GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2018
Quiz #2

July 16, 2018

NAME: ______________ (FIRST) ________________ (LAST)   GT username: ________________ (e.g., gtxyz123)

To avoid losing 3 points, circle your recitation section:

10:05 – 11:55am	L01 (Beck)
12:30 – 2:20pm	L02 (Beck)
2:35 – 4:25pm	L03 (Bhattacharjea)

Important Notes:

- Do not unstaple the test.
- Two double-sided page (8.5” $\times$ 11”) of hand-written notes permitted.
- Calculators are allowed, but no smartphones/WiFI/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	15	
2	20	
3	20	
4	20	
5	25	
Total		

PROB. Su18-Q2.1.

(a) If $x[n] = 3\delta[n] + 2\delta[n-2]$, and if $y[n] = x[n] * x[n]$ is the convolution of $x[n]$ with itself, then find numerical values for the following:

$y[0]$	$y[1]$	$y[2]$	$y[3]$	$y[4]$	$y[5]$

(b) The inverse DTFT of $X(e^{j\hat{\omega}}) = \dfrac{10e^{-8j\hat{\omega}}}{10 - e^{-j\hat{\omega}}}$ is $x[n] =$ $\boxed{}$.

(Give an equation that is valid for all n.)

PROB. Su18-Q2.2.

Consider an LTI system defined by the difference equation $y[n] = -x[n] + Bx[n-1] - x[n-2]$, where B is an unspecified constant that may be different for each part below.

(a) If the output in response to the constant signal $x[n] = \frac{1}{3}$ (for all n) is $y[n] = 0$ (for all n), then

$B = \boxed{}$.

(b) If the output in response to the signal $x[n] = 4\cos(\pi n)$ is $y[n] = 20\cos(\pi n)$, then

$B = \boxed{}$.

(c) If $B = -1.1756$,
and if the output in response to the constant-plus-sinusoidal signal $x[n] = 100 + \cos(\hat{\omega}_0 n)$
is the constant signal $y[n] = C$, then it must be that:

$C = \boxed{}$,

$\hat{\omega}_0 = \boxed{}$

PROB. Su18-Q2.3. Consider three LTI filters whose impulse responses are as follows:

$$h_1[n] = \frac{\sin(0.6\pi n)}{\pi n},$$

$$h_2[n] = \delta[n] - \frac{\sin(0.2\pi n)}{\pi n},$$

$$h_3[n] = 2\cos(0.5\pi n)\frac{\sin(0.4\pi n)}{\pi n}.$$

(a) Indicate what kind of filter each is by writing LPF, BPF, or HPF into each answer box above.

(b) Listed below are six different impulse responses $h[n]$, where the symbol "$*$" denotes convolution. Match each to its corresponding magnitude response $|H(e^{j\hat{\omega}})|$ shown to the right (A to F). Indicate your answers by writing a letter from {A, ... F} into each answer box.

(i) $h_1[n]$

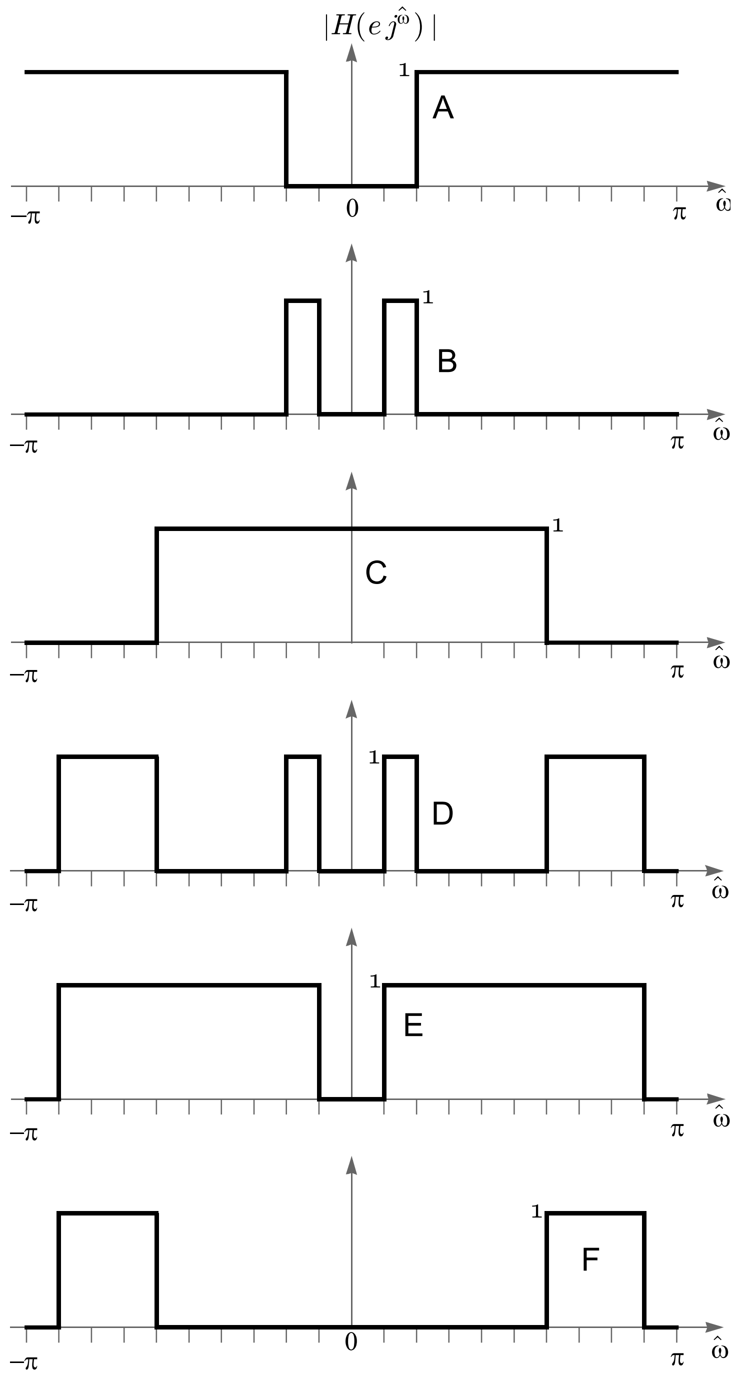
(ii) $h_2[n]$

(iii) $h_3[n]$

(iv) $(h_1[n] - h_2[n]) * h_3[n]$

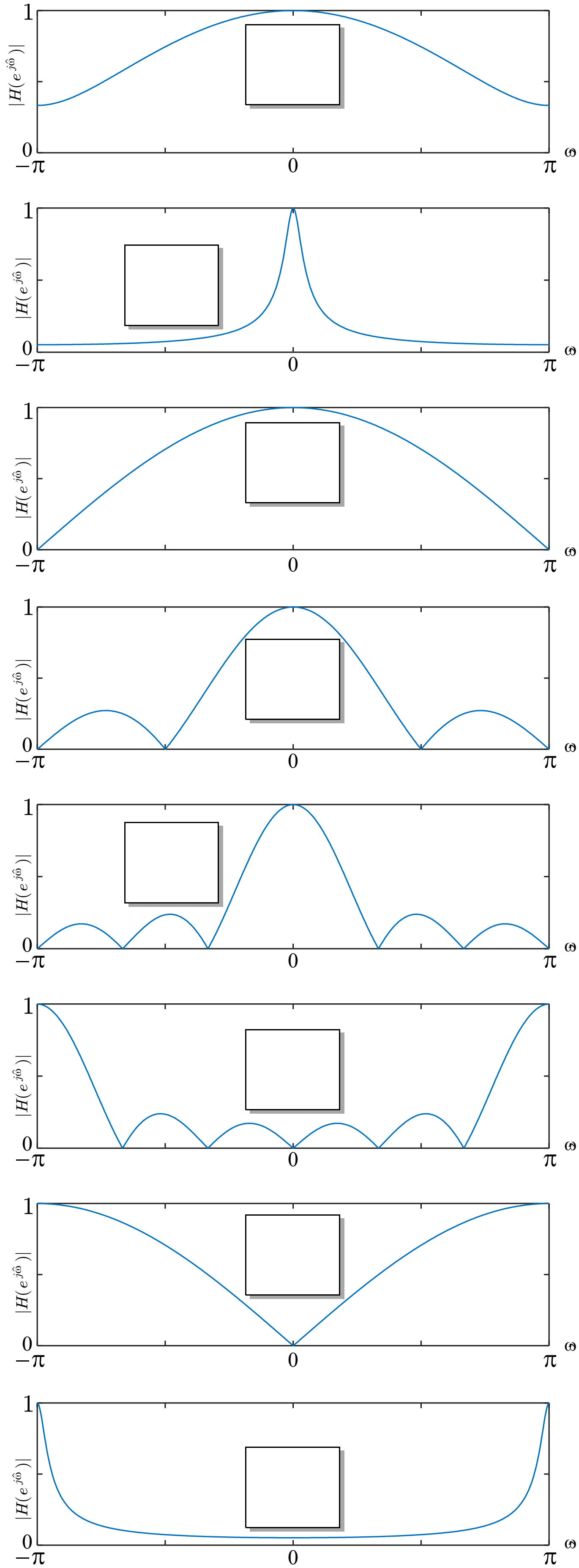
(v) $(h_1[n] - h_3[n]) * h_2[n]$

(vi) $(h_2[n] - h_3[n]) * h_1[n]$



PROB. Su18-Q2.4.

Below are a list of LTI filters (labeled A to H) specified in the time domain: either by their impulse response $h[n]$, their difference equation, or their MATLAB implementation. Match each filter to its corresponding magnitude response $|H(e^{j\hat{\omega}})|$ shown on the right. Indicate your answers by writing a letter from {A, B, ... H} in each answer box.



(A) $y[n] = 0.5x[n] - 0.5x[n-1]$.

(B) `y = conv(x,[2/3, 1/3]);`

(C) $h[n] = 0.5x[n-5] + 0.5x[n-6]$

(D) $y[n] = 0.1\sum_{k=0}^{99}(0.9)^k x[n-k]$

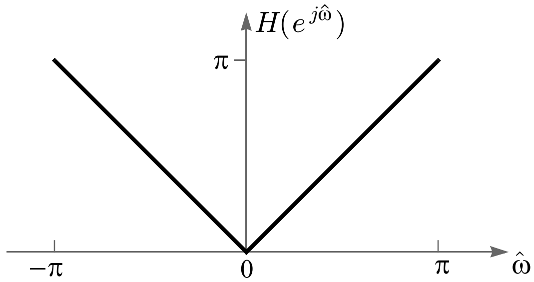
(E) $y[n] = 0.1\sum_{k=0}^{99}(-0.9)^k x[n-k]$

(F) $y[n] = 0.25\sum_{k=0}^{3} x[n-k]$

(G) $h[n] = \frac{1}{6}\sum_{k=0}^{5}\delta[n-k]$

(H) `y = conv(x,cos(pi*(0:5))/6);`

PROB. Su18-Q2.5. Consider an LTI filter whose real-valued frequency response satisfies $H(e^{j\hat{\omega}}) = |\,\hat{\omega}\,|$, for $|\,\hat{\omega}\,| < \pi$, as illustrated below:



(a) The DC gain of the filter is $\boxed{}$.

(b) If the input is the sinusoid $x[n] = \cos(0.5\pi n)$, then the output at $n = 18$ will be $y[18] = \boxed{}$.

(c) If the output of this system in response to the input $x[n] = \cos(\hat{\omega}_0 n) + \cos(\hat{\omega}_0(n-1))$ is $y[n] = \cos(\hat{\omega}_0 n + \varphi)$, then

$\hat{\omega}_0 = \boxed{}$

$\varphi = \boxed{}$

Table of DTFT Pairs	
Time-Domain: $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\hat{\omega}n_0}$
$u[n]-u[n-L]$	$\dfrac{\sin(\frac{1}{2}L\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}\,e^{-j\hat{\omega}(L-1)/2}$
$\dfrac{\sin(\hat{\omega}_b n)}{\pi n}$	$u(\hat{\omega}+\hat{\omega}_b)-u(\hat{\omega}-\hat{\omega}_b)=\begin{cases}1 & \lvert\hat{\omega}\rvert\le\hat{\omega}_b\\ 0 & \hat{\omega}_b<\lvert\hat{\omega}\rvert\le\pi\end{cases}$
$a^n u[n] \quad (\lvert a\rvert<1)$	$\dfrac{1}{1-ae^{-j\hat{\omega}}}$

Table of DTFT Properties		
Property Name	*Time-Domain:* $x[n]$	*Frequency-Domain:* $X(e^{j\hat{\omega}})$
Periodic in $\hat{\omega}$		$X(e^{j(\hat{\omega}+2\pi)})=X(e^{j\hat{\omega}})$
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(e^{j\hat{\omega}})+bX_2(e^{j\hat{\omega}})$
Conjugate Symmetry	$x[n]$ is real	$X(e^{-j\hat{\omega}})=X^*(e^{j\hat{\omega}})$
Conjugation	$x^*[n]$	$X^*(e^{-j\hat{\omega}})$
Time-Reversal	$x[-n]$	$X(e^{-j\hat{\omega}})$
Delay (d=integer)	$x[n-d]$	$e^{-j\hat{\omega}d}X(e^{j\hat{\omega}})$
Frequency Shift	$x[n]e^{j\hat{\omega}_0 n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_0)})$
Modulation	$x[n]\cos(\hat{\omega}_0 n)$	$\frac{1}{2}X(e^{j(\hat{\omega}-\hat{\omega}_0)})+\frac{1}{2}X(e^{j(\hat{\omega}+\hat{\omega}_0)})$
Convolution	$x[n]*h[n]$	$X(e^{j\hat{\omega}})H(e^{j\hat{\omega}})$
Parseval's Theorem	$\displaystyle\sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2$	$\displaystyle\frac{1}{2\pi}\int_{-\pi}^{\pi}\lvert X(e^{j\hat{\omega}})\rvert^2 d\hat{\omega}$

Date: 28-Apr-2013

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PROB. Su18-Q2.1.

(a) If $x[n] = 3\delta[n] + 2\delta[n-2]$, and if $y[n] = x[n] * x[n]$ is the convolution of $x[n]$ with itself, then find numerical values for the following:

$y[0]$	$y[1]$	$y[2]$	$y[3]$	$y[4]$	$y[5]$
9	0	12	0	4	0

x[n]	3	0	2		
x[0]x[n]	9	0	6		
x[1]x[n-1]		0	0	0	
x[2]x[n-2]			6	0	4
	9	0	12	0	4

(b) The inverse DTFT of $X(e^{j\hat{\omega}}) = \dfrac{10e^{-8j\hat{\omega}}}{10 - e^{-j\hat{\omega}}}$ is $x[n] = \boxed{(0.1)^{n-8}u[n-8]}$.

(Give an equation that is valid for all n.)

Simplifies to $X(e^{j\hat{\omega}}) = \dfrac{e^{-8j\hat{\omega}}}{1 - 0.1e^{-j\hat{\omega}}} = e^{-8j\hat{\omega}} \dfrac{1}{1 - 0.1e^{-j\hat{\omega}}}$

From Table 1, the second factor is the DTFT of $(0.1)^n u[n]$.

From Table 2, multiplying the DTFT by $e^{-8j\hat{\omega}}$ delays by 8.

PROB. Su18-Q2.2.

Consider an LTI system defined by the difference equation $y[n] = -x[n] + Bx[n-1] - x[n-2]$, where B is an unspecified constant that may be different for each part below.

(a) If the output in response to the constant signal $x[n] = \frac{1}{3}$ (for all n) is $y[n] = 0$ (for all n), then

$B = \boxed{2}$.

The DC gain is zero

$$\Rightarrow \quad -1 + B - 1 = 0$$

$$\Rightarrow \quad B = 2$$

(b) If the output in response to the signal $x[n] = 4\cos(\pi n)$ is $y[n] = 20\cos(\pi n)$, then

$B = \boxed{-7}$.

The gain at π is five

$$\Rightarrow \quad -1 - B - 1 = 5$$

$$\Rightarrow \quad B = -7$$

(c) If $B = -1.1756$,
and if the output in response to the constant-plus-sinusoidal signal $x[n] = 100 + \cos(\hat{\omega}_0 n)$
is the constant signal $y[n] = C$, then it must be that:

$C = \boxed{-317.56}$,

$\hat{\omega}_0 = \boxed{0.7\pi}$

The frequency response of this $[-1,\ B,\ -1]$ filter is

$$\begin{aligned} H(e^{j\hat{\omega}}) &= -1 + Be^{-j\hat{\omega}} - e^{-2j\hat{\omega}} \\ &= e^{-j\hat{\omega}}(-e^{j\hat{\omega}} + B - e^{-j\hat{\omega}}) \\ &= e^{-j\hat{\omega}}(B - 2\cos(\hat{\omega})) \end{aligned}$$

The nulled frequency must therefore satisfy $2\cos(\hat{\omega}_0) = B$

$$\Rightarrow \quad \hat{\omega}_0 = \cos^{-1}(B/2) = \cos^{-1}(-1.1756/2) = 0.7\pi$$

Equating the DC gain $B - 2$ to the ratio $C/100$ yields:

$$C/100 = B - 2 = -3.1756$$

$$\Rightarrow C = -317.56$$

PROB. Su18-Q2.3. Consider three LTI filters whose impulse responses are as follows:

$\boxed{\text{LPF}}$ $\quad h_1[n] = \dfrac{\sin(0.6\pi n)}{\pi n}$,

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(i) $\boxed{\text{C}}$ $\quad h_1[n]$

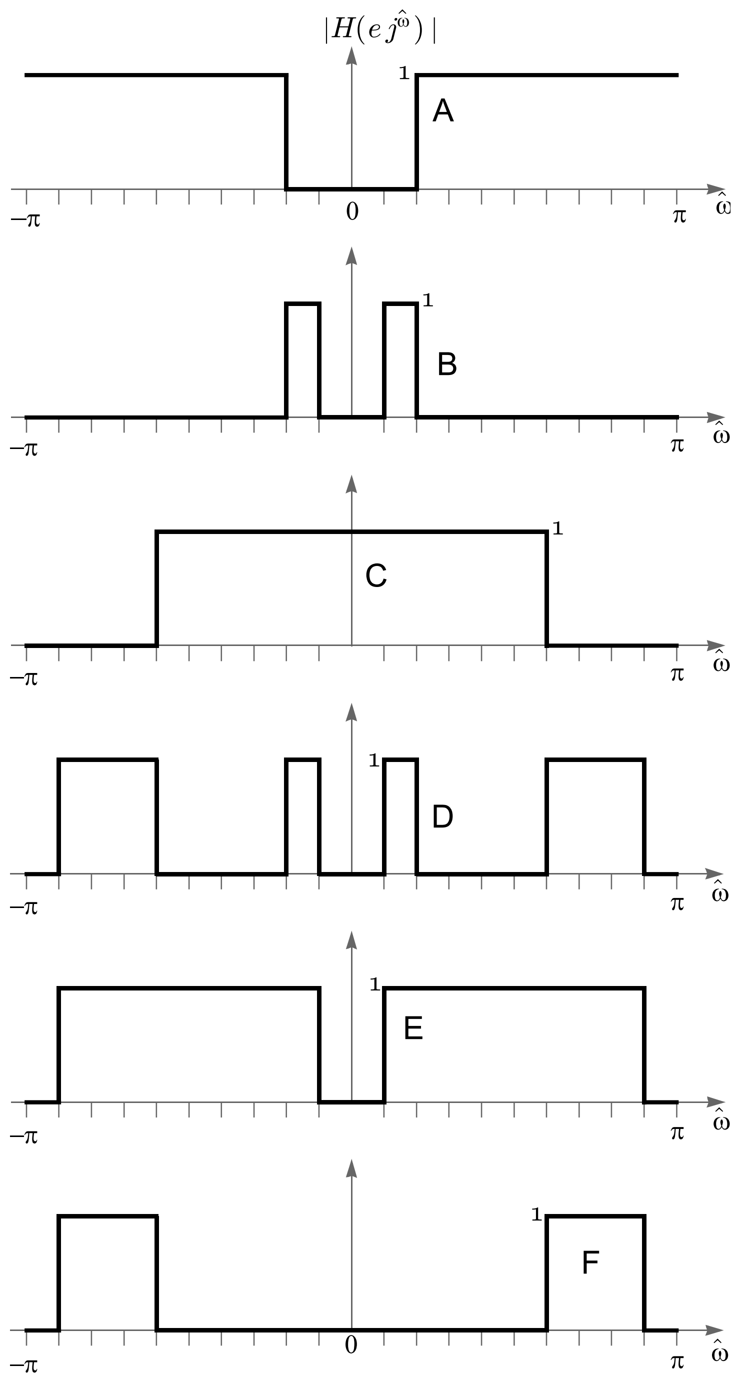
(ii) $\boxed{\text{A}}$ $\quad h_2[n]$

(iii) $\boxed{\text{E}}$ $\quad h_3[n]$

(iv) $\boxed{\text{D}}$ $\quad (h_1[n] - h_2[n]) * h_3[n]$

(v) $\boxed{\text{F}}$ $\quad (h_1[n] - h_3[n]) * h_2[n]$

(vi) $\boxed{\text{B}}$ $\quad (h_2[n] - h_3[n]) * h_1[n]$



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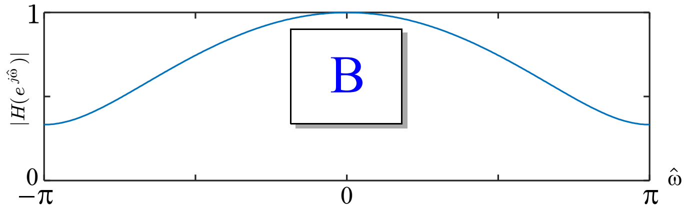
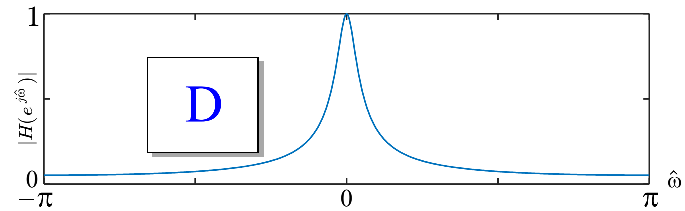
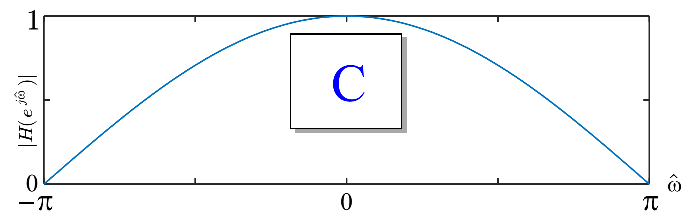
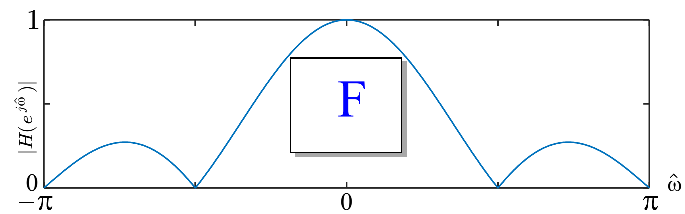
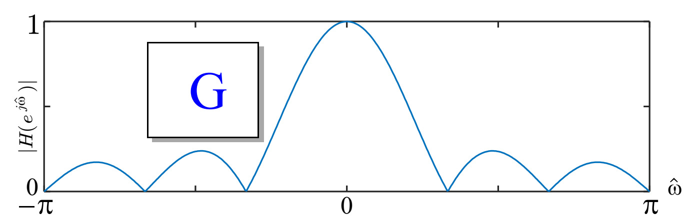
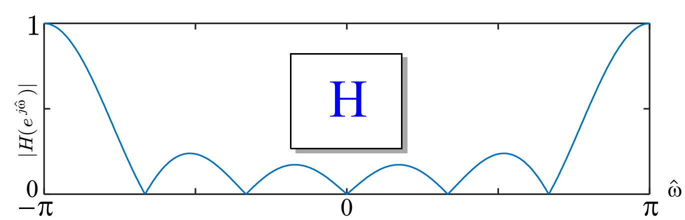


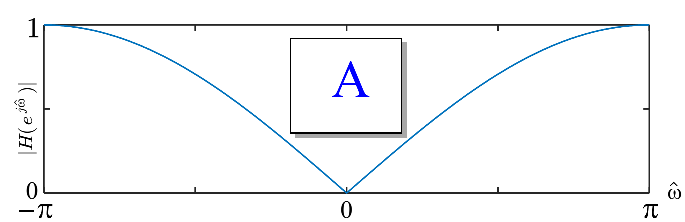


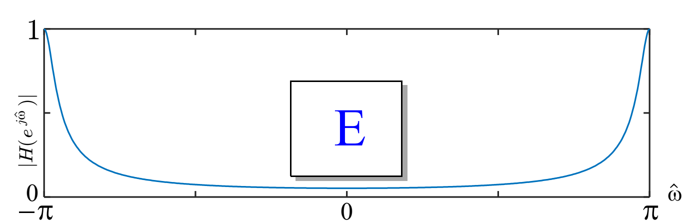




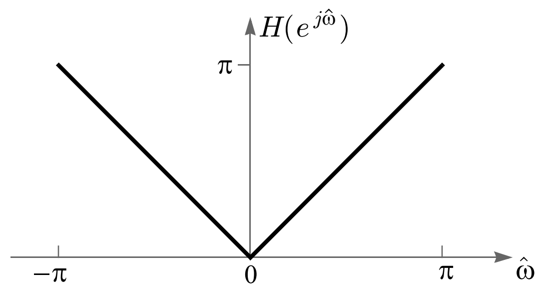








PROB. Su18-Q2.5. Consider an LTI filter whose real-valued frequency response satisfies $H(e^{j\hat{\omega}}) = |\hat{\omega}|$, for $|\hat{\omega}| < \pi$, as illustrated below:



(a) The DC gain of the filter is $\boxed{0}$.

(b) If the input is the sinusoid $x[n] = \cos(0.5\pi n)$, then the output at $n = 18$ will be $y[18] = \boxed{-0.5\pi}$.

The freq response at the sinusoid frequency $H(e^{j0.5\pi}) = 0.5\pi$,

$$\Rightarrow y[n] = r\cos(0.5\pi n + \theta) = 0.5\pi\cos(0.5\pi n)$$

$$\Rightarrow y[18] = -0.5\pi$$

polar form $re^{j\theta}$ where $r = 0.5\pi$, $\theta = 0$

(c) If the output of this system in response to the input $x[n] = \cos(\hat{\omega}_0 n) + \cos(\hat{\omega}_0(n-1))$ is $y[n] = \hat{\omega}_0\cos(\hat{\omega}_0 n + \varphi)$, then

$\hat{\omega}_0 = \boxed{\frac{2\pi}{3}}$

$\varphi = \boxed{\frac{-\pi}{3}}$

From phasor addition, input simplifies to

$Ae^{j\theta} = 1 + e^{-j\hat{\omega}_0}$
$\Rightarrow A = 2\cos(\hat{\omega}_0/2),\ \theta = -\hat{\omega}_0/2$

$$x[n] = 2\cos(\hat{\omega}_0/2)\cos(\hat{\omega}_0 n - \hat{\omega}_0/2).$$

Filter scales by $\hat{\omega}_0$, no phase change

$\Rightarrow$ input amplitude must be 1

$$\Rightarrow \quad 2\cos(\hat{\omega}_0/2) = 1$$

$$\Rightarrow \quad \hat{\omega}_0 = 2\pi/3$$

alternative solution:

$\hat{\omega}_0 = 0$
$\varphi =$ anything