GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2024
Quiz #2

July 17, 2024

NAME: ______________ (FIRST) ________________ (LAST)   GT username: ________________ (e.g., gtxyz123)

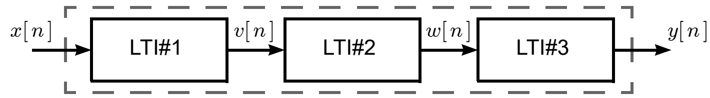
Important Notes:

- Closed book, except for two double-sided pages (8.5” × 11”) of hand-written notes.
- No calculators or other electronics (no smartphones/readers/watches/tablets/laptops/etc.)
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
- Do not write on the backs of pages, only the fronts will be graded.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
Total		

PROB. Su24-Q2.1.

Suppose that an "overall" system (dashed box) is constructed by cascading three FIR filters as follows:



where:

- the first system has frequency response $H_1(e^{j\hat{\omega}}) = 2 - e^{-j\hat{\omega}}$;
- the second system has impulse response $h_2[n] = 2\delta[n] + \delta[n-1]$;
- the third system has difference equation $y[n] = \alpha w[n] + \beta w[n-2]$, with α and β unspecified:

(a) Find the output of the *first* system *at time* 2026 when the input is $x[n] = \cos(0.5\pi n)$:

$v[2026] =$ [ ].

(b) Find the *overall* output $y[n]$ when the input is a constant $x[n] = 1$ (for all n), and assuming $\alpha = \beta = 2$:

$y[n] =$ [ ].

(c) If the *overall* impulse response (of the dashed box) is $h[n] = A\delta[n] - \delta[n-d]$, and if $\beta \neq 0$, then it must be that:

$\alpha =$ [ ],

$\beta =$ [ ],

$A =$ [ ],

$d =$ [ ].

PROB. Su24-Q2.2. Consider a 5-point averager with input $x[n]$ and output $y[n] = \frac{1}{5}\sum_{k=0}^{4} x[n-k]$.

(a) Which of the following best describes this filter? [HPF][LPF][BPF][NOTCH][ALL-PASS].

(b) The dc gain of this filter is ______ .

(c) Specify two distinct frequencies $\hat{\omega}_1 \in [0, \pi]$ that are *nulled*, so that $y[n] = 0$ when $x[n] = \cos(\hat{\omega}_1 n)$:

$\hat{\omega}_1 =$ ______ $\in[0, \pi]$,

or

$\hat{\omega}_1 =$ ______ $\in[0, \pi]$.

(d) If the input is $x[n] = -5\delta[n] + 15\delta[n-2]$, the outputs at times $n \in \{0, 1, \ldots 6\}$ are:

$y[0] =$ ______ ,

$y[1] =$ ______ ,

$y[2] =$ ______ ,

$y[3] =$ ______ ,

$y[4] =$ ______ ,

$y[5] =$ ______ ,

$y[6] =$ ______ .

(e) If a sinusoidal input $x[n] = A\cos(\hat{\omega}_0 n + \varphi)$ in standard form results in the output $y[n] = \cos(0.5\pi n)$, then it must be that:

$\hat{\omega}_0 =$ ______ $\in[0, \pi]$,

$A =$ ______ > 0,

$\varphi =$ ______ $\in(-\pi, \pi]$.

PROB. Su24-Q2.3. Match each impulse response to its corresponding magnitude response by writing a letter (from A to M) in each answer box:
(the y-axis scale is not specified, only the shapes matter)

1. $h[n] = \dfrac{\sin(0.3\pi n)}{\pi n}$

2. $h[n] = \delta[n] - \dfrac{\sin(0.1\pi n)}{\pi n}$

3. $h[n] = \delta[n] - \dfrac{\sin(0.8\pi n)}{\pi n}$

4. $h[n] = \cos(0.6\pi n)\,\dfrac{\sin(0.1\pi n)}{\pi n}$

5. $h[n] = \cos(0.5\pi n)\dfrac{\sin(0.3\pi n)}{\pi n}$

6. $h[n] = \delta[n] + \dfrac{\sin(0.2\pi n)}{\pi n}$

7. $h[n] = \delta[n] - \cos(0.5\pi n)\,\dfrac{\sin(0.2\pi n)}{\pi n}$

8. $\dfrac{\sin(0.7\pi n)}{\pi n} - \cos(0.6\pi n)\dfrac{\sin(0.1\pi n)}{\pi n}$

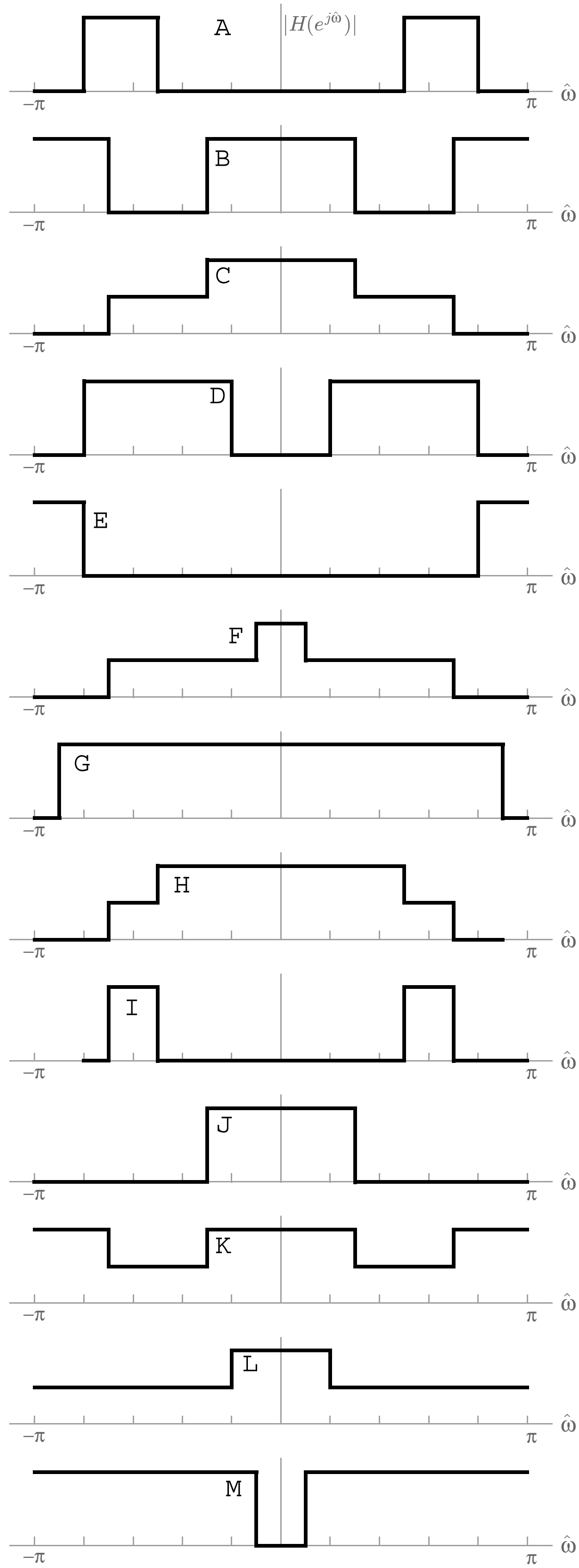
9. $h[n] = \delta[n] - \cos(\pi n)\,\dfrac{\sin(0.1\pi n)}{\pi n}$

10. $\left(\dfrac{\sin(0.8\pi n)}{\pi n} - \dfrac{\sin(0.2\pi n)}{\pi n}\right) * \left(\dfrac{\sin(0.9\pi n)}{\pi n} - \dfrac{\sin(0.5\pi n)}{\pi n}\right)$

11. $h[n] = \dfrac{\sin(0.1\pi n)}{\pi n} + \dfrac{\sin(0.7\pi n)}{\pi n}$

12. $h[n] = \cos(0.2\pi n)\dfrac{\sin(0.5\pi n)}{\pi n}$

13. $h[n] = \delta[n] - 2\cos(0.5\pi n)\dfrac{\sin(0.2\pi n)}{\pi n}$



PROB. Su24-Q2.4.

(a) The 6-point DFT of $[x[0], \dots x[3]] = [1, 0, 0, 1]$ is:

$X[0] =$ ☐ ,

$X[1] =$ ☐ ,

$X[2] =$ ☐ ,

$X[3] =$ ☐ ,

$X[4] =$ ☐ ,

$X[5] =$ ☐ .

(b) Let a and b be unspecified real numbers.
If the 4-point DFT of $[a, b, \ 1]$ is $[X[0], \dots X[3]] = [C, 2 - j, D, 2 + j]$, then:

$a =$ ☐ ,

$b =$ ☐ ,

$C =$ ☐ ,

$D =$ ☐ .

Table of DTFT Pairs	
Time-Domain: $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\hat{\omega}n_0}$
$r_L[n] = u[n] - u[n-L]$	$\dfrac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}\, e^{-j\hat{\omega}(L-1)/2}$
$r_L[n]e^{j\hat{\omega}_0 n}$	$\dfrac{\sin(\frac{1}{2}L(\hat{\omega}-\hat{\omega}_0))}{\sin(\frac{1}{2}(\hat{\omega}-\hat{\omega}_0))}\, e^{-j(\hat{\omega}-\hat{\omega}_0)(L-1)/2}$
$\dfrac{\sin(\hat{\omega}_b n)}{\pi n}$	$u(\hat{\omega}+\hat{\omega}_b) - u(\hat{\omega}-\hat{\omega}_b) = \begin{cases} 1 & \lvert\hat{\omega}\rvert \le \hat{\omega}_b \\ 0 & \hat{\omega}_b < \lvert\hat{\omega}\rvert \le \pi \end{cases}$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\dfrac{1}{1 - ae^{-j\hat{\omega}}}$

Table of DTFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
Periodic in $\hat{\omega}$		$X(e^{j(\hat{\omega}+2\pi)}) = X(e^{j\hat{\omega}})$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\hat{\omega}}) + bX_2(e^{j\hat{\omega}})$
Conjugate Symmetry	$x[n]$ is real	$X(e^{-j\hat{\omega}}) = X^*(e^{j\hat{\omega}})$
Conjugation	$x^*[n]$	$X^*(e^{-j\hat{\omega}})$
Time-Reversal	$x[-n]$	$X(e^{-j\hat{\omega}})$
Delay (n_d=integer)	$x[n-n_d]$	$e^{-j\hat{\omega}n_d}X(e^{j\hat{\omega}})$
Frequency Shift	$x[n]e^{j\hat{\omega}_0 n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_0)})$
Modulation	$x[n]\cos(\hat{\omega}_0 n)$	$\frac{1}{2}X(e^{j(\hat{\omega}-\hat{\omega}_0)}) + \frac{1}{2}X(e^{j(\hat{\omega}+\hat{\omega}_0)})$
Convolution	$x[n] * h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
Autocorrelation	$x[-n] * x[n]$	$\lvert X(e^{j\hat{\omega}})\rvert^2$
Parseval's Theorem	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$	$\dfrac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\hat{\omega}})\rvert^2 d\hat{\omega}$

Date: 28-Apr-2013

Table of Pairs for N-point DFT

Time-Domain: $x[n], \quad n=0,1,2,\ldots,N-1$	*Frequency-Domain:* $X[k], \quad k=0,1,2,\ldots,N-1$
If $x[n]$ is finite length, i.e., $x[n] \neq 0$ only when $n \in [0, N-1]$ and the DTFT of $x[n]$ is $X(e^{j\hat{\omega}})$	$X[k] = X(e^{j\hat{\omega}})\Big\vert_{\hat{\omega}=2\pi k/N}$ (frequency sampling the DTFT)
$\delta[n]$	1
1	$N\delta[k]$
$\delta[n-n_0]$	$e^{-j(2\pi k/N)n_0}$
$e^{j(2\pi n/N)k_0}$	$N\delta[k-k_0]$, when $k_0 \in [0, N-1]$
$r_L[n] = u[n] - u[n-L]$, when $L \leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi k/N))}{\sin(\frac{1}{2}(2\pi k/N))}\, e^{-j(2\pi k/N)(L-1)/2}$
$r_L[n]e^{j(2\pi k_0/N)n}$, when $L \leq N$	$\dfrac{\sin(\frac{1}{2}L(2\pi(k-k_0)/N))}{\sin(\frac{1}{2}(2\pi(k-k_0)/N))}\, e^{-j(2\pi(k-k_0)/N)(L-1)/2}$
$\dfrac{\sin(\frac{1}{2}L(2\pi n/N))}{\sin(\frac{1}{2}(2\pi n/N))}\, e^{j(2\pi n/N)(L-1)/2}$	$N(u[k]-u[k-L])$, when $L \leq N$

Table of DFT Properties

Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X[k]$
Periodic	$x[n] = x[n+N]$	$X[k] = X[k+N]$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$
Conjugate Symmetry	$x[n]$ is real	$X[N-k] = X^*[k]$
Conjugation	$x^*[n]$	$X^*[N-k]$
Time-Reversal	$x[N-n]$	$X[N-k]$
Delay (PERIODIC)	$x[n-n_d]$	$e^{-j(2\pi k/N)n_d}X[k]$
Frequency Shift	$x[n]e^{j(2\pi k_0/N)n}$	$X[k-k_0]$
Modulation	$x[n]\cos((2\pi k_0/N)n)$	$\frac{1}{2}X[k-k_0] + \frac{1}{2}X[k+k_0]$
Convolution (PERIODIC)	$x[n] * h[n] = \sum_{m=0}^{N-1} h[m]x[n-m]$	$X[k]H[k]$
Parseval's Theorem	$\displaystyle\sum_{n=0}^{N-1} \lvert x[n]\rvert^2 = \frac{1}{N}\sum_{k=0}^{N-1} \lvert X[k]\rvert^2$	

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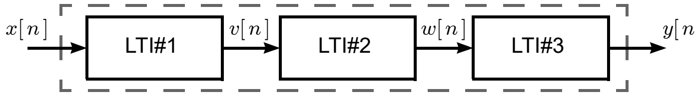
PROB. Su24-Q2.1.

Suppose that an "overall" system (dashed box) is constructed by cascading three FIR filters as follows:



where:

- the first system has frequency response $H_1(e^{j\hat{\omega}}) = 2 - e^{-j\hat{\omega}}$;
- the second system has impulse response $h_2[n] = 2\delta[n] + \delta[n-1]$;
- the third system has difference equation $y[n] = \alpha w[n] + \beta w[n-2]$, with α and β unspecified:

(a) Find the output of the *first* system *at time* 2026 when the input is $x[n] = \cos(0.5\pi n)$:

$$H_1(e^{j\hat{\omega}}) = 2 - e^{-j\hat{\omega}}$$
$$\Rightarrow v[n] = 2x[n] - x[n-1]$$
$$\Rightarrow v[2026] = 2x[2026] - x[2025]$$
$$= \quad -2 \quad - \quad 0$$

$v[2026] =$ -2 .

(b) Find the *overall* output $y[n]$ when the input is a constant $x[n] = 1$ (for all n), and assuming $\alpha = \beta = 2$:

$y[n] =$ 12 .

Overall dc gain $=$ (dc gain #1)(dc gain #2)(dc gain #3)
$= (1)(3)(4)$
$= 12$

(c) If the *overall* impulse response (of the dashed box) is $h[n] = A\delta[n] - \delta[n-d]$, and if $\beta \neq 0$, then it must be that:

The impulse response of cascade of just first two is:

$$(\underline{2} \quad -1) * (\underline{2} \quad 1) = \underline{4} \quad 0 \quad -1$$

Convolving this with $(\underline{\alpha} \quad 0 \quad \beta)$ yields overall $h[n]$:

$$\begin{array}{ccccc} \underline{4\alpha} & 0 & -\alpha & & \\ & & 4\beta & 0 & -\beta \\ \hline \underline{4\alpha} & 0 & (-\alpha + 4\beta) & 0 & -\beta \end{array}$$

❸ $A = 16$

❷ $\alpha = 4\beta = 4$

❶ $\beta = 1,\ d = 4$

$\alpha =$ 4 ,

$\beta =$ 1 ,

$A =$ 16 ,

$d =$ 4 .

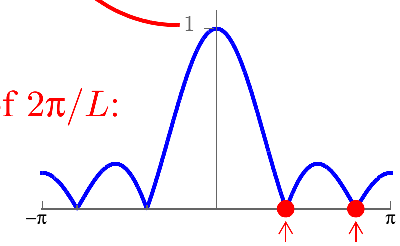
PROB. Su24-Q2.2. Consider a 5-point averager with input $x[n]$ and output $y[n] = \frac{1}{5}\sum_{k=0}^{4} x[n-k]$.

(a) Which of the following best describes this filter? [HPF][LPF][BPF][NOTCH][ALL-PASS].

(b) The dc gain of this filter is $\boxed{1}$.

(c) Specify two distinct frequencies $\hat{\omega}_1 \in [0, \pi]$ that are *nulled*, so that $y[n] = 0$ when $x[n] = \cos(\hat{\omega}_1 n)$:

Integer multiples of $2\pi/L$:



$\hat{\omega}_1 = \boxed{0.4\pi}$ $\in[0, \pi]$,

or

$\hat{\omega}_1 = \boxed{0.8\pi}$ $\in[0, \pi]$.

(d) If the input is $x[n] = -5\delta[n] + 15\delta[n-2]$, the outputs at times $n \in \{0, 1, \ldots 6\}$ are:

Convolve 0.2 0.2 0.2 0.2 0.2 with -5 0 15:

```
-1  -1  -1  -1  -1
          3   3   3   3   3
-1  -1    2   2   2   3   3
```

$y[0] = \boxed{-1}$,

$y[1] = \boxed{-1}$,

$y[2] = \boxed{2}$,

$y[3] = \boxed{2}$,

$y[4] = \boxed{2}$,

$y[5] = \boxed{3}$,

$y[6] = \boxed{3}$.

(e) If a sinusoidal input $x[n] = A\cos(\hat{\omega}_0 n + \varphi)$ in standard form results in the output $y[n] = \cos(0.5\pi n)$, then it must be that:

When $\hat{\omega} = 0.5\pi$, the frequency response becomes a polynomial in $e^{-j0.5\pi} = -j$:

$\Rightarrow H(e^{j0.5\pi}) = 0.2(1 - j + j^2 - j^3 + j^4) = 0.2$

$\Rightarrow$ No phase change, attenuates by factor of 0.2

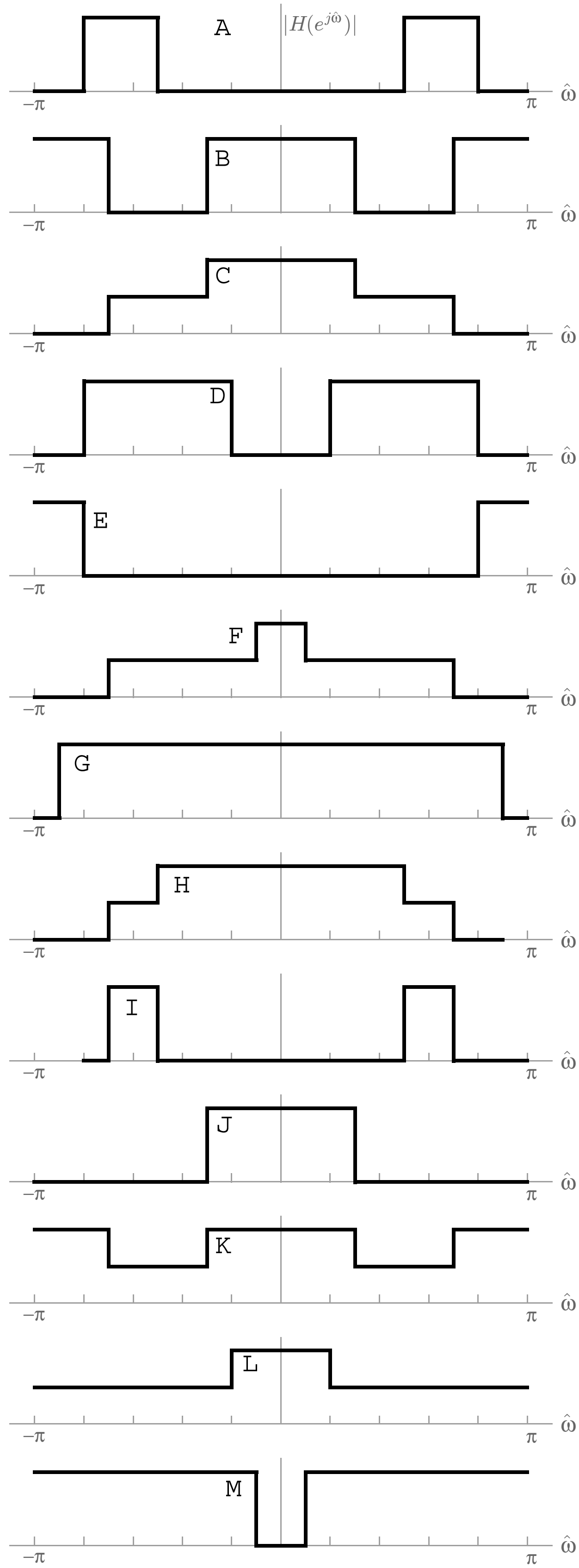
$\hat{\omega}_0 = \boxed{0.5\pi}$ $\in[0, \pi]$,

$A = \boxed{5}$ > 0,

$\varphi = \boxed{0}$ $\in(-\pi, \pi]$.

PROB. Su24-Q2.3. Match each impulse response to its corresponding magnitude response by writing a letter (from A to M) in each answer box:
(the y-axis scale is not specified, only the shapes matter)

J	$h[n] = \dfrac{\sin(0.3\pi n)}{\pi n}$
M	$h[n] = \delta[n] - \dfrac{\sin(0.1\pi n)}{\pi n}$
E	$h[n] = \delta[n] - \dfrac{\sin(0.8\pi n)}{\pi n}$
I	$h[n] = \cos(0.6\pi n)\dfrac{\sin(0.1\pi n)}{\pi n}$
D	$h[n] = \cos(0.5\pi n)\dfrac{\sin(0.3\pi n)}{\pi n}$
L	$h[n] = \delta[n] + \dfrac{\sin(0.2\pi n)}{\pi n}$
K	$h[n] = \delta[n] - \cos(0.5\pi n)\dfrac{\sin(0.2\pi n)}{\pi n}$
H	$\dfrac{\sin(0.7\pi n)}{\pi n} - \cos(0.6\pi n)\dfrac{\sin(0.1\pi n)}{\pi n}$
G	$h[n] = \delta[n] - \cos(\pi n)\dfrac{\sin(0.1\pi n)}{\pi n}$
A	$\left(\dfrac{\sin(0.8\pi n)}{\pi n} - \dfrac{\sin(0.2\pi n)}{\pi n}\right) * \left(\dfrac{\sin(0.9\pi n)}{\pi n} - \dfrac{\sin(0.5\pi n)}{\pi n}\right)$
F	$h[n] = \dfrac{\sin(0.1\pi n)}{\pi n} + \dfrac{\sin(0.7\pi n)}{\pi n}$
C	$h[n] = \cos(0.2\pi n)\dfrac{\sin(0.5\pi n)}{\pi n}$
B	$h[n] = \delta[n] - 2\cos(0.5\pi n)\dfrac{\sin(0.2\pi n)}{\pi n}$



PROB. Su24-Q2.4.

(a) The 6-point DFT of $[x[0],\ \dots\ x[3]] = [1,\ 0,\ 0,\ 1]$ is:

The DTFT is $X(e^{j\hat{\omega}}) = 1 + e^{-j3\hat{\omega}}$

$\Rightarrow$ sampling at $k(2\pi/6) = k\pi/3$ yields:

$$X[k] = 1 + e^{-jk\pi}$$

$$= \begin{cases} 2 & \text{when } k \text{ is even,} \\ 0 & \text{when } k \text{ is odd.} \end{cases}$$

$X[0] =$ 2 ,

$X[1] =$ 0 ,

$X[2] =$ 2 ,

$X[3] =$ 0 ,

$X[4] =$ 2 ,

$X[5] =$ 0 .

(b) Let a and b be unspecified real numbers.
If the 4-point DFT of $[a,\ b,\ \ 1]$ is $[X[0],\ \dots\ X[3]] = [C,\ 2-j,\ D,\ 2+j]$, then:

Sampling $X(e^{j\hat{\omega}}) = a + be^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$ at $2\pi/4 = 0.5\pi$ yields

$$X[1] = a - jb - 1.$$

Equate with given $2 - j \quad \Rightarrow a = 3,\ b = 1$

Similarly, sampling at $0(0.5\pi)$ and $2(0.5\pi)$ yields:

$$C = X[0] = a + b + 1 = 5,$$

$$D = X[2] = a - b + 1 = 3.$$

$a =$ 3 ,

$b =$ 1 ,

$C =$ 5 ,

$D =$ 3 .