[3]}$ $e^{-t}u(t) * u(t-4)$

(h) $\boxed{[1]}$ $\dfrac{d}{dt}\left\{e^{-t}u(t-4)\right\}$

Each of the expressions above is equivalent to one (and only one) of the expressions below:

[1] $-e^{-t}u(t-4) + e^{-4}\delta(t-4)$

[2] $(t-4)u(t-4)$

[3] $\left(1-e^{-t+4}\right)u(t-4)$

[4] $e^{-(t-4)}u(t-4)$

[5] $e^{-4}\delta(t-4)$

[6] 0

[7] $\delta(t-4)$

[8] 1

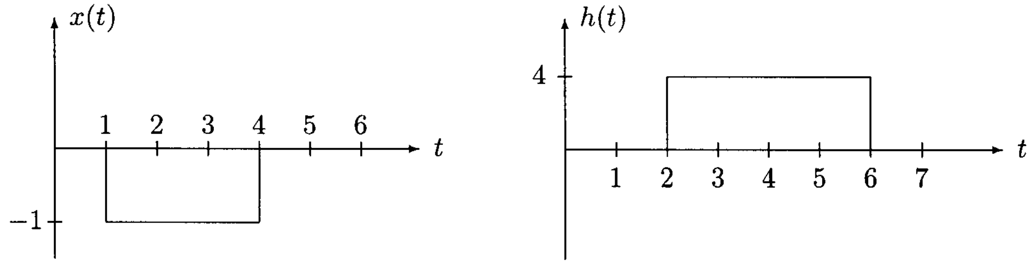
[9] e^{-4}

[10] $-e^{-t}u(t-4)$

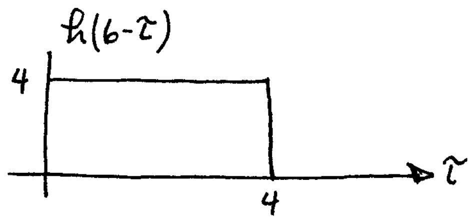
[11] $u(t-4)$

PROBLEM fa-04-Q.3.3:

The following figure shows the signal $x(t) = -\,u(t-1) + u(t-4)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right) is $h(t) = 4u(t-2) - 4u(t-6)$.



(a) Sketch $h(6-\tau)$ as a function of τ in the space below.



(b) Determine the value of the output of the LTI system, $y(t)$, at $t = 6$; that is, determine $y(6)$. It is not necessary to evaluate $y(t)$ for all t, only for $t = 6$. Note: This problem may be answered without performing any integration.

$$y(6) = \int_{-\infty}^{\infty} x(\tau)\,h(6-\tau)\,d\tau = -12$$

$$\boxed{y(6) = -12}$$

(c) $y(t)$ reaches its minimum value for $T_1 \le t \le T_2$. Find the minimum value, y_{min} and also the values for T_1 and T_2.

$y_{min} =$ -12	
$T_1 =$ 6	sec
$T_2 =$ 7	sec

PROBLEM fa-04-Q.3.4:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = -\frac{4}{3}e^{-2t/3}u(t) + 2\delta(t)$

$\boxed{[0]}$

(b) $x(t) = \frac{4}{3}e^{(-2+j3)t}u(t)$

$\boxed{[9]}$

(c) $x(t) = \delta(t-3)\,\sin(\pi t)$

$\boxed{[4]}$

(d) $x(t) = u(t-2) - u(t-4)$

$\boxed{[5]}$

(e) $x(t) = \Im m\{\delta(t-4) * e^{j\pi t}\}$

$\boxed{[7]}$

Each of the time signals above has a Fourier transform that can be found in the list below.

[0] $X(j\omega) = \dfrac{j6\omega}{2+j3\omega}$

[1] $X(j\omega) = 2e^{-j3\omega}\dfrac{\sin(3\omega)}{\omega}$

[2] $X(j\omega) = \dfrac{-9}{2+j3\omega}$

[3] $X(j\omega) = j2e^{-j3\omega}\sin(3\omega)$

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[6] $X(j\omega) = \dfrac{\sin(\omega)}{\omega/2}$

[7] $X(j\omega) = j\pi\delta(\omega+\pi) - j\pi\delta(\omega-\pi)$

[8] $X(j\omega) = \frac{1}{2\pi}e^{-j3\omega} * [j\pi u(\omega+\pi) - j\pi u(\omega-\pi)]$

[9] $X(j\omega) = \dfrac{4/3}{2+j(\omega-3)}$