GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

# ECE 2026 — Summer 2022
# Quiz #2

July 20, 2022

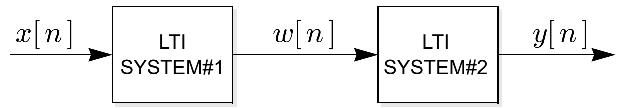
NAME: ____________ ______________ GT username: ______________
(FIRST) (LAST) (e.g., gtxyz123)

**Important Notes**:

- Do not unstaple the test.
- Closed book, except for one two-sided page (8.5” × 11”) of hand-written notes.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of $\pi$. For example, write $0.1\pi$ as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

| Problem | Value | Score Earned |
|---|---|---|
| 1 | 25 | |
| 2 | 25 | |
| 3 | 25 | |
| 4 | 25 | |
| Total | | |

**PROB. Su22-Q2.1.** Two LTI systems are connected in serial cascade, with the output of the first system fed as the input to the second system, as shown below:




- the first system is a 3-point averager (with difference equation $w[n] = \frac{1}{3}\sum_{k=0}^{2} x[n-k]$).
- the second system is a first-difference filter (with impulse response $\delta[n] - \delta[n-1]$).

If the input to the first system is $x[n] = 3\delta[n] + 18\delta[n-1] + 6\delta[n-2]$ (*i.e.* $\ldots 0\ 0\ \underline{3}\ 18\ 6\ 0\ 0 \ldots$), find numerical values for the following outputs after the second system:

| $y[0]$ | $y[1]$ | $y[2]$ | $y[3]$ | $y[4]$ | $y[5]$ | $y[6]$ |
|---|---|---|---|---|---|---|
| | | | | | | |

**PROB. Su22-Q2.2.** Consider an LTI system described by the difference equation:

$$y[n] = 0.5x[n] + 2x[n-2] + 0.5x[n-4].$$

(The input $x[n]$ in part (b) is different from that in part (c).)

(a) TRUE ☐ FALSE ☐ The system is FIR.

(b) If the input is $x[n] = u[n] - u[n-10]$, so that the number of nonzero values in the input list is 10, then the number of nonzero values in the *output* list is ☐.

(c) If the output of this system in response to the input

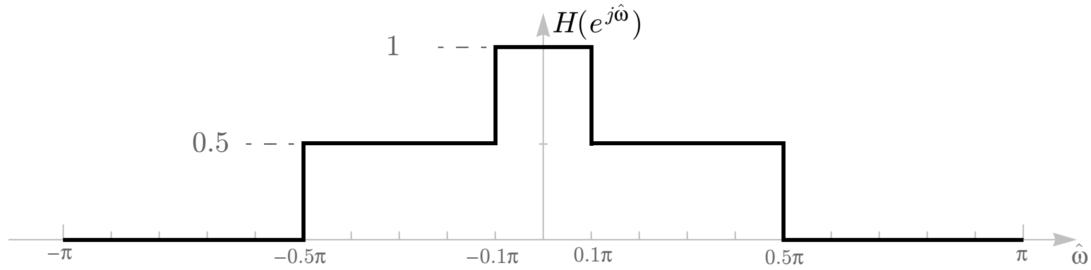
$$x[n] = 1 + \cos(\hat{\omega}_0 n) \quad \text{is} \quad y[n] = A + \cos(\hat{\omega}_0(n-2)),$$

where $\hat{\omega}_0$ satisfies $0 < \hat{\omega}_0 \leq \pi$, then it must be that:

$\hat{\omega}_0 =$ ☐

$A =$ ☐

**PROB. Su22-Q2.3.** Shown below is the real-valued frequency response of an LTI filter:




(a) TRUE ☐ FALSE ☐ The filter is FIR.

(b) Give an expression for the filter output $y[n]$ when the filter input is $x[n] = (\cos(0.3\pi n))^2$
(simplify as much as possible):

$y[n] =$ ☐ .

The filter's impulse response $h[n]$ can be written in the following four different ways:

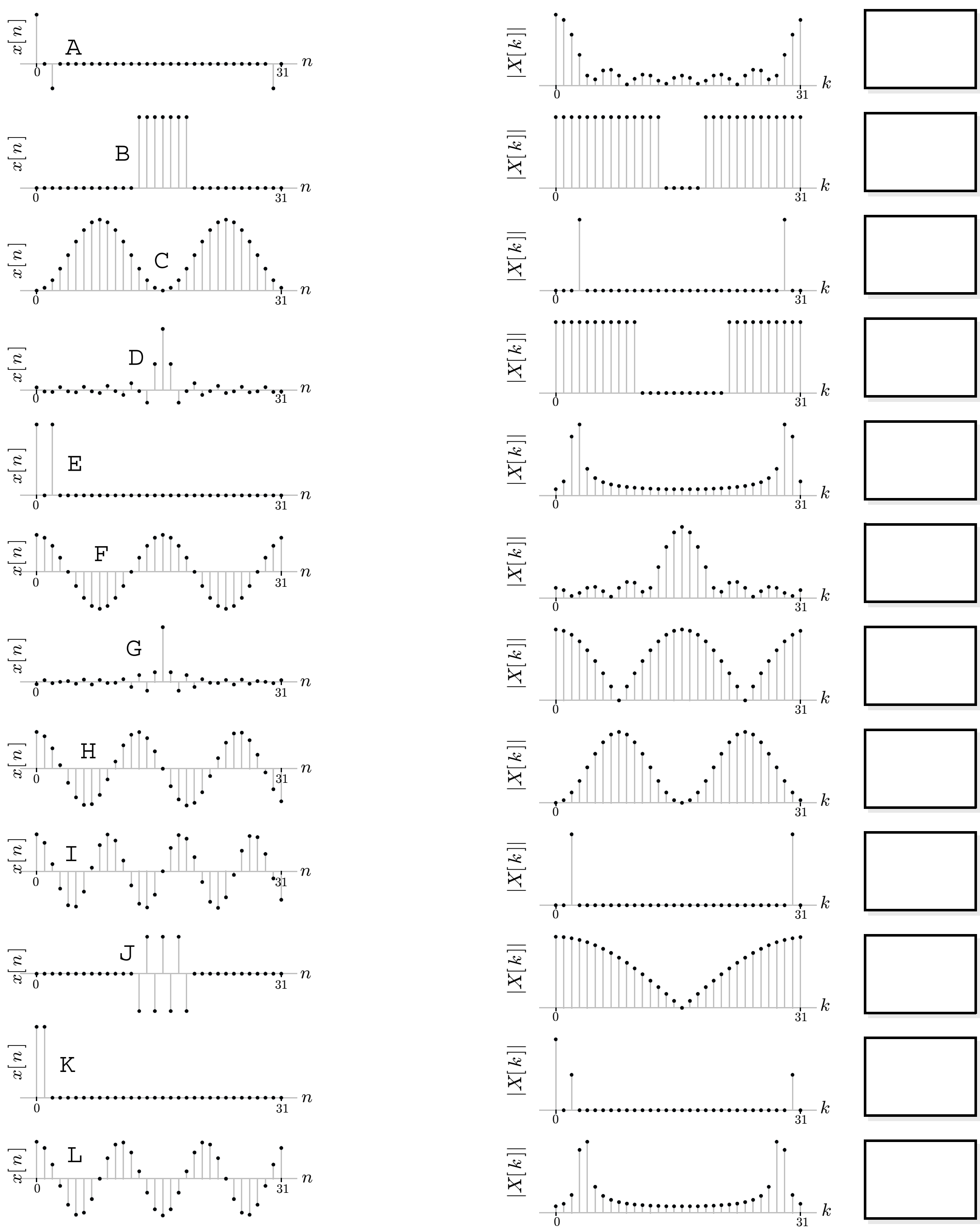
(c) $h[n] = A(\frac{\sin(B\pi n)}{\pi n} + \frac{\sin(C\pi n)}{\pi n})$, where $A =$ ☐ , $B =$ ☐ , $C =$ ☐ .

(d) $h[n] = \cos(D\pi n)\frac{\sin(E\pi n)}{\pi n}$, where $D =$ ☐ and $E =$ ☐ .

(e) $h[n] = \frac{\sin(0.5\pi n)}{\pi n} - \cos(F\pi n)\frac{\sin(G\pi n)}{\pi n}$, where $F =$ ☐ and $G =$ ☐ .

(f) $h[n] = \frac{\sin(0.1\pi n)}{\pi n} + \cos(K\pi n)\frac{\sin(L\pi n)}{\pi n}$, where $K =$ ☐ and $L =$ ☐ .

**PROB. Su22-Q2.4.** Shown below on the left are the plots of 12 different signal segments $[x[0], \ldots x[31]]$, labeled A through L, where each $x[n]$ is plotted versus $n \in \{0, 1, \ldots 31\}$. Let $[X[0], \ldots X[31]]$ be the $N = 32$-point DFT of $[x[0], \ldots x[31]]$. Shown on the right are the corresponding plots of the DFT magnitudes $|X[k]|$ versus $k \in \{0, 1, \ldots 31\}$, but in a scrambled order. Match each DFT magnitude plot to its corresponding signal segment by writing a letter (from A through L) into each of the 12 answer boxes.
(None of the y-axis scales are specified, they are not needed, only the shapes matter.)



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**ECE 2026 — Summer 2022**
**Quiz #2**

July 20, 2022

NAME: ANSWER KEY (FIRST) ______ (LAST) GT username: A (e.g., gtxyz123)

**Important Notes**:

A

- Do not unstaple the test.
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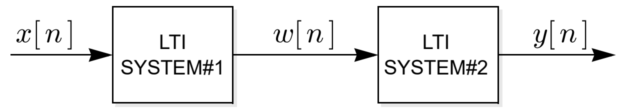
| 3 | 25 | |
| 4 | 25 | |
| Total | | |

**PROB. Su22-Q2.1.** Two LTI systems are connected in serial cascade, with the output of the first system fed as the input to the second system, as shown below:




- the first system is a 3-point averager (with difference equation $w[n] = \frac{1}{3}\sum_{k=0}^{2} x[n-k]$).
- the second system is a first-difference filter (with impulse response $\delta[n] - \delta[n-1]$).

If the input to the first system is $x[n] = 3\delta[n] + 18\delta[n-1] + 6\delta[n-2]$ (*i.e.* ... 0 0 $\underline{3}$ 18 6 0 0 ... ), find numerical values for the following outputs after the second system:

| $y[0]$ | $y[1]$ | $y[2]$ | $y[3]$ | $y[4]$ | $y[5]$ | $y[6]$ |
|---|---|---|---|---|---|---|
| 1 | 6 | 2 | –1 | –6 | –2 | 0 |

The first impulse response is $\underline{\frac{1}{3}}\ \frac{1}{3}\ \frac{1}{3}$, the second impulse response is 1, –1.

Convolving these yields overall impulse response: $\underline{\frac{1}{3}}\quad 0\quad 0\quad \frac{-1}{3}$

$$\Rightarrow y[n] = \frac{1}{3}x[n] - \frac{1}{3}x[n-3]$$

$$= \frac{1}{3}(\underline{3}\ \ 18\ \ 6\ \ 0\ \ 0\ \ 0\ \ \ldots) - \frac{1}{3}(\underline{0}\ \ 0\ \ 0\ \ 3\ \ 18\ \ 6\ \ldots)$$

$$= (\underline{1}\ \ 6\ \ 2\ \ 0\ \ 0\ \ 0\ \ldots) - (\underline{0}\ \ 0\ \ 0\ \ 1\ \ 6\ \ 2\ \ldots)$$

$$= \underline{1}\ \ 6\ \ 2\ \ {-1}\ \ {-6}\ \ {-2}\ \ldots$$

**PROB. Su22-Q2.2.** Consider an LTI system described by the difference equation:

$$y[n] = 0.5x[n] + 2x[n-2] + 0.5x[n-4].$$

(The input $x[n]$ in part (b) is different from that in part (c).)

(a) TRUE [X] FALSE [ ] The system is FIR.

(b) If the input is $x[n] = u[n] - u[n-10]$, so that the number of nonzero values in the input list is 10, then the number of nonzero values in the *output* list is **14**.

Output length = input length + filter order
= 10 + 4

(c) If the output of this system in response to the input

$$x[n] = 1 + \cos(\hat{\omega}_0 n) \quad \text{is} \quad y[n] = A + \cos(\hat{\omega}_0(n-2)),$$

where $\hat{\omega}_0$ satisfies $0 < \hat{\omega}_0 \le \pi$, then it must be that:

$\hat{\omega}_0 =$ **$0.5\pi$**

$A =$ **3**

Freq response $H(e^{j\hat{\omega}}) = 0.5 + 2e^{-2j\hat{\omega}} + 0.5e^{-4j\hat{\omega}}$

$= e^{-2j\hat{\omega}}(0.5e^{2j\hat{\omega}} + 2 + 0.5e^{-2j\hat{\omega}})$

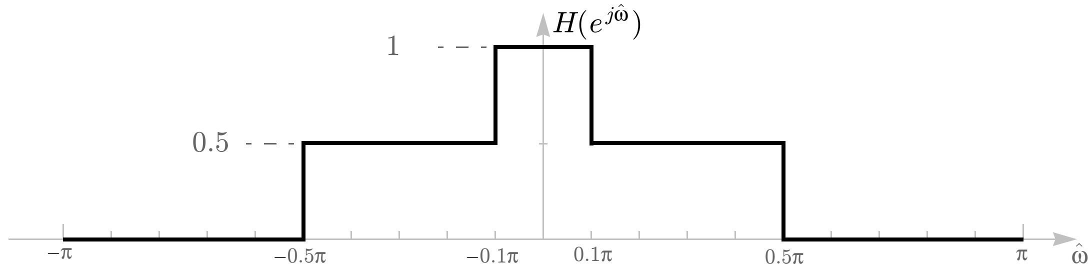
$= e^{-2j\hat{\omega}}(2 + \cos(2\hat{\omega}))$

$\Rightarrow$ magn. response $|H(e^{j\hat{\omega}})| = 2 + \cos(2\hat{\omega})$

Sinusoid amplitude *unchanged* by filter $\Rightarrow$ $|H(e^{j\hat{\omega}})| = 1$ $\Rightarrow$ $\hat{\omega}_0 = 0.5\pi$

DC gain is $H(e^{j0}) = 0.5 + 2 + 0.5 = A$ $\Rightarrow A = 3$

**PROB. Su22-Q2.3.** Shown below is the real-valued frequency response of an LTI filter:




(a) TRUE ☐ FALSE ☒ The filter is FIR.

(b) Give an expression for the filter output $y[n]$ when the filter input is $x[n] = (\cos(0.3\pi n))^2$ (simplify as much as possible):

$y[n] =$ 0.5 .

Euler ⇒ input reduces to $x[n] = 0.5 + 0.5\cos(0.6\pi n)$ — rejected by filter

The filter's impulse response $h[n]$ can be written in the following four different ways:

(c) $h[n] = A(\frac{\sin(B\pi n)}{\pi n} + \frac{\sin(C\pi n)}{\pi n})$, where $A =$ 0.5 , $B =$ 0.5 , $C =$ 0.1 .

(d) $h[n] = \cos(D\pi n)\frac{\sin(E\pi n)}{\pi n}$, where $D =$ 0.2 and $E =$ 0.3 .

(e) $h[n] = \frac{\sin(0.5\pi n)}{\pi n} - \cos(F\pi n)\frac{\sin(G\pi n)}{\pi n}$, where $F =$ 0.3 and $G =$ 0.2 .

(f) $h[n] = \frac{\sin(0.1\pi n)}{\pi n} + \cos(K\pi n)\frac{\sin(L\pi n)}{\pi n}$, where $K =$ 0.3 and $L =$ 0.2 .

**PROB. Su22-Q2.4.** Shown below on the left are the plots of 12 different signal segments $[x[0], \dots x[31]]$, labeled A through L, where each $x[n]$ is plotted versus $n \in \{0, 1, \dots 31\}$. Let $[X[0], \dots X[31]]$ be the $N = 32$-point DFT of $[x[0], \dots x[31]]$. Shown on the right are the corresponding plots of the DFT magnitudes $|X[k]|$ versus $k \in \{0, 1, \dots 31\}$, but in a scrambled order. Match each DFT magnitude plot to its corresponding signal segment by writing a letter (from A through L) into each of the 12 answer boxes.
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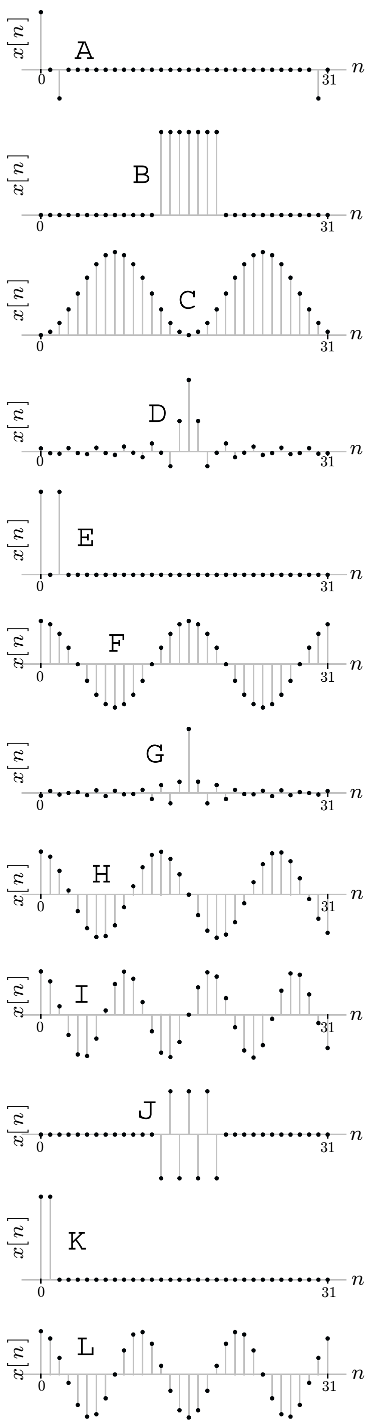


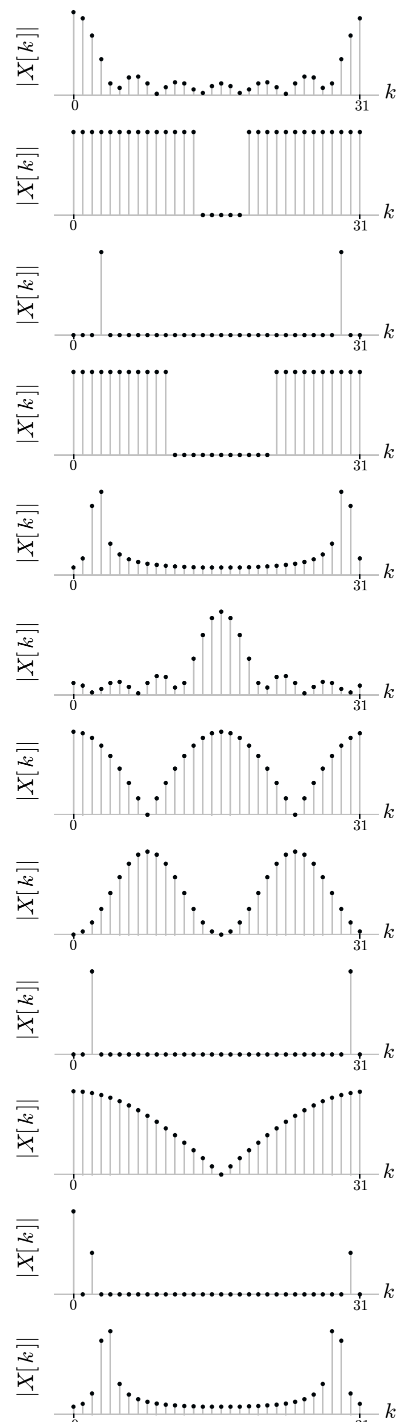


B

G

L

D

H

J

E

A

F

K

C

I