GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2014
Quiz #2

July 9, 2014

ANSWER KEY

NAME: ______________ ______________ GT username: ______________
(FIRST) (LAST) (e.g., gtxyz123)

Circle your recitation section (otherwise you lose 3 points!):

	Mon	Tue
10 – 11:45am		L02 (Moore)
12 – 1:45pm		L03 (Moore)
2:40 – 3:50pm		L04 (Davis)
4 – 5:45pm	L01 (Barry)	

Important Notes:

- Circle your recitation above.
- DO NOT unstaple the test.
- One two-sided page (8.5” $\times$ 11”) of hand-written notes permitted.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	20	
2	20	
3	20	
4	20	
5	20	
No/Wrong Rec	–3	
Total		

PROB. Su14-Q2.1.

(a) The acronym *FIR* stands for finite impulse response.

(b) The acronym *LTI* stands for linear (and) time-invariant.

(c) The acronym *DTFT* stands for discrete-time Fourier transform.

(d) The acronym *DFT* stands for discrete Fourier transform.

(e) The acronym *FFT* stands for fast Fourier transform.

(f) Convolving a length-5 signal with a length-10 signal yields a length- 14 (discrete-time) signal.

(g) The response of an LTI system with frequency response $H(e^{j\hat{\omega}}) = 20\cos(4\hat{\omega})$ to a sinusoidal input of

$x[n] = \sin(\pi n/12)$ is $y[n] =$ $10\sin(\pi n/12)$.

$@\ \hat{\omega} = \pi/12,\ H(e^{j\hat{\omega}}) = 20\cos(\pi/30) = 10$

(h) There are 6 values of $\hat{\omega}$ in the range $0 < \hat{\omega} < \pi$ for which the frequency response $H(e^{j\hat{\omega}})$ of a 13-point averager is *zero* (i.e., for which $H(e^{j\hat{\omega}}) = 0$).

$H(e^{j\hat{\omega}}) = e^{-j6\hat{\omega}}\frac{\sin(13\hat{\omega}/2)}{13\sin(\hat{\omega}/2)}$ is zero when numerator is zero;

i.e. when $13\hat{\omega}/2 = k\pi$ for $k = 1, 2, \ldots 6$.

(Other values of k fall outside the 0-to-π range.)

(i) The inverse DTFT of $X(e^{j\hat{\omega}}) = \frac{e^{-j\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}}$ is $x[n] =$ $0.8^{n-1}u[n-1]$.

(Give an equation that is valid for all n.)

(j) TRUE ☐ FALSE ☒ If $x[n]$ is real then its DFT $X[k]$ is also real.

(k) After taking the 16-point DFT of $[x[0], x[1], x[2], x[3]] = [2, 4, 6, 8]$, the zero-th DFT coefficient is:

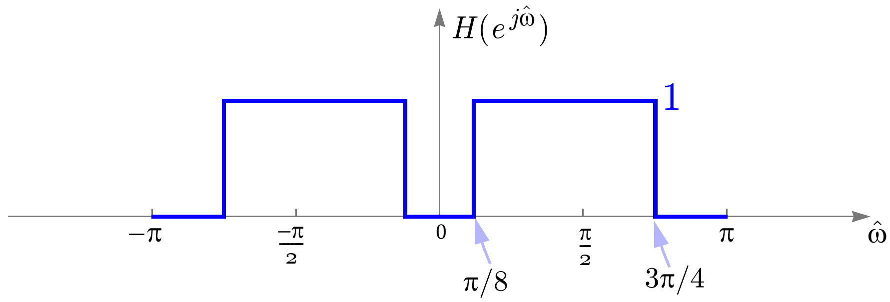
$X[0] =$ 20.

$X[0] = x[0] + x[1] + \ldots = 2 + 4 + 6 + 8 = 20$

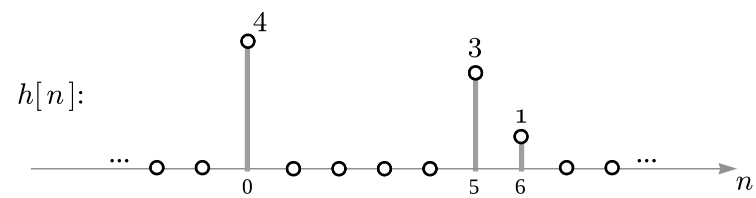
PROB. Su14-Q2.2. (The three parts of this problem are unrelated to eachother.)

(a) In the space below, sketch the frequency response of an LTI system whose impulse response is

$$h[n] = \frac{\sin(3\pi n/4)}{\pi n} - \frac{\sin(\pi n/8)}{\pi n}:$$



(b) Write an equation for the frequency response $H(e^{j\hat{\omega}})$ of an LTI system when the impulse response $h[n]$ is shown in the stem plot below:



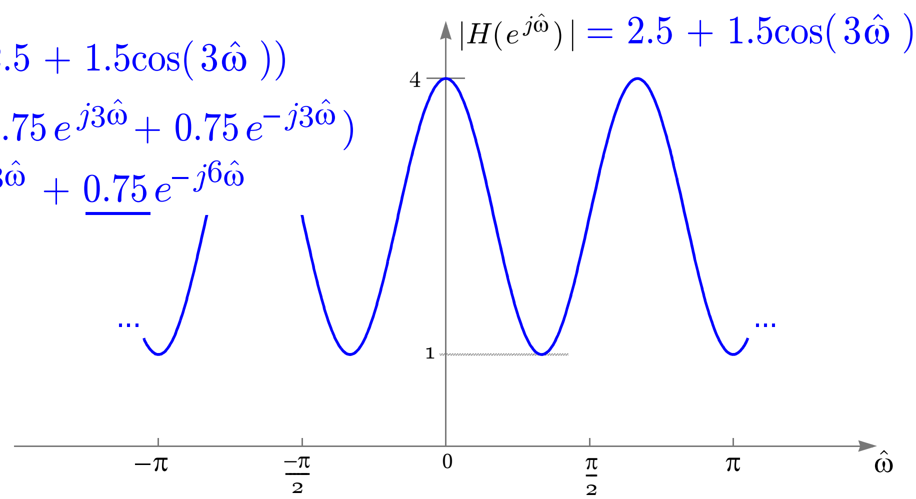
$\Rightarrow$ $H(e^{j\hat{\omega}}) = 4 + 3e^{-5j\hat{\omega}} + e^{-6j\hat{\omega}}$

(c) Consider an LTI system whose frequency response is $H(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}}|H(e^{j\hat{\omega}})|$, where $|H(e^{j\hat{\omega}})|$ oscillates sinusoidally between 1 and 4, with a period of $2\pi/3$, as shown below:

$$\Rightarrow H(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}}(2.5 + 1.5\cos(3\hat{\omega}))$$

$$= e^{-j3\hat{\omega}}(2.5 + 0.75\, e^{j3\hat{\omega}} + 0.75\, e^{-j3\hat{\omega}})$$

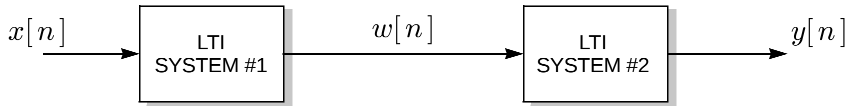
$$= \underline{0.75} + \underline{2.5}\, e^{-j3\hat{\omega}} + \underline{0.75}\, e^{-j6\hat{\omega}}$$



Find numerical values for the impulse response $h[n]$ in time domain, at times $n \in \{0, 1, 2, \ldots 6\}$:

$h[0]$	$h[1]$	$h[2]$	$h[3]$	$h[4]$	$h[5]$	$h[6]$
0.75	0	0	2.5	0	0	0.75

PROB. Su14-Q2.3. The figure depicts a *cascade* connection of two LTI systems, where the output $w[n]$ of system#1 is the input to system#2, and the output of system#2 is the *overall* output:



LTI system#1 is defined by the difference equation $w[n] = x[n] + 2x[n-2] + x[n-4]$.

LTI system#2 is defined by the impulse response $h_2[n] = \delta[n] + \delta[n-1]$.

(a) The DC gain of system#1 is **4** since $H_1(e^{j0}) = 1 + 2 + 1 = 4.$

(b) In response to $x[n] = \cos(\hat{\omega}_1 n)$,
the output of system#1 will be *zero* for all time (i.e., $w[n] = 0$) when the input frequency is:

$\hat{\omega}_1 =$ **0.5π** radians.

$$H_1(e^{j\hat{\omega}}) = 1 + 2e^{-2j\hat{\omega}} + e^{-4j\hat{\omega}}$$
$$= e^{-2j\hat{\omega}}(e^{2j\hat{\omega}} + 2 + e^{-2j\hat{\omega}})$$
$$= e^{-2j\hat{\omega}}(2 + 2\cos(2\hat{\omega}))$$
$$= 0 \qquad \text{... when } \hat{\omega} = 0.5\pi$$

(c) The difference equation for the *overall* cascade system (relating $x[n]$ to $y[n]$) can be written as:

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3] + b_4 x[n-4] + b_5 x[n-5] + b_6 x[n-6],$$

where the coefficients are:

b_0	b_1	b_2	b_3	b_4	b_5	b_6
1	1	2	2	1	1	0

convolving $\mathbf{h}_1 = [\,1\ 0\ 2\ 0\ 1\,]$ with $\mathbf{h}_2 = [\,1\ 1\,]$ yields $\mathbf{h} = [1\ 1\ 2\ 2\ 1\ 1\,]$

PROB. Su14-Q2.4. We have seen that LTI systems can be represented in a variety of ways. For each of the LTI systems below, provide the missing representations. (The frequency response can be given in any convenient form; you need not simplify.)

(a)

difference equation:	$y[n] = x[n] + x[n-1]$
impulse response:	$h[n] = \delta[n] + \delta[n-1]$
frequency response:	$H(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}}$

(b)

extra factor of $3e^{-j2\hat{\omega}}$ beyond 3-pt averager
$\Rightarrow$ scale by 3 and delay by 2

frequency response:	$H(e^{j\hat{\omega}}) = \left(\dfrac{\sin(1.5\hat{\omega})}{\sin(0.5\hat{\omega})}\right) e^{-j3\hat{\omega}}$
difference equation:	$y[n] = x[n-2] + x[n-3] + x[n-4]$
impulse response:	$h[n] = \delta[n-2] + \delta[n-3] + \delta[n-4]$

$$= u[n-2] - u[n-5]$$

(c)

MATLAB code:	`yy = conv(xx,[1,0,0,0,-1]);`
impulse response:	$h[n] = \delta[n] - \delta[n-4]$
frequency response:	$H(e^{j\hat{\omega}}) = 1 - e^{-j4\hat{\omega}}$

PROB. Su14-Q2.5. Find the 10-point DFT $\{X[0], X[1], \ldots X[9]\}$ of the length-10 signal segment:

$x[0] = 1$
$x[1] = -1$
$x[2] = 1$
$x[3] = -1$
$x[4] = 1$
$x[5] = -1$
$x[6] = 1$
$x[7] = -1$
$x[8] = 1$
$x[9] = -1$

The given $x[n] = (-1)^n$ can be rewritten as a complex exponential:

$$x[n] = e^{j\pi n}$$

which is the same as prescribed by the inverse 10-pt DFT equation:

$$x[n] = \frac{1}{10}\sum_{k=0}^{9} X[k]e^{jkn2\pi/10}$$

when only $X[5] = 10$ is nonzero.

$X[0] =$	0
$X[1] =$	0
$X[2] =$	0
$X[3] =$	0
$X[4] =$	0
$X[5] =$	10
$X[6] =$	0
$X[7] =$	0
$X[8] =$	0
$X[9] =$	0

Hint: Answers are all real integers.