ECE 2026

QUIZ 2

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY

JULY 20, 2026

Name: ______________________ FIRST ______________________ LAST ______________________ GTID

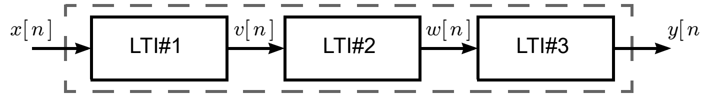
1) The quiz is closed book, closed notes,
   except for two double-sided handwritten sheets of notes.
2) No electronics (no watches/calculators/readers/phones/etc.)
3) Final answers must be entered into the answer box.
4) Correct answers *must be accompanied by concise justifications* to receive full credit.
5) Do not attach additional sheets. A blank sheet is attached in case you need more room.
6) Only the *front* side of each page will be scanned and graded, use the back sides as scrap only.
7) Express all angles as a fraction of π. For example, write 0.1π instead of $18°$ or 0.3142 radians.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
TOTAL:	100	

Since you do not have a calculator, this table may or may not be useful:

θ	$0°$	$30°$	$45°$	$60°$	$90°$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Prob. Su26-Q2.1. An "overall" system (dashed box) is constructed by cascading three FIR filters as follows:



where:

- the first system is a first-order FIR filter with filter coefficients $b_0 = 1$ and $b_1 = 2$;
- the second system has frequency response $H_2(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(9 + 6\cos(\hat{\omega}))$;
- the third system has impulse response $h_3[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$.

(a) Find the *overall* output $y[n]$ when the input is the constant $x[n] = 1$ (for all n):

$y[n] =$ ________ .

(b) If the input $x[n] = \cos(\hat{\omega}_0 n)$ results in the *overall* output $y[n] = 0$ (for all n), then it must be that

$\hat{\omega}_0 =$ ________ $\in [0, \pi]$.

(c) Let $h[n]$ denote the *overall* impulse response (of the dashed box). Find values for the following:

$h[0] =$ ________ ,

$h[1] =$ ________ ,

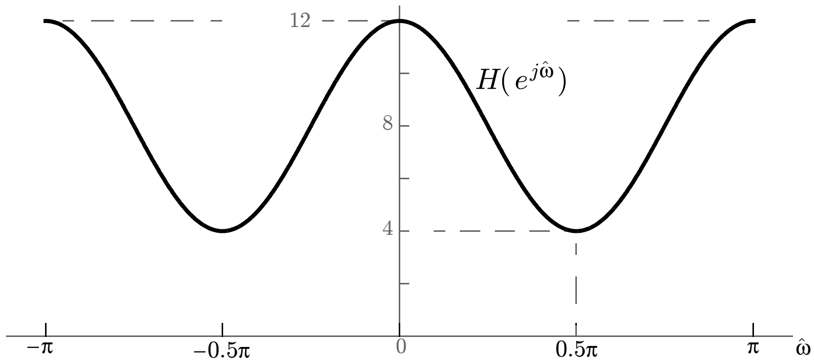
$h[2] =$ ________ ,

$h[3] =$ ________ ,

$h[4] =$ ________ ,

$h[5] =$ ________ .

Prob. Su26-Q2.2. Consider an LTI filter whose real-valued frequency response $H(e^{j\hat{\omega}})$ has the sinusoidal shape shown below:



(a) If the input $x[n] = 1 + B\cos(0.5\pi n) + \cos(0.75\pi n)$

results in the output $y[n] = A + \cos(0.5\pi n) + C\cos(0.75\pi n)$,

then the unspecified positive and real constants must be:

$A =$ ______ > 0,

$B =$ ______ > 0,

$C =$ ______ > 0.

(b) How many of the corresponding impulse-response coefficients $h[n]$ are nonzero?

______

(c) Specify numerical values for the following:

$h[-2] =$ ______,

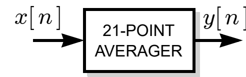
$h[-1] =$ ______,

$h[0] =$ ______,

$h[1] =$ ______,

$h[2] =$ ______.

Prob. Su26-Q2.3. All parts of this problem concern a 21-point averager with impulse response $h[n]$ and frequency response $H(e^{j\hat{\omega}})$.



(a) The sum of the impulse response coefficients is $\sum_{n=-\infty}^{\infty} h[n] = \boxed{}$.

(b) The frequency response evaluated at zero frequency is $H(e^{j0}) = \boxed{}$.

(c) Circle the shape that best describes a plot of the magnitude response $|H(e^{j\hat{\omega}})|$: [rectangle][triangle][sinc function][trapezoid][Dirichlet]

(d) Circle the shape that best describes a stem plot of $h[n]$: [rectangle][triangle][sinc function][trapezoid][Dirichlet]

(e) Circle the shape that best describes a stem plot of $g[n] = \sum_{k=-\infty}^{\infty} h[k]h[n-k]$: [rectangle][triangle][sinc function][trapezoid][Dirichlet]

(f) If the input $x[n] = A\cos\left(\frac{10\pi}{35}n\right) + 2A\cos\left(\frac{2\pi}{3}n\right) + 7A\cos(\pi n) + 8A$

results in the output $y[n] = \cos(\hat{\omega}_0 n) + B$,

then the unspecified positive and real constants must be:

$A = \boxed{}_{>0}$,

$B = \boxed{}_{>0}$,

$\hat{\omega}_0 = \boxed{}_{\in[0,\,\pi]}$.

Prob. Su26-Q2.4.

The top plot shows the frequency response $H(e^{j\hat{\omega}})$ of a bandpass filter whose impulse response is $h[n]$:

The remaining ten plots (labeled A through J) show the DTFT $X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\hat{\omega}}$ of:

$$x[n] = \cos(\hat{\omega}_0 n)h[n]$$

for different values of $\hat{\omega}_0$.

Match each $\hat{\omega}_0$ value below to its corresponding plot. (Write a letter from {A ... J} into each answer box.)

(i) $\hat{\omega}_0 = 0.05\pi$

(ii) $\hat{\omega}_0 = 0.1\pi$

(iii) $\hat{\omega}_0 = 0.15\pi$

(iv) $\hat{\omega}_0 = 0.2\pi$

(v) $\hat{\omega}_0 = 0.3\pi$

(vi) $\hat{\omega}_0 = 0.4\pi$

(vii) $\hat{\omega}_0 = 0.5\pi$

(viii) $\hat{\omega}_0 = 0.6\pi$

(ix) $\hat{\omega}_0 = 0.9\pi$

(x) $\hat{\omega}_0 = \pi$

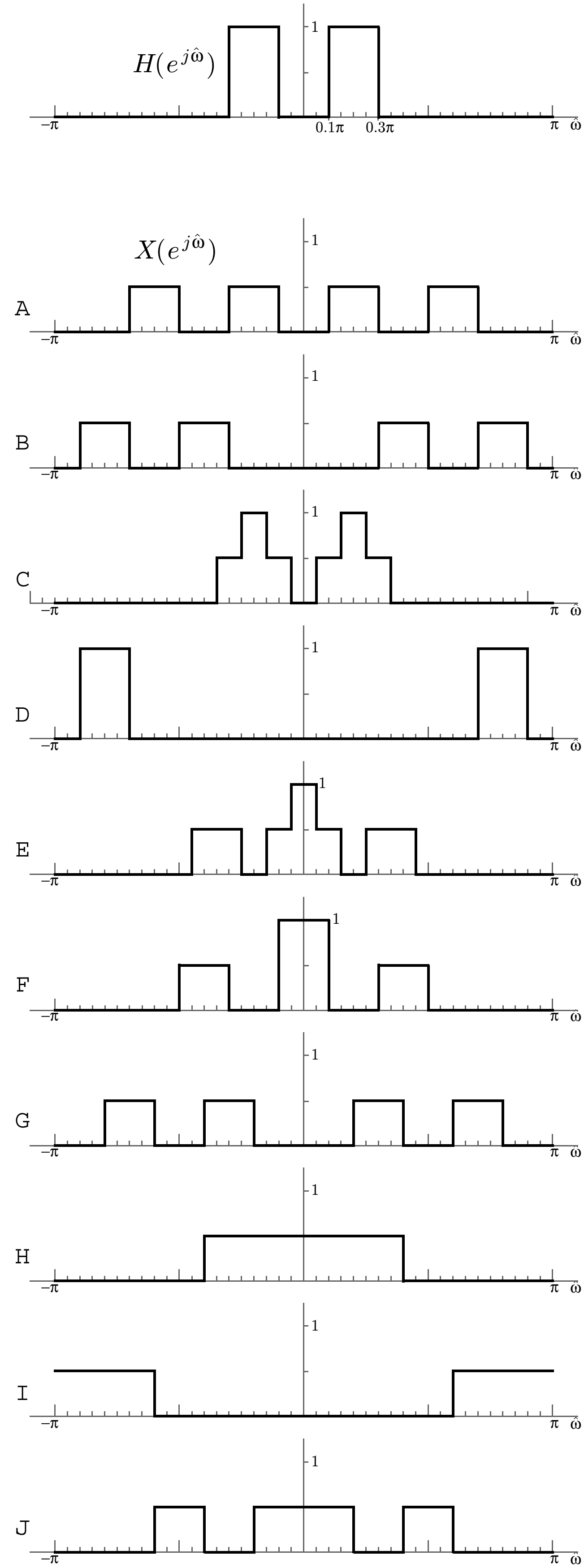


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Table of DTFT Pairs

Time-Domain: $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\hat{\omega}n_0}$
$r_L[n] = u[n] - u[n-L]$	$\dfrac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}\, e^{-j\hat{\omega}(L-1)/2}$
$r_L[n]e^{j\hat{\omega}_0 n}$	$\dfrac{\sin(\frac{1}{2}L(\hat{\omega}-\hat{\omega}_0))}{\sin(\frac{1}{2}(\hat{\omega}-\hat{\omega}_0))}\, e^{-j(\hat{\omega}-\hat{\omega}_0)(L-1)/2}$
$\dfrac{\sin(\hat{\omega}_b n)}{\pi n}$	$u(\hat{\omega}+\hat{\omega}_b) - u(\hat{\omega}-\hat{\omega}_b) = \begin{cases} 1 & \lvert\hat{\omega}\rvert \le \hat{\omega}_b \\ 0 & \hat{\omega}_b < \lvert\hat{\omega}\rvert \le \pi \end{cases}$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\dfrac{1}{1 - ae^{-j\hat{\omega}}}$

Table of DTFT Properties

Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
Periodic in $\hat{\omega}$		$X(e^{j(\hat{\omega}+2\pi)}) = X(e^{j\hat{\omega}})$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\hat{\omega}}) + bX_2(e^{j\hat{\omega}})$
Conjugate Symmetry	$x[n]$ is real	$X(e^{-j\hat{\omega}}) = X^*(e^{j\hat{\omega}})$
Conjugation	$x^*[n]$	$X^*(e^{-j\hat{\omega}})$
Time-Reversal	$x[-n]$	$X(e^{-j\hat{\omega}})$
Delay (n_d=integer)	$x[n-n_d]$	$e^{-j\hat{\omega}n_d}X(e^{j\hat{\omega}})$
Frequency Shift	$x[n]e^{j\hat{\omega}_0 n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_0)})$
Modulation	$x[n]\cos(\hat{\omega}_0 n)$	$\frac{1}{2}X(e^{j(\hat{\omega}-\hat{\omega}_0)}) + \frac{1}{2}X(e^{j(\hat{\omega}+\hat{\omega}_0)})$
Convolution	$x[n] * h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
Autocorrelation	$x[-n] * x[n]$	$\lvert X(e^{j\hat{\omega}})\rvert^2$
Parseval's Theorem	$\displaystyle\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$	$\displaystyle\frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\hat{\omega}})\rvert^2 d\hat{\omega}$

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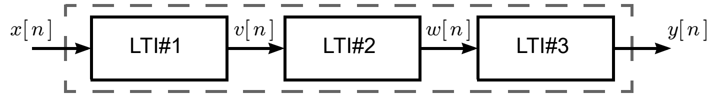
Prob. Su26-Q2.1. An "overall" system (dashed box) is constructed by cascading three FIR filters as follows:

where:



- the first system is a first-order FIR filter with filter coefficients $b_0 = 1$ and $b_1 = 2$;
- the second system has frequency response $H_2(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(9 + 6\cos(\hat{\omega}))$;
- the third system has impulse response $h_3[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$.

(a) Find the *overall* output $y[n]$ when the input is the constant $x[n] = 1$ (for all n):

$y[n] = \boxed{45}$.

Multiply frequency responses and evaluate at $\hat{\omega} = 0$:

$$\text{dc gain} = H(e^{j0}) = H_1(e^{j0})H_2(e^{j0})H_3(e^{j0})$$
$$= (1+2)(9+6)(1) = 45$$

(b) If the input $x[n] = \cos(\hat{\omega}_0 n)$ results in the *overall* output $y[n] = 0$ (for all n), then it must be that

$\hat{\omega}_0 = \boxed{\frac{2\pi}{3}}_{\in[0,\,\pi]}$.

The first two frequency responses are never zero.

The third is a 3-point averager, which has a single spectral null at $\frac{2\pi}{3}$

(c) Let $h[n]$ denote the *overall* impulse response (of the dashed box). Find values for the following:

$h[0] = \boxed{1}$, $h[1] = \boxed{6}$, $h[2] = \boxed{13}$, $h[3] = \boxed{14}$, $h[4] = \boxed{9}$, $h[5] = \boxed{2}$.

Euler $\Rightarrow H_2(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(9 + 3e^{j\hat{\omega}} + 3e^{-j\hat{\omega}})$

$$\Rightarrow h_2[n] = \underline{3} \quad 9 \quad 3$$

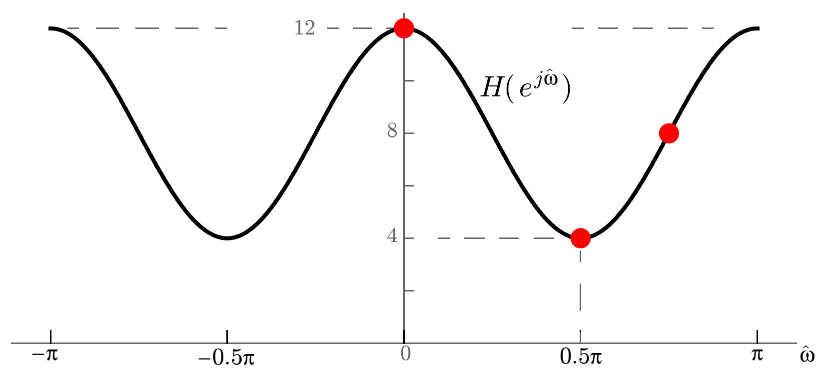
Convolve: $h_1[n] * h_2[n]$:

$$\begin{array}{cccc} \underline{3} & 9 & 3 & \\ & 6 & 18 & 6 \\ \hline \underline{3} & 15 & 21 & 6 \end{array}$$

Convolve answer with $h_3[n]$:

$$\begin{array}{cccccc} \underline{1} & 5 & 7 & 2 & & \\ & 1 & 5 & 7 & 2 & \\ & & 1 & 5 & 7 & 2 \\ \hline \underline{1} & 6 & 13 & 14 & 9 & 2 \end{array}$$

Prob. Su26-Q2.2. Consider an LTI filter whose real-valued frequency response $H(e^{j\hat{\omega}})$ has the sinusoidal shape shown below:



(a) If the input $x[n] = 1 + B\cos(0.5\pi n) + \cos(0.75\pi n)$

results in the output $y[n] = A + \cos(0.5\pi n) + C\cos(0.75\pi n)$,

then the unspecified positive and real constants must be:

$A =$ 12 $_{>0}$,

$B =$ 0.25 $_{>0}$,

$C =$ 8 $_{>0}$.

Evaluate freq response at frequencies 0, 0.5π, 0.75π:

$$\text{dc gain} = H(e^{j0}) = 12$$
$$H(e^{j0.5\pi}) = 4$$
$$H(e^{j0.75\pi}) = 8$$

(b) How many of the corresponding impulse-response coefficients $h[n]$ are nonzero?

3

(c) Specify numerical values for the following:

$h[-2] =$ 2,

$h[-1] =$ 0,

$h[0] =$ 8,

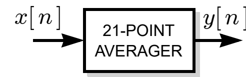
$h[1] =$ 0,

$h[2] =$ 2.

Write equation for frequency response, then use Euler:

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 8 + 4\cos(2\hat{\omega}) \\ &= 8 + 2e^{j2\hat{\omega}} + 2e^{-j2\hat{\omega}} \\ &= h[0] + h[-2]e^{j2\hat{\omega}} + h[2]e^{-j2\hat{\omega}} \end{aligned}$$

Prob. Su26-Q2.3. All parts of this problem concern a 21-point averager with impulse response $h[n]$ and frequency response $H(e^{j\hat{\omega}})$.



(a) The sum of the impulse response coefficients is $\sum_{n=-\infty}^{\infty} h[n] = \boxed{1}$.

(b) The frequency response evaluated at zero frequency is $H(e^{j0}) = \boxed{1}$.

(c) Circle the shape that best describes a plot of the magnitude response $|H(e^{j\hat{\omega}})|$: [rectangle][triangle][sinc function][trapezoid][Dirichlet]

(d) Circle the shape that best describes a stem plot of $h[n]$: [rectangle][triangle][sinc function][trapezoid][Dirichlet]

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results in the output $\quad y[n] = \cos(\hat{\omega}_0 n) + B,$

then the unspecified positive and real constants must be:

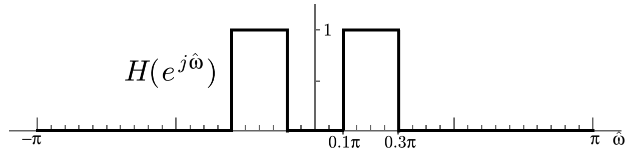
$A = \boxed{3}_{>0}$,

$B = \boxed{24}_{>0}$,

$\hat{\omega}_0 = \boxed{\pi}_{\in[0,\,\pi]}$.

Prob. Su26-Q2.4.

The top plot shows the frequency response $H(e^{j\hat{\omega}})$ of a bandpass filter whose impulse response is $h[n]$:



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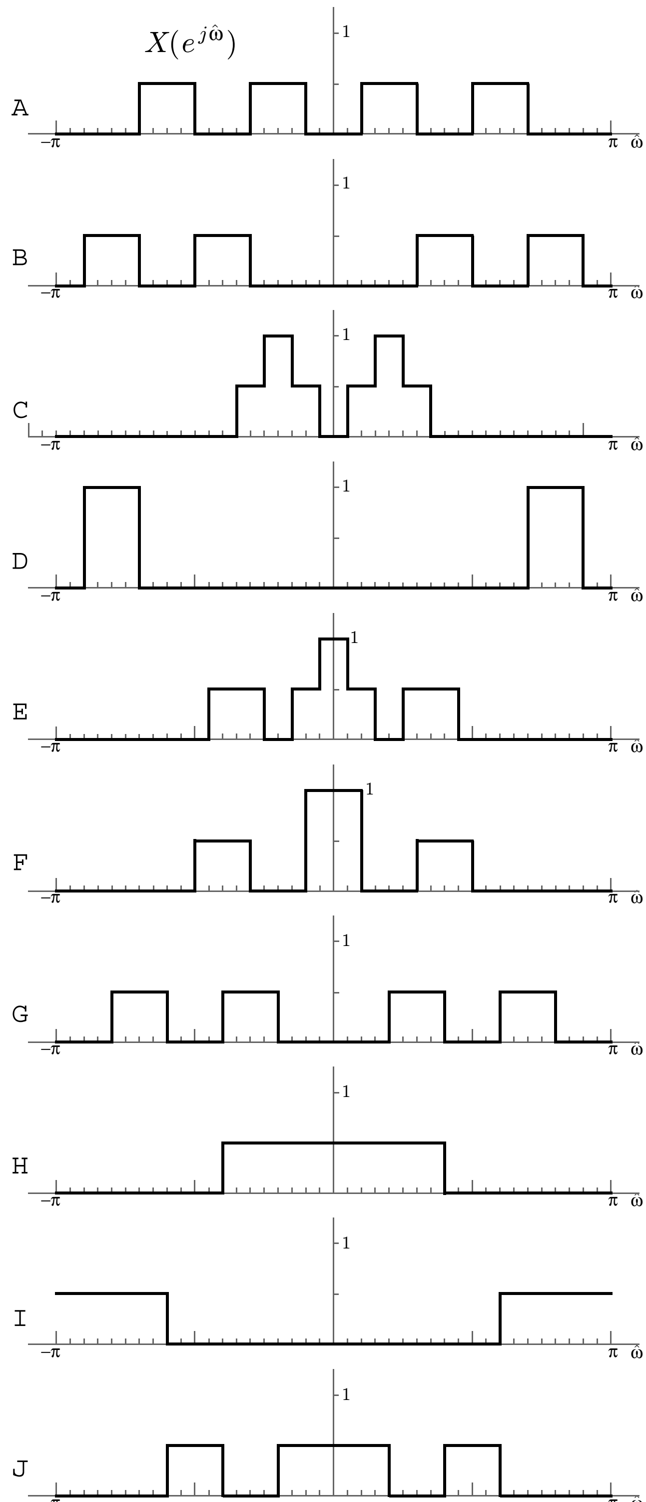
$$x[n] = \cos(\hat{\omega}_0 n)h[n]$$

for different values of $\hat{\omega}_0$.

Match each $\hat{\omega}_0$ value below to its corresponding plot. (Write a letter from {`A` ... `J`} into each answer box.)



(i) C $\hat{\omega}_0 = 0.05\pi$

(ii) H $\hat{\omega}_0 = 0.1\pi$

(iii) E $\hat{\omega}_0 = 0.15\pi$

(iv) F $\hat{\omega}_0 = 0.2\pi$

(v) J $\hat{\omega}_0 = 0.3\pi$

(vi) A $\hat{\omega}_0 = 0.4\pi$

(vii) G $\hat{\omega}_0 = 0.5\pi$

(viii) B $\hat{\omega}_0 = 0.6\pi$

(ix) I $\hat{\omega}_0 = 0.9\pi$

(x) D $\hat{\omega}_0 = \pi$

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