GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2016
Quiz #2

July 13, 2016

NAME: ANSWER KEY ________ ________ GT username: ________
(FIRST) (LAST) (e.g., gtxyz123)

To avoid losing 3 points, circle your recitation section:

	Tue
10 – 11:45am	L01 (Stuber)
12 – 1:45pm	L02 (Stuber)
2 – 3:45pm	L03 (Zhang)
4 – 5:45pm	L04 (Zhang)

Important Notes:

- DO NOT unstaple the test.
- One two-sided page (8.5” $\times$ 11”) of hand-written notes permitted.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	25	
2	25	
3	25	
4	25	
No/Wrong Rec	–3	
Total		

PROB. Su16-Q2.1.

Shown below are the relationships between the input $x[n]$ and output $y[n]$ of several systems. Specify whether each is linear, and whether each is time-invariant: (*Brief* explanations are OK!)

	Linear?	Time-Invariant?
(a) $y[n] = 3x[n+1]$	YES ☒ NO ☐ Response to $\alpha x_1[n] + \beta x_2[n]$ is $\alpha y_1[n] + \beta y_2[n]$	YES ☒ NO ☐ Response to $x_1[n - n_0]$ is $y_1[n - n_0]$
(b) $y[n] = 3x[n] + 1$	YES ☐ NO ☒ Fails zero-in, zero-out test: Response to 0 is 1	YES ☒ NO ☐ Response to $x_1[n - n_0]$ is $y_1[n - n_0]$
(c) $y[n] = \cos(\pi x[n])$	YES ☐ NO ☒ Fails zero-in, zero-out test: Response to 0 is 1	YES ☒ NO ☐ Response to $x_1[n - n_0]$ is $y_1[n - n_0]$
(d) $y[n] = x[\cos(\pi n)]$	YES ☒ NO ☐ Response to $\alpha x_1[n] + \beta x_2[n]$ is $\alpha y_1[n] + \beta y_2[n]$	YES ☐ NO ☒ Response to $\delta[n]$ is $y_1[n] = 0$, while response to $\delta[n-1]$ is ...101010... $\neq y_1[n-1]$
(e) $y[n] = \dfrac{1}{x[n-1]}$	YES ☐ NO ☒ Fails zero-in, zero-out test: Response to 0 is ∞	YES ☒ NO ☐ Response to $x_1[n - n_0]$ is $y_1[n - n_0]$

PROB. Su16-Q2.2. (The different parts are unrelated. No integrals or tedious calculations required!)

(a) The difference equation of an LTI system whose frequency response is $H(e^{j\hat{\omega}}) = 7 + 3e^{-j88\hat{\omega}}$ is:

$$y[n] = \boxed{7x[n] + 3x[n-88]}.$$

(b) Simplify

$$\sum_{k=-\infty}^{\infty} \sin(0.2\pi k)\frac{\sin(0.3\pi(n-k))}{\pi(n-k)} = \boxed{\sin(0.2\pi n)}.$$

a function of n

This is the equation for the output of a LPF with cutoff 0.3π
when the input is a sinusoid with frequency 0.2π.
Since the sinusoid frequency is below cutoff, the output is the same as the input.

(c) Simplify

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^{-j2\hat{\omega}}}{1-0.2e^{-j\hat{\omega}}}\, e^{jn\hat{\omega}} d\hat{\omega} = \boxed{0.2^{(n-2)}u[n-2]}.$$

a function of n

We *recognize* this as the inverse DTFT of $\dfrac{e^{-j2\hat{\omega}}}{1-0.2e^{-j\hat{\omega}}}$
From the DTFT *pairs* table:

$$\frac{1}{1-0.2e^{-j\hat{\omega}}} \quad \Longleftrightarrow \quad 0.2^n u[n]$$

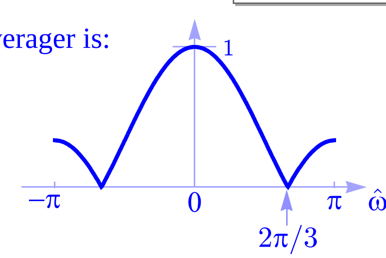
From the DTFT *properties* table, the extra factor of $e^{-j2\hat{\omega}}$
corresponds to a delay by 2 in the time domain.

(d) If the output of a causal three-point running average filter is zero ($y[n] = 0$ for all n) when the input is $x[n] = \cos(\hat{\omega}_0 n)$, then $\hat{\omega}_0 = \boxed{2\pi/3}$.

From the DTFT *pairs* table, the magnitude response of the 3-pt averager is:

$$\left|\frac{\sin(\hat{\omega}3/2)}{3\sin(\hat{\omega}/2)}\right|$$

It is zero when the argument of the $\sin(\,\cdot\,)$ in the numerator is π.



(e) If a unit step ($x[n] = u[n]$) is fed as an input to an LTI system whose impulse response is $h[n]=3\delta[n-1]+\delta[n-2]-\delta[n-3]$, the output at time $n = 2$ will be $y[2] = \boxed{4}$.

$$y[n] = 3x[n-1] + x[n-2] - x[n-3]$$

$$\Rightarrow y[2] = 3x[1] + x[0] - x[-1]$$

$$= 3(1) + (1) - (0) = 4.$$

PROB. Su16-Q2.3.

Consider an LTI system described by the difference equation:

$$y[n] = \beta x[n] + x[n-1] + \beta x[n-2],$$

for some constant β to be determined. If the output of this system in response to the input

$$x[n] = 8 + 6\cos(\hat{\omega}_0 n) \qquad \text{is} \qquad y[n] = 6\cos(\hat{\omega}_0(n-1)),$$

then it must be that: $\beta = \boxed{-0.5}$ and $\hat{\omega}_0 = \boxed{0.5\pi}$.

The freq response is $H(e^{j\hat{\omega}}) = \beta + e^{-j\hat{\omega}} + \beta e^{-2j\hat{\omega}}$

Because the system eliminates the DC (constant) component of the input:

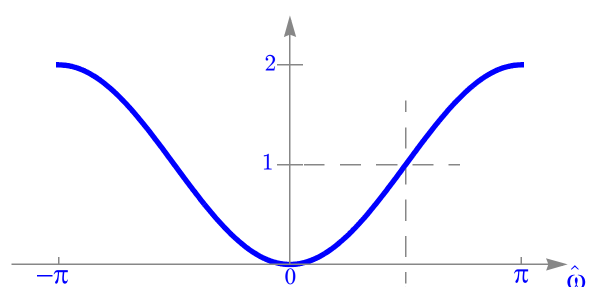
$\Rightarrow$ the DC gain (i.e. the freq response evaluated at $\hat{\omega} = 0$) is zero:

$\Rightarrow$ $H(e^{j0}) = \beta + 1 + \beta = 0 \quad \Rightarrow \quad \beta = -0.5$

With $\beta = -0.5$, the freq response simplifies:

$$\begin{aligned} H(e^{j\hat{\omega}}) &= -0.5 + e^{-j\hat{\omega}} - 0.5e^{-2j\hat{\omega}} \\ &= e^{-j\hat{\omega}}(-0.5e^{j\hat{\omega}} + 1 - 0.5e^{-j\hat{\omega}}) \\ &= e^{-j\hat{\omega}}(1 - \cos(\hat{\omega})), \end{aligned}$$

so that the magnitude response is $|H(e^{j\hat{\omega}})| = 1 - \cos(\hat{\omega})$:



Because the sinusoid output has the *same* amplitude of the sinusoidal component of the input, the magnitude response must be 1 at $\hat{\omega}_0$:

$\hat{\omega}_0 = 0.5\pi$

We can get the same answer without the plot:

$$\begin{aligned} |H(e^{j\hat{\omega}_0})| &= 1 - \cos(\hat{\omega}_0) = 1 \\ \Rightarrow \quad & \cos(\hat{\omega}_0) = 0 \\ \Rightarrow \quad & \hat{\omega}_0 = 0.5\pi \end{aligned}$$

PROB. Su13-Q2.4. On the left below are several discrete-time signals. On the right are the corresponding DTFT, in a scrambled order. Match each signal to its corresponding DTFT by writing a letter {A, B, ... K} into each answer box.

C	$x[n] = \delta[n] - \dfrac{\sin(0.4\pi n)}{\pi n}$
D	$x[n] = \dfrac{\sin(0.3\pi n)}{\pi n} - \dfrac{\sin(0.2\pi n)}{\pi n}$
B	$x[n] = \cos(\pi n)\dfrac{\sin(0.4\pi n)}{\pi n}$
H	$x[n] = \dfrac{\sin(0.3\pi n)}{\pi n} * \dfrac{\sin(0.2\pi n)}{\pi n}$ (CONVOLUTION)
K	$x[n] = \delta[n] - \dfrac{\sin(0.8\pi n)}{\pi n} + \dfrac{\sin(0.2\pi n)}{\pi n}$
A	$x[n] = (\cos(\pi n) + 1)\dfrac{\sin(0.3\pi n)}{\pi n}$
E	$x[n] = \dfrac{\sin(0.3\pi n)}{\pi n} + \dfrac{\sin(0.2\pi n)}{\pi n}$
F	$x[n] = 2\cos(0.3\pi n)\dfrac{\sin(0.2\pi n)}{\pi n}$
J	$x[n] = \dfrac{\sin(0.3\pi n)}{\pi n}$
G	$x[n] = 2\cos(0.2\pi n)\dfrac{\sin(0.3\pi n)}{\pi n}$