GEORGIA INSTITUTUE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
EXAM 2

DATE: 8-July-15 COURSE: ECE-2026

NAME: SOLUTIONS GTUserID: ________________
LAST, FIRST (Log-in ID)

Circle your correct **recitation section** number - failing to do so will cost you 3 points

L01(MW) 4-545: BARRY	L02(TTH) 10-1145: ZHANG	L03 (TTH) 12-145: ZHANG

- Write your name on the front page ONLY. **DO NOT unstaple the test**
- Closed book, but a calculator is permitted.
- One page $\left(8\frac{1}{2}''\times 11''\right)$ of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **SHOW ALL YOUR WORK TO RECEIVE CREDIT**
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. **Circle** your answers, or write them in the **boxes/spaces** provided. If more space is needed for scratch work, use the backs of previous pages.
- **WRITE ANY RADIAN ANSWERS AS A FRACTION OF PI. (i.e., write 0.4π instead of 1.257)**
- **ALL RADIAN ANSWERS SHOULD BE IN THE RANGE (-π, π].**
- **TABLES FOR DTFT PAIRS AND PROPERTIES ARE AT THE BACK OF THE EXAM AND CAN BE REMOVED FOR YOUR REFERENCE**

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

PROBLEM 1:

All parts can be solved independently of each other.

(a) Assume that an LTI system is defined with the following impulse response:

$$h[n] = u[n-100] - u[n-300]$$

The input to the system is $x[n] = u[n-5] - u[n-15]$ and the output is defined as $y[n] = x[n] * h[n]$ (where $y[n] = x[n] * h[n]$ denotes $x[n]$ convolved with $h[n]$).

(i). Find the *first* (n_f) and *last* (n_l) non-zero samples for $y[n]$ (i.e., $y[n] = 0$ for $n < n_f$ and $n > n_l$) (5 points)

x[n] has boundaries: {5 -> 14}

h[n] has boundaries: {100 -> 299}

$n_f = 5 + 100 = 105$

$n_l = 14 + 299 = 313$

n_f = 105

n_l = 313

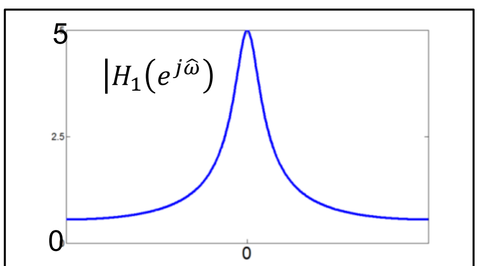
(ii) Find the **maximum value** (A) for $y[n]$ (i.e., $y[n] = A$ for some n and $y[n] \le A$ for all n) (5 points)

$y[n] = \sum_{k=5}^{14} x[k]h[n-k]$. When $x[n]$ completely overlaps $h[n]$

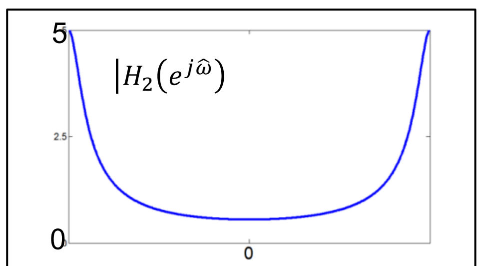
$$\sum_{k=5}^{14} (1)(1) = 10$$

A =10

(b) The magnitudes for the frequency responses of two LTI systems $h_1[n]$ and $h_2[n]$ are shown below.



$-\pi$ π



$-\pi$ π

Circle the relationship between $h_1[n]$ and $h_2[n]$ that is TRUE. (5 points)

(1) $h_2[n] = \delta[n-\pi] * h_1[n]$ (2) $h_2[n] = \delta[n-\pi]h_1[n]$ (3) $h_2[n] = (-1)^n * h_1[n]$ (4) $h_2[n] = (-1)^n h_1[n]$

This is a frequency shift by **π**

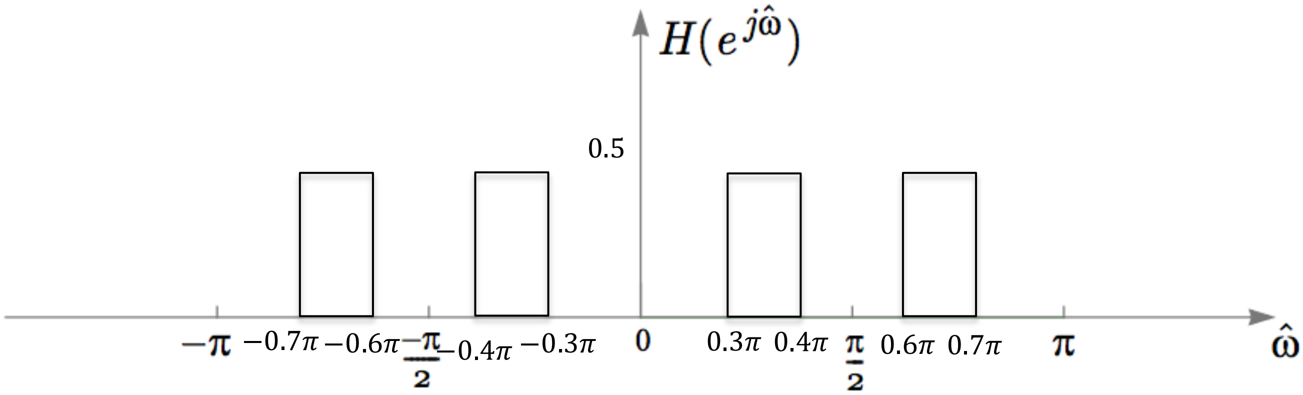
(c) Find $x[n]$, the inverse DTFT of $X(e^{j\hat{\omega}}) = \frac{5e^{-j3\hat{\omega}}}{10-e^{-j\hat{\omega}}}$ (5 points)

$$X(e^{j\hat{\omega}}) = \frac{5e^{-j3\hat{\omega}}}{10-e^{-j\hat{\omega}}} = \frac{0.5e^{-j3\hat{\omega}}}{1-0.1e^{-j\hat{\omega}}} = 0.5\frac{1}{1-0.1e^{-j\hat{\omega}}}e^{-j3\hat{\omega}} \to x[n] = 0.5(0.1)^{n-3}u[n-3]$$

$x[n] = 0.5(0.1)^{n-3}u[n-3]$

PROBLEM 2:
All parts can be solved independently of each other.

(a) Label and plot the frequency response $H(e^{j\hat{\omega}})$ when $h[n] = \left(\frac{\sin(0.2\pi n)}{\pi n} - \frac{\sin(0.1\pi n)}{\pi n}\right)\cos(0.5\pi n)$
(5 points)



(b) The continuous time signal $x(t) = A\cos(2\pi f_0 t)$ has a fundamental frequency of f_0. Find the **smallest** sampling frequency, f_s (where $x[n] = x(n/f_s)$) that (i) avoids aliasing and (ii) creates a fundamental period of $N_0 = 7$ (i.e., $x[n] = x[n + N_0] = x[n + 7]$. (NOTE: Your answer will be expressed in terms of f_0). (5 points)

To avoid aliasing we need at least $f_s > 2f_0$.

Recall for periodic signals: $f_s = \frac{N_0}{M} f_0$ for $x[n] = x[n + N_0]$

Therefore the smallest sampling frequency without aliasing is: $f_s = \frac{7}{3} f_0$

$$f_s = \frac{7}{3} f_0$$

(c) An engineer wants to design a system with an impulse response: $h[n] = \delta[n+1] - \delta[n-1]$. Can his system be implemented in real-time? Circle ONE answer (YES or NO) and briefly explain.
(5 points)

YES NO

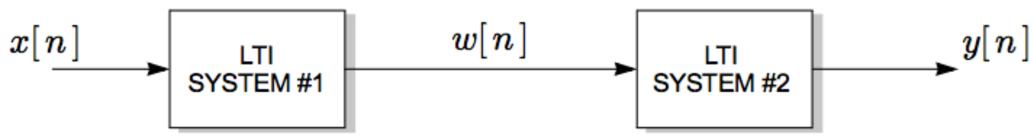
This system is not causal

(d) If the 4-point DFT $X[k]$ of the signal $x[n]$ has the values $X[0] = 0, X[1] = 1, X[2] = 0, X[3] = 0$ for $k = 0,1,2,$ and 3, is $x[n]$ a real signal? Circle ONE answer (YES or NO) and briefly explain. (5 points)

YES NO

$X[1]$ is the only non-zero value of the DFT which corresponds to frequency $\hat{\omega}_k = \frac{2\pi}{4}(1) = \frac{\pi}{2}$. This indicates there is no conjugate symmetry and therefore x[n] is NOT a real signal.

PROBLEM 3:
The figure depicts a *cascade* connection of two LTI systems, where the output w[n] of system#1 is the input to system#2, and the output of system#2 is the *overall* output:



LTI System #1 is defined as: $w[n] = \sum_{k=0}^{2} x[n-k]$
LTI System #2 is defined as: $h_2[n] = \delta[n-2] - \delta[n-3]$

(a) The DC Gain (*DC*) of LTI System #1 is: (4 points)

LTI System #1 is an L-point running sum with a DC gain of L (where L = 3 in this case) therefore DC Gain = 3

$DC = 3$

(b) If $x[n] = \cos(\hat{\omega}_0 n)$, find *all* values of $\hat{\omega}_0$ such that $w[n] = 0$. (6 points)

The nulls of an L-point running sum are located at $\hat{\omega} = \frac{2\pi}{L}k$

In this case, there is only one null point less than **π**

$$\hat{\omega}_0 = \pm\frac{2\pi}{3}$$

(c) Obtain and plot the overall impulse response $h[n]$ in the space below as a properly labeled stem plot. (NOTE: You must include labels on both the *x*- and *y*- axes of the plot to get full credit. (10 points)

$$\begin{aligned} h[n] = h_1[n] * h_2[n] &= (\delta[n] + \delta[n-1] + \delta[n-2]) * (\delta[n-2] - \delta[n-3]) \\ &= (\delta[n-2] + \delta[n-3] + \delta[n-4]) - \big((\delta[n-3] + \delta[n-4] + \delta[n-5])\big) \\ &= \delta[n-2] - \delta[n-5] \end{aligned}$$

PROBLEM 4:

We have seen that LTI systems can be represented in a variety of ways. For each of the LTI systems below, provide the missing representations. (Frequency response formulas can be given in any convenient form; you need not simplify them.)

(a) (5 points)

Matlab Code:	`yn = conv(xn, [0,0,2,0,8,0,2]);`
Frequency Response Magnitude:	$\lvert H(e^{j\hat{\omega}})\rvert = 8 + 4\cos(4\hat{\omega})$

$$h[n] = 2\delta[n-2] + 8\delta[n-4] + 2\delta[n-6] \rightarrow H(e^{j\hat{\omega}}) = 2e^{-j2\hat{\omega}} + 8e^{-j4\hat{\omega}} + 2e^{-j6\hat{\omega}}$$
$$= (8 + 4\cos(2\hat{\omega}))e^{-j4\hat{\omega}}$$

(b) (5 points)

Frequency Response:	$H(e^{j\hat{\omega}}) = \dfrac{\sin(5.5\hat{\omega})}{11\sin(0.5\hat{\omega})}e^{-j\hat{\omega}10}$
Impulse Response:	$h[n] = u[n-5] - u[n-16]$

$$H(e^{j\hat{\omega}}) = \left(\frac{\sin(5.5\hat{\omega})}{11\sin(0.5\hat{\omega})}e^{-j\hat{\omega}5}\right)e^{-j\hat{\omega}5} \rightarrow h[n] = \frac{1}{11}(u[n-5] - u[n-16])$$

(c) (5 points)

Impulse response	$h[n] = \delta[n] + 3\delta[n-4] + \delta[n-10]$
Difference Equation:	$y[n] = x[n] + 3x[n-4] + x[n-10]$

(d) (5 points)

Difference Equation:	$y[n] = \frac{1}{3}\sum_{k=0}^{2} x[n-k] - \frac{1}{2}\sum_{k=3}^{4} x[n-k]$
Impulse Response:	$h[n] = \frac{1}{3}\sum_{k=0}^{2} \delta[n-k] - \frac{1}{2}\sum_{k=3}^{4} \delta[n-k]$

PROBLEM 5:

Find the 16-point DFT ($X[k]$) of the sampled segment $x[n] = 2 + 4\cos\left(\frac{3\pi}{8}n + \frac{\pi}{6}\right)$ for $n = 0, \ldots, 15$. Enter the values in the table below. (20 points)

k	$X[k]$
0	32
1	0
2	0
3	$32e^{\frac{j\pi}{6}}$
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	$32e^{\frac{-j\pi}{6}}$
14	0
15	0

$x[n] = 2 + 4\cos\left(\frac{3\pi}{8}n\right)$ is periodic with a period of 16 samples (i.e. $\hat{\omega}_0 = \frac{2\pi}{16}$), . Since a 16-point DFT is an integer multiple of that period, the DFT coefficients will be directly related to the DFS coefficients (c_k) by $X[k] = Nc_k$.

The DFS coefficients are: $c_0 = 2$, $c_3 = 2e^{\frac{j\pi}{6}}$, $c_{-3} = 2e^{\frac{-j\pi}{6}}$

Therefore, the DFT coefficients are: $X[0] = 32$,
$X[3] = 32e^{\frac{j\pi}{6}}$,$X[-3] = X[16-3] = X[13] = 32e^{\frac{-j\pi}{6}}$