GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2023
Quiz #2

July 19, 2023

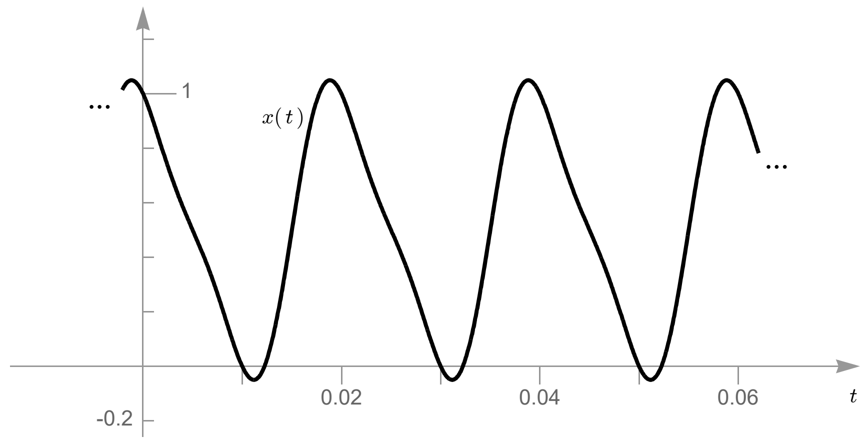
NAME: ______________ ________________ GT username: ________________
(FIRST) (LAST) (e.g., gtxyz123)

Important Notes:

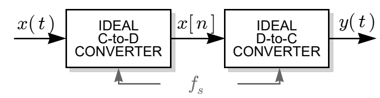
- Do not unstaple the test.
- Closed book, except for one two-sided page (8.5” × 11”) of hand-written notes.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
Only these answers will be graded. Write your answers in the provided answer boxes.
If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
Total		

PROB. Su23-Q2.1. Consider the signal $x(t) = \frac{1}{2} + \frac{\cos(100\pi t)}{1 + e^{\sin(100\pi t)}}$ shown here:



This signal is fed to the ideal sampling and reconstruction system shown here, where the same f_s parameter is used by the C-to-D and D-to-C converters, producing a continuous-time output $y(t)$:



(a) When the sampling rate is $f_s = 50$ Hz, the output is (simplify as much as possible):

$y(t) =$ ______ .

(b) When the sampling rate is $f_s = 100$ Hz, the output can be written as $y(t) = B + A\cos(2\pi f_0 t + \varphi)$, where (in standard form):

$A =$ ______ > 0,

$B =$ ______ > 0,

$f_0 =$ ______ Hz > 0,

$\varphi =$ ______ $\in(-\pi, \pi]$.

PROB. Su23-Q2.2. Consider an LTI system with output $y[n]$ whose frequency response is

$$H(e^{j\hat{\omega}}) = e^{-2j\hat{\omega}}\left(1 + 2\cos(\hat{\omega}) + 2\cos(2\hat{\omega})\right).$$

(a) Specify the following outputs when the filter input is $x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$ (*i.e.* ... 0 0 1 2 3 0 0 ...):

$y[0]$	$y[1]$	$y[2]$	$y[3]$	$y[4]$	$y[5]$	$y[6]$

(b) If the filter *output* is the sinusoid $y[n] = 3\cos(\frac{\pi}{3}n)$, the *input* must be $x[n] = A\cos(\hat{\omega}_0 n + \varphi)$, where (in standard form):

$A =$ ☐ > 0,

$\hat{\omega}_0 =$ ☐ $\in [0, \pi]$,

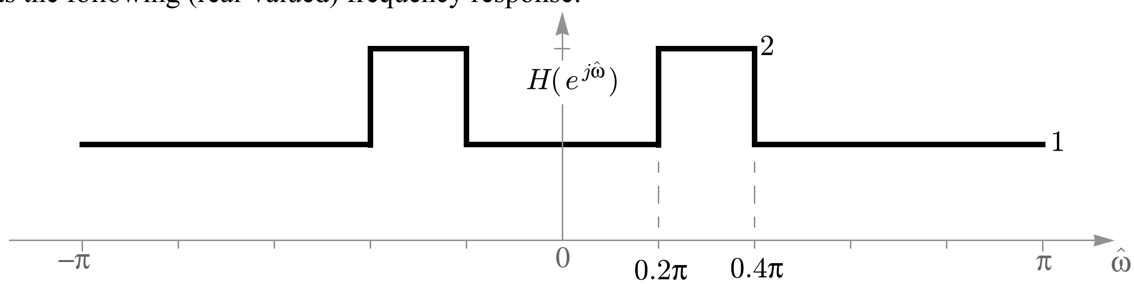
$\varphi =$ ☐ $\in(-\pi, \pi]$.

PROB. Su23-Q2.3.

(a) Find positive numeric values for the constants A, B and C so that a filter with impulse response:

$$h[n] = \frac{\sin(A\pi n)}{\pi n} + B\cos(\pi n)\frac{\sin(C\pi n)}{\pi n}:$$

has the following (real-valued) frequency response:



$A =$ ______ > 0,

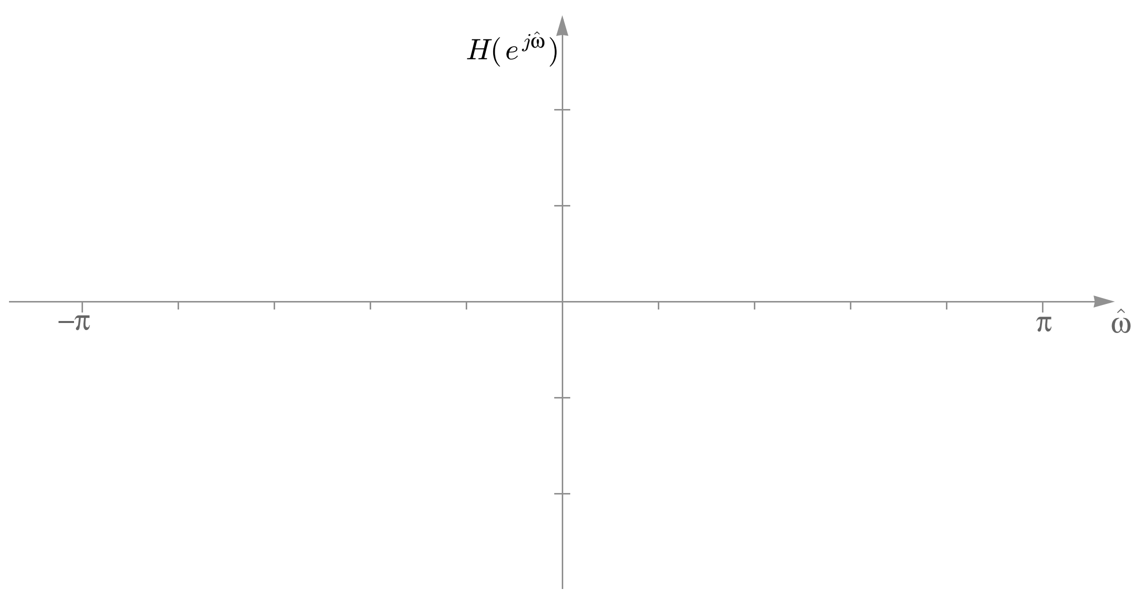
$B =$ ______ > 0,

$C =$ ______ > 0.

(b) Sketch the (real-valued) frequency response $H(e^{j\hat{\omega}})$ of an LTI filter whose impulse response is:

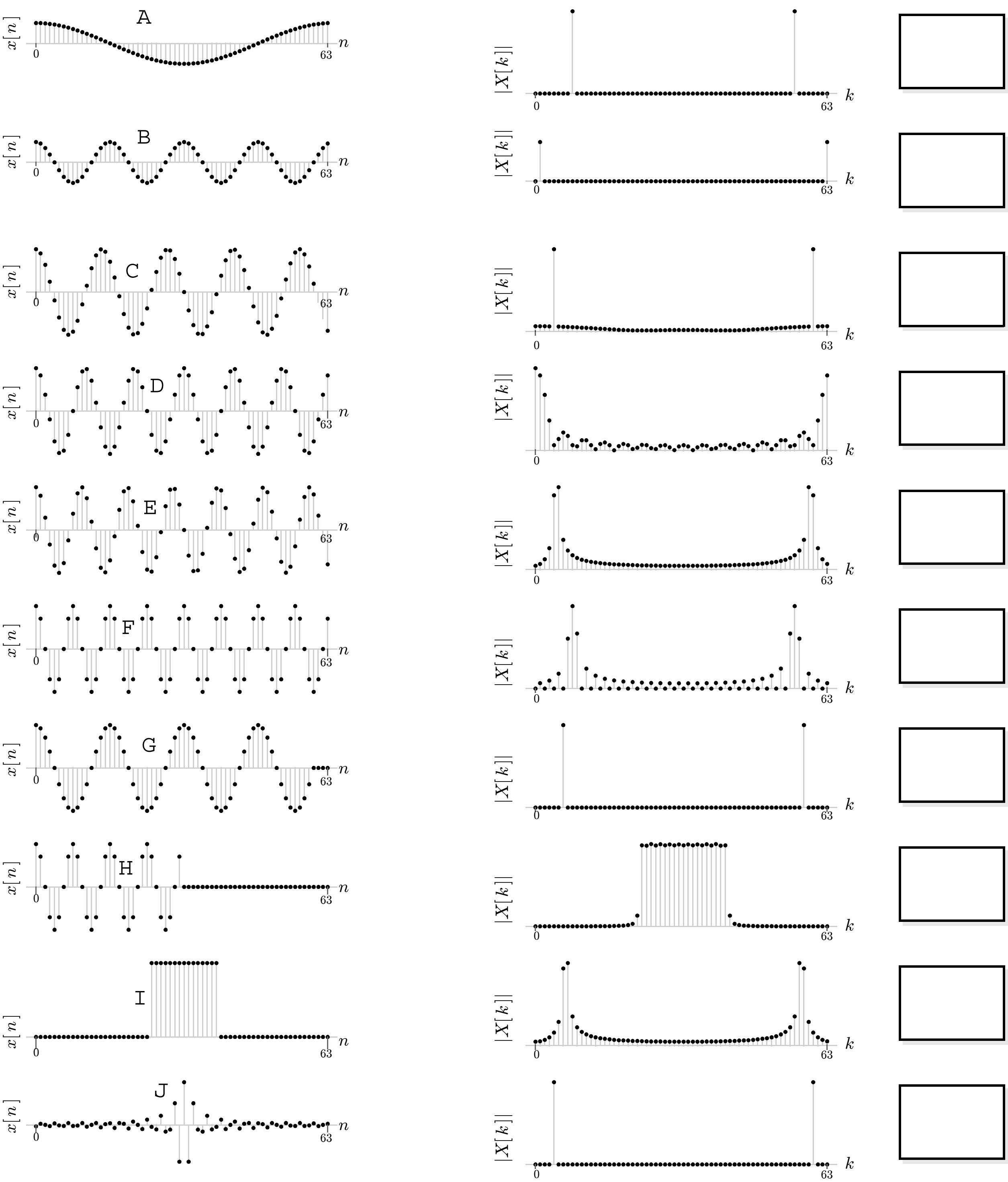
$$h[n] = \delta[n] - \frac{\sin(0.2\pi n)}{\pi n} - \cos(0.5\pi n)\frac{\sin(0.1\pi n)}{\pi n},$$

(carefully label all important heights and frequencies):



PROB. Su23-Q2.4. Shown below on the left are the plots of 10 different signal segments $[x[0], \ldots x[63]]$, labeled `A` through `J`, where each $x[n]$ is plotted versus $n \in \{0, 1, \ldots 63\}$. Let $[X[0], \ldots X[63]]$ be the $N = 64$-point DFT of $[x[0], \ldots x[63]]$. Shown on the right are the corresponding plots of the DFT magnitudes $|X[k]|$ versus $k \in \{0, 1, \ldots 63\}$, but in a scrambled order. Match each DFT magnitude plot to its corresponding signal segment by writing a letter (from `A` through `J`) into each of the 10 answer boxes.

(None of the y-axis scales are specified, they are not needed, only the shapes matter.)



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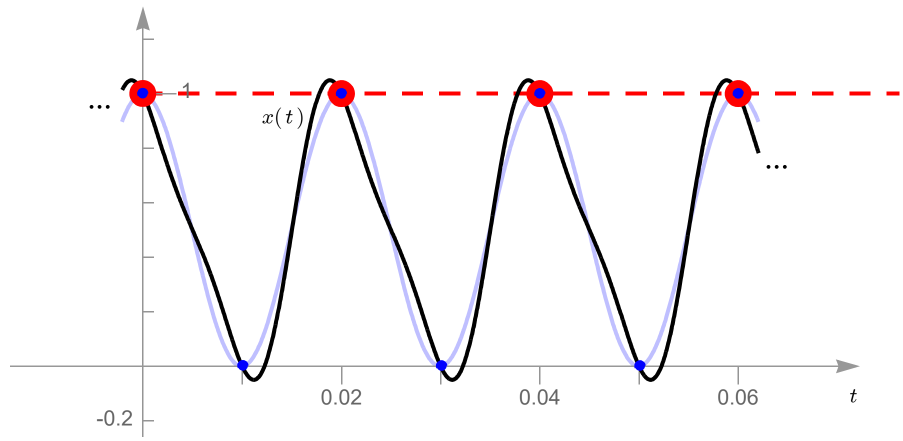
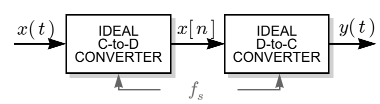
PROB. Su23-Q2.1. Consider the signal $x(t) = \frac{1}{2} + \frac{\cos(100\pi t)}{1 + e^{\sin(100\pi t)}}$ shown here:



This signal is fed to the ideal sampling and reconstruction system shown here, where the same f_s parameter is used by the C-to-D and D-to-C converters, producing a continuous-time output $y(t)$:



(a) When the sampling rate is $f_s = 50$ Hz, the output is (simplify as much as possible):

$y(t) =$ 1 .

See red dots: Every sample is 1, $x[n] = 1$ for all n, the D-C connects the dots to give the constant 1 for all time t (see red dashed line above)

(b) When the sampling rate is $f_s = 100$ Hz, the output can be written as $y(t) = B + A\cos(2\pi f_0 t + \varphi)$, where (in standard form):

$A =$ 0.5 > 0,

$B =$ 0.5 > 0,

$f_0 =$ 50 Hz > 0,

$\varphi =$ 0 $\in(-\pi, \pi]$.

See blue dots:

$x[n] = 1$ when n is even, $x[n] = 0$ when n is odd.

In other words:

$x[n] = 0.5 + 0.5\cos(\pi n)$.

The D-C substitutes $n = f_s t$ $\Rightarrow$ $y(t) = 0.5 + 0.5\cos(100\pi t)$

(see blue sinusoid in figure)

PROB. Su23-Q2.2. Consider an LTI system with output $y[n]$ whose frequency response is

$$H(e^{j\hat{\omega}}) = e^{-2j\hat{\omega}}\big(1 + 2\cos(\hat{\omega}) + 2\cos(2\hat{\omega})\big)$$

$$= e^{-2j\hat{\omega}}(1 + e^{j\hat{\omega}} + e^{-j\hat{\omega}} + e^{j2\hat{\omega}} + e^{-j2\hat{\omega}})$$

$$= 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}} \Rightarrow h[n] = \underline{1}\ 1\ 1\ 1\ 1$$

(a) Specify the following outputs when the filter input is $x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$
(*i.e.* ... 0 0 $\underline{1}$ 2 3 0 0 ...):

$y[0]$	$y[1]$	$y[2]$	$y[3]$	$y[4]$	$y[5]$	$y[6]$
1	3	6	6	6	5	3

Convolve $\underline{1}$ 2 3 with $\underline{1}$ 1 1 1 1:

1	1	1	1	1		
	2	2	2	2	2	
		3	3	3	3	3
1	3	6	6	6	5	3

(b) If the filter *output* is the sinusoid $y[n] = 3\cos(\frac{\pi}{3}n)$, the *input* must be $x[n] = A\cos(\hat{\omega}_0 n + \varphi)$, where (in standard form):

$A =$ 3 > 0,

$\hat{\omega}_0 =$ $\frac{\pi}{3}$ $\in [0, \pi]$,

$\varphi =$ $\frac{2\pi}{3}$ $\in (-\pi, \pi]$.

Sin-in, sin-out $\Rightarrow \quad \hat{\omega}_0 = \frac{\pi}{3}$

Evaluate $H(e^{j\hat{\omega}})$ at $\frac{\pi}{3}$ $\quad \Rightarrow H(e^{j\pi/3}) = e^{-j2\pi/3}(1 + 1 - 1) = e^{-j2\pi/3}$

Magnitude is $1 \Rightarrow A = 3$

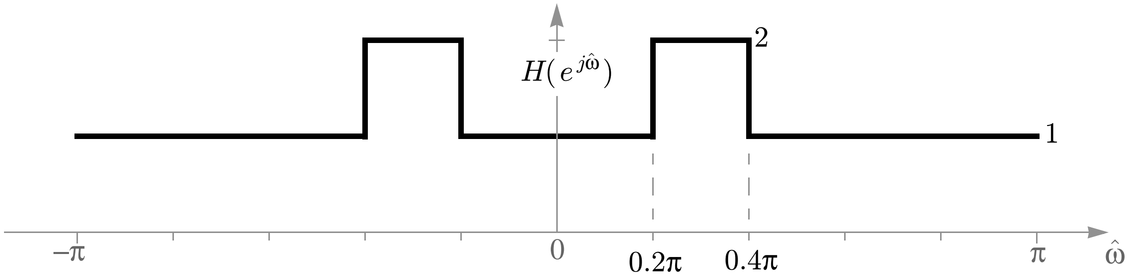
phase is $-2\pi/3 \Rightarrow \varphi = 2\pi/3$

PROB. Su23-Q2.3.

(a) Find positive numeric values for the constants A, B and C so that a filter with impulse response:

$$h[n] = \frac{\sin(A\pi n)}{\pi n} + B\cos(\pi n)\frac{\sin(C\pi n)}{\pi n}:$$

has the following (real-valued) frequency response:



Write $h[n] = h_1[n] + h_2[n]$, where:

$H_1(e^{j\hat{\omega}})$ is a LPF with cutoff $A\pi$

$H_2(e^{j\hat{\omega}})$ is a HPF with cutoff $\pi - C\pi$ when $B = 1$

Setting the LPF cutoff to 0.4π
and the HPF cutoff to 0.2π yields desired shape.

$A =$ 0.4 > 0,

$B =$ 1 > 0,

$C =$ 0.8 > 0.

(b) Sketch the (real-valued) frequency response $H(e^{j\hat{\omega}})$ of an LTI filter whose impulse response is:

$$h[n] = \delta[n] - \frac{\sin(0.2\pi n)}{\pi n} - \cos(0.5\pi n)\frac{\sin(0.1\pi n)}{\pi n},$$

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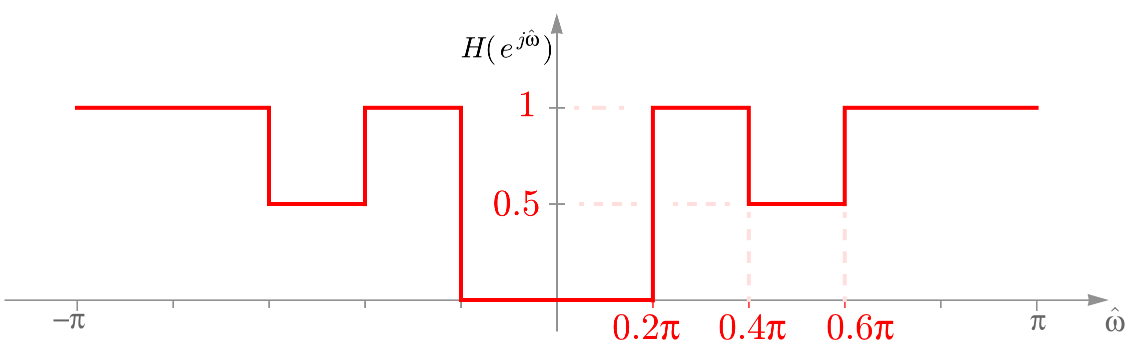

Write $h[n] = h_1[n] - h_2[n]$, where:

$H_1(e^{j\hat{\omega}})$ is a HPF with cutoff 0.2π

$H_2(e^{j\hat{\omega}})$ is half of a BPF centered at 0.5π

Subtracting half of the BPF shape from the HPF shape yields above result.

PROB. Su23-Q2.4. Shown below on the left are the plots of 10 different signal segments $[x[0], \dots x[63]]$, labeled `A` through `J`, where each $x[n]$ is plotted versus $n \in \{0, 1, \dots 63\}$. Let $[X[0], \dots X[63]]$ be the $N = 64$-point DFT of $[x[0], \dots x[63]]$. Shown on the right are the corresponding plots of the DFT magnitudes $|X[k]|$ versus $k \in \{0, 1, \dots 63\}$, but in a scrambled order. Match each DFT magnitude plot to its corresponding signal segment by writing a letter (from `A` through `J`) into each of the 10 answer boxes.

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