GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2017
Quiz #2

July 20, 2017

NAME: ______________________________

Important Notes:

- DO NOT unstaple the test.
- Two two-sided page (8.5” $\times$ 11”) of hand-written notes permitted.
- Calculators are allowed, but no smartphones/WiFI/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	20	
2	20	
3	20	
4	20	
5	20	
Total		

PROB. Su17-Q2.1. Consider an LTI system described by the difference equation:

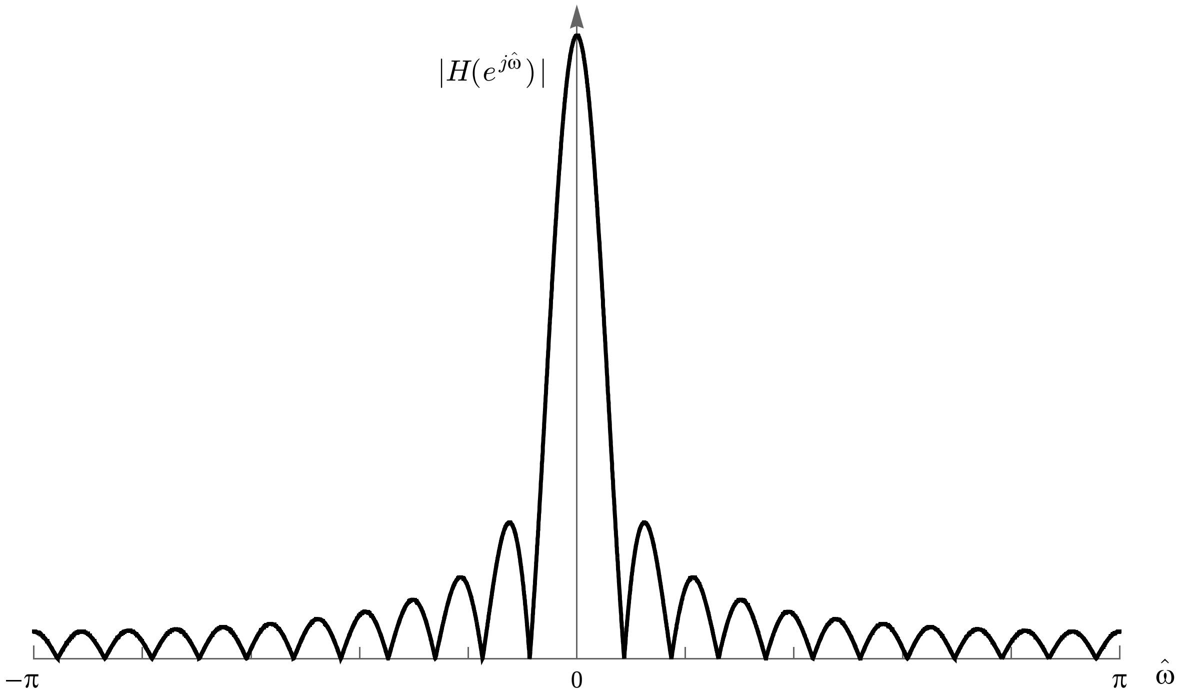
$$y[n] = x[n] + Kx[n-1] + x[n-2],$$

for some constant K to be determined. If the output of this system in response to the input

$$x[n] = 2017 + 2026\cos(\hat{\omega}_0 n) \qquad \text{is} \qquad y[n] = 2017 \quad \text{for all } n,$$

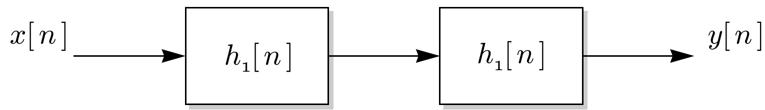
then it must be that: $K = \boxed{}$ and $\hat{\omega}_0 = \boxed{}$.

PROB. Su17-Q2.2. Suppose an L-point averaging LTI filter has the following magnitude response:



From this picture we conclude that the value of L must be $L = \boxed{}$.

PROB. Su17-Q2.3. Consider the following serial cascade of two LTI filters, where the output of the first is the input to the second:



Both filters have the same impulse response $h_1[n]$, which satisfies:

$$\sum_{n=-\infty}^{\infty} h_1[n] e^{-j\hat{\omega}n} = 0.5 + e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} + 4e^{-j4\hat{\omega}} .$$

(The "overall" impulse response $h[n]$ of the cascade is, by definition, the output $y[n]$ of the second filter when the input to the first filter is the unit impulse $x[n] = \delta[n]$.)

Specify numeric values for the "overall" impulse response $h[n]$ for $n \in \{0, 1, \ldots 9\}$:

$h[0] =$

$h[1] =$

$h[2] =$

$h[3] =$

$h[4] =$

$h[5] =$

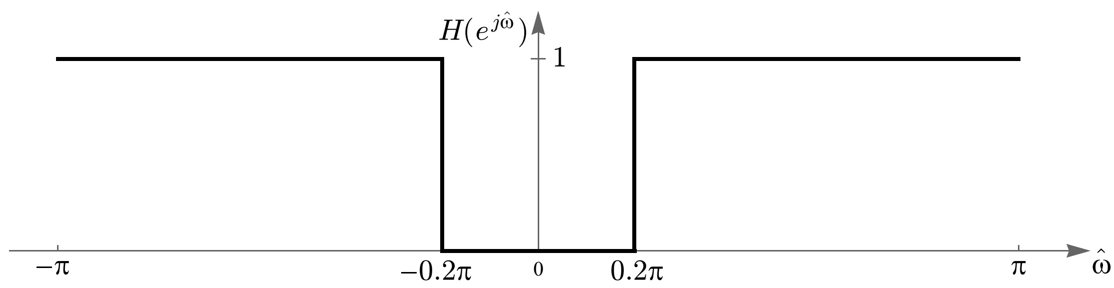
$h[6] =$

$h[7] =$

$h[8] =$

$h[9] =$

PROB. Su17-Q2.4. Shown below is the frequency response $H(e^{j\hat{\omega}})$ of an ideal high-pass filter:



Its impulse response $h[n]$ can be written in the following four different ways;
find numerical values for all of the unspecified constants:

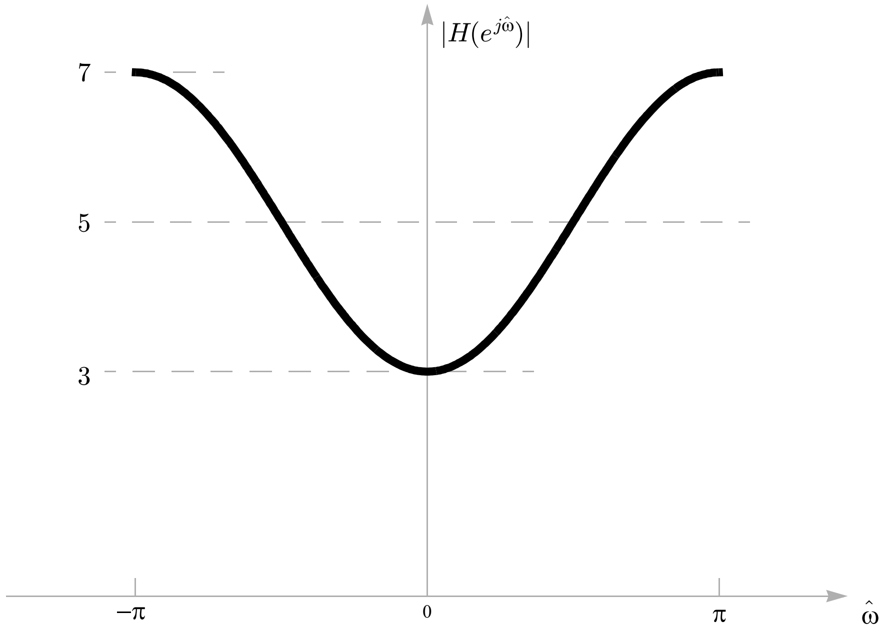
$$h[n] = \delta[n] - \frac{\sin(An)}{\pi n}, \quad \Rightarrow \quad A = \boxed{}.$$

$$h[n] = \delta[n] - B\cos(Cn)\frac{\sin(0.1\pi n)}{\pi n} \quad \Rightarrow \quad B = \boxed{}, \; C = \boxed{}.$$

$$h[n] = D\cos(\pi n)\frac{\sin(En)}{\pi n} \quad \Rightarrow \quad D = \boxed{}, \; E = \boxed{}.$$

$$h[n] = F\cos(0.6\pi n)\frac{\sin(Gn)}{\pi n} \quad \Rightarrow \quad F = \boxed{}, \; G = \boxed{}.$$

PROB. Su17-Q2.5. Shown below is the magnitude response of an FIR filter:



If the difference equation for this FIR system is:

$$y[n] = x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3],$$

with $b_0 = 1$, then the remaining FIR filter coefficients are:

$b_1 =$ ☐ , $b_2 =$ ☐ , $b_3 =$ ☐ .

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2017
Quiz #2

July 20, 2017

NAME: SOLUTIONS

Important Notes:

- DO NOT unstaple the test.
- Two two-sided page (8.5” $\times$ 11”) of hand-written notes permitted.
- Calculators are allowed, but no smartphones/WiFI/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	20	
2	20	
3	20	
4	20	
5	20	
Total		

PROB. Su17-Q2.1. Consider an LTI system described by the difference equation:

$$y[n] = x[n] + Kx[n-1] + x[n-2],$$

for some constant K to be determined. If the output of this system in response to the input

$$x[n] = 2017 + 2026\cos(\hat{\omega}_0 n) \qquad \text{is} \qquad y[n] = 2017 \quad \text{for all } n,$$

then it must be that: $K = \boxed{K = -1}$ and $\hat{\omega}_0 = \boxed{\pi/3}$.

The freq response is $H(e^{j\hat{\omega}}) = 1 + Ke^{-j\hat{\omega}} + e^{-2j\hat{\omega}}$

Because the system doesn't change the DC (constant) component of the input:

$\Rightarrow$ the DC gain (i.e. the freq response evaluated at $\hat{\omega} = 0$) is *one*:

$\Rightarrow$ $H(e^{j0}) = 1 + K + 1 = 1 \quad \Rightarrow \quad K = -1$

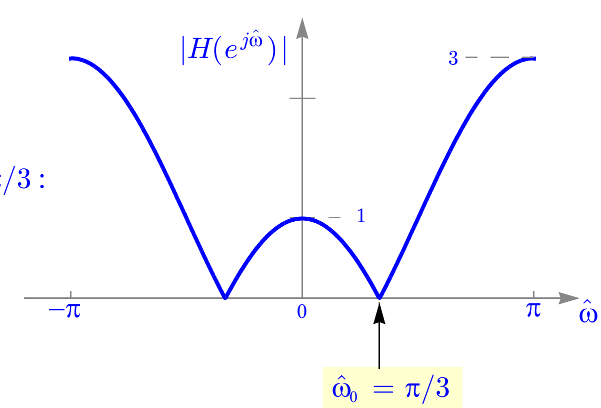
With $K = -1$, the freq response simplifies:

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 - e^{-j\hat{\omega}} + e^{-2j\hat{\omega}} \\ &= e^{-j\hat{\omega}}(e^{-j\hat{\omega}} - 1 + e^{-j\hat{\omega}}) \\ &= e^{-j\hat{\omega}}(2\cos(\hat{\omega}) - 1). \end{aligned}$$

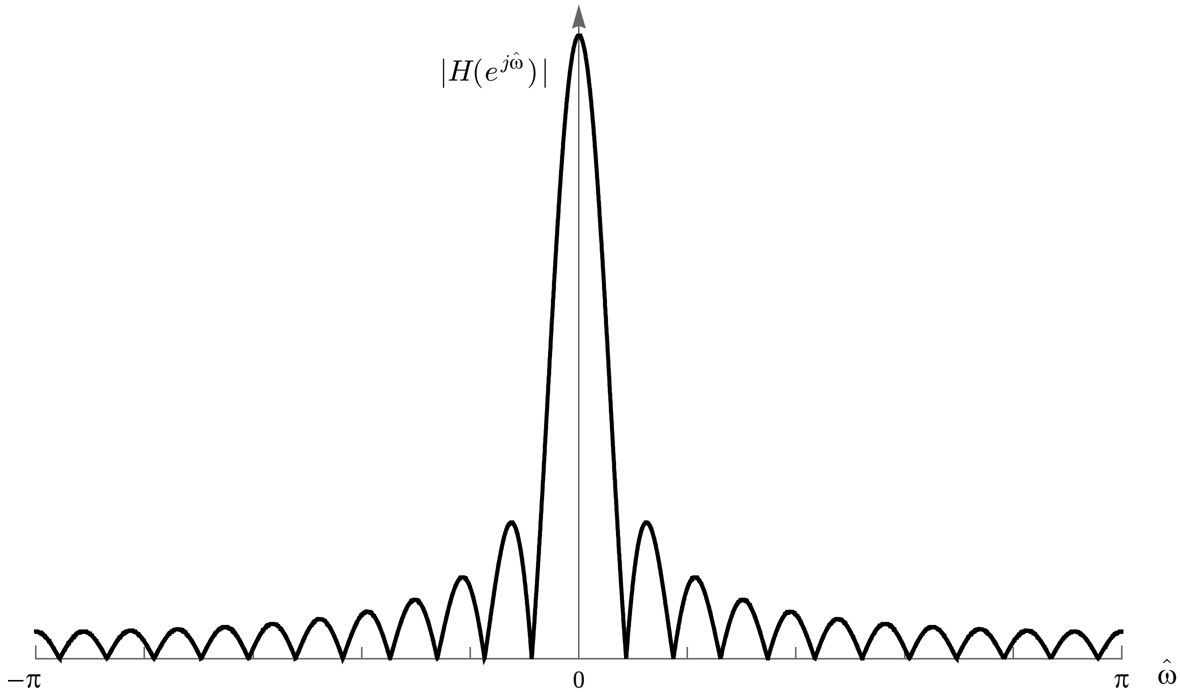
Because the sinusoid with freq $\hat{\omega}_0$ is *rejected*, the freq response must be 0 at $\hat{\omega}_0$:

$$\begin{aligned} &H(e^{j\hat{\omega}_0}) = 0 \\ &\Rightarrow 2\cos(\hat{\omega}) - 1 = 0 \\ &\Rightarrow \cos(\hat{\omega}_0) = 0.5 \\ &\Rightarrow \hat{\omega}_0 = \pi/3 \end{aligned}$$

Here's a picture showing the zero at $\hat{\omega}_0 = \pi/3$:



PROB. Su17-Q2.2. Suppose an L-point averaging LTI filter has the following magnitude response:

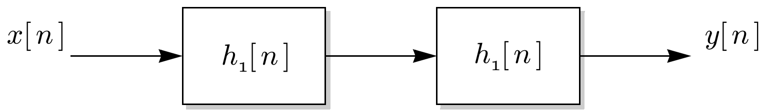


From this picture we conclude that the value of L must be $L = \boxed{23}$.

The number of spectral nulls (where the magnitude response is zero) of the L-point moving average filter in the range $\hat{\omega} \in (-\pi, \pi]$ is $L - 1$.

Counting, we see $L - 1 = 22$ nulls $\Rightarrow L = 23$.

PROB. Su17-Q2.3. Consider the following serial cascade of two LTI filters, where the output of the first is the input to the second:



Both filters have the same impulse response $h_1[n]$, which satisfies:

$$\sum_{n=-\infty}^{\infty} h_1[n]e^{-j\hat{\omega}n} = 0.5 + e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} + 4e^{-j4\hat{\omega}}\,.$$

(The "overall" impulse response $h[n]$ of the cascade is, by definition, the output $y[n]$ of the second filter when the input to the first filter is the unit impulse $x[n] = \delta[n]$.)

Specify numeric values for the "overall" impulse response $h[n]$ for $n \in \{0, 1, \ldots 9\}$:

The equivalent question is to convolve the sequence $[0.5\ 1\ 2\ 0\ 4]$ with itself:

0.25	0.5	1	0	2				
	0.5	1	2	0	4			
		1	2	4	0	8		
				2	4	8	0	16
0.25	1	3	4	8	8	16	0	16

$h[0] =$ 0.25

$h[1] =$ 1

$h[2] =$ 3

$h[3] =$ 4

$h[4] =$ 8

$h[5] =$ 8

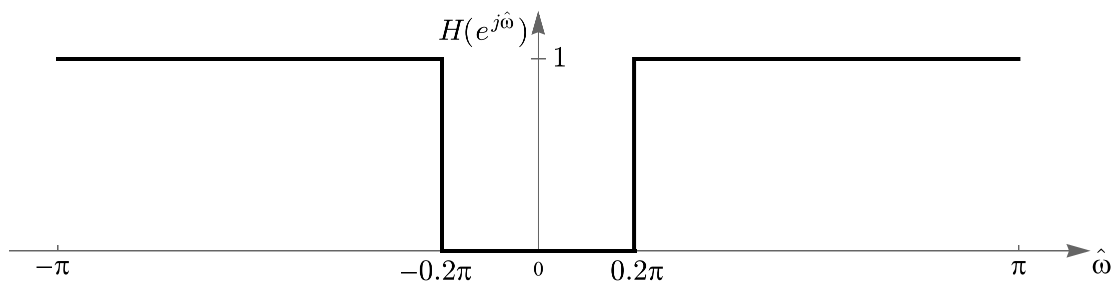
$h[6] =$ 16

$h[7] =$ 0

$h[8] =$ 16

$h[9] =$ 0

PROB. Su17-Q2.4. Shown below is the frequency response $H(e^{j\hat{\omega}})$ of an ideal high-pass filter:



Its impulse response $h[n]$ can be written in the following four different ways;
find numerical values for all of the unspecified constants:

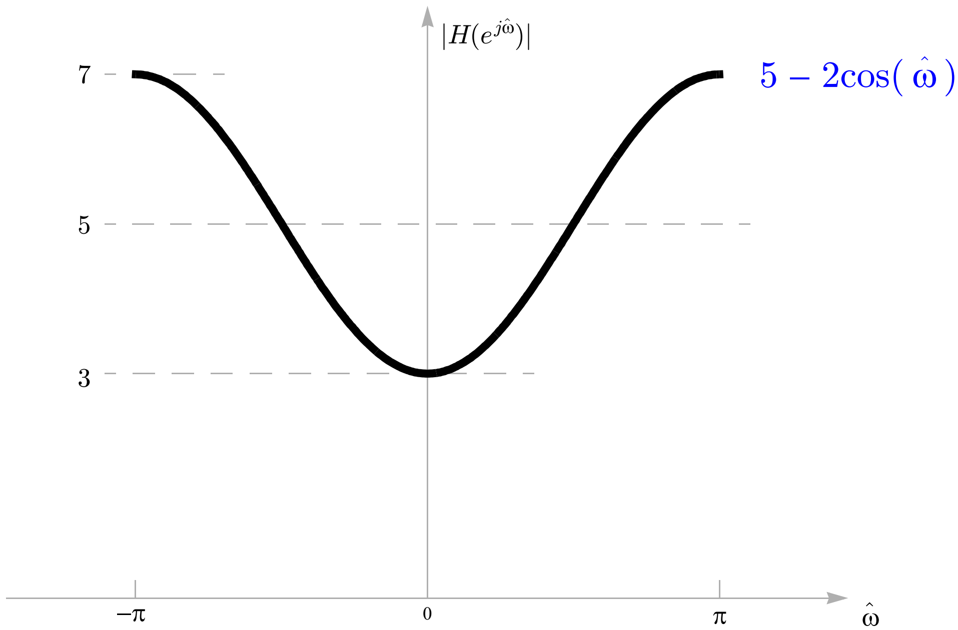
$$h[n] = \delta[n] - \frac{\sin(An)}{\pi n}, \quad \Rightarrow \quad A = \boxed{0.2\pi}.$$

$$h[n] = \delta[n] - B\cos(Cn)\frac{\sin(0.1\pi n)}{\pi n} \quad \Rightarrow \quad B = \boxed{2}, \; C = \boxed{0.1\pi}.$$

$$h[n] = D\cos(\pi n)\frac{\sin(En)}{\pi n} \quad \Rightarrow \quad D = \boxed{1}, \; E = \boxed{0.8\pi}.$$

$$h[n] = F\cos(0.6\pi n)\frac{\sin(Gn)}{\pi n} \quad \Rightarrow \quad F = \boxed{2}, \; G = \boxed{0.4\pi}.$$

PROB. Su17-Q2.5. Shown below is the magnitude response of an FIR filter:



If the difference equation for this FIR system is:

$$y[n] = x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3],$$

with $b_0 = 1$, then the remaining FIR filter coefficients are:

$b_1 = \boxed{-5}$, $b_2 = \boxed{1}$, $b_3 = \boxed{0}$.

From the plot: $|H(e^{j\hat{\omega}})| = 5 - 2\cos(\hat{\omega}) = 5 - e^{j\hat{\omega}} - e^{-j\hat{\omega}}$

which would arise from: $h[-1] = -1,\quad h[0] = 5,\quad h[1] = -1$

$\Rightarrow$ delay and negate (neither changes magnitude response) to get causal system:

$$h[0] = 1,\quad h[1] = -5,\quad h[2] = 1$$