GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Summer 2013
Quiz #2

July 10, 2013

NAME: VERSION A ______________ ______________ GT username: ______________
(FIRST) (LAST) (e.g., gtxyz123)

Circle your recitation section (otherwise you lose 3 points!):

	Mon	Tue
10 – 11:45am		L02 (Elliot)
12 – 1:45pm		L03 (Elliot)
4 – 5:45pm	L01 (Barry)	

Important Notes:

- DO NOT unstaple the test.
- One two-sided page (8.5" × 11") of hand-written notes permitted.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	20	
2	20	
3	20	
4	20	
5	20	
No/Wrong Rec	–3	
Total		

PROB. Su13-Q2.1. Consider an LTI system with the following property: For any value of $\hat{\omega}_0$, the output in response to $x[n] = \cos(\hat{\omega}_0 n)$ is $y[n] = \cos(\hat{\omega}_0)\cos(\hat{\omega}_0(n-1))$.

$$x[n] = \cos(\hat{\omega}_0 n) \longrightarrow \boxed{\text{LTI}} \longrightarrow y[n] = \cos(\hat{\omega}_0)\cos(\hat{\omega}_0(n-1))$$

In other words, this system does two things to such a sinusoidal input:
it delays by 1; and it scales (by a factor $\cos(\hat{\omega}_0)$ that depends on its frequency).

(a) The output of this system in response to $x[n] = \cos(\pi n/2)$ is $y[n] =$ $\boxed{0}$.

(b) The output of this system in response to a constant input $x[n] = \sqrt{2}$ (for all n) is $y[n] =$ $\boxed{\sqrt{2}}$.

(c) The output of this system in response to $x[n] = (-1)^n$ is $y[n] =$ $\boxed{(-1)^n}$.

$$\cos(\pi)(-1)^{n-1} = (-1)^n$$

(d) This system can be described by the difference equation

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3],$$

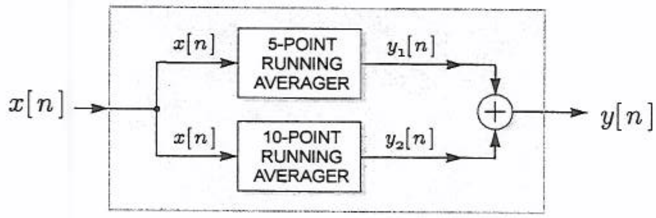
where $b_0 =$ $\boxed{\frac{1}{2}}$, $b_1 =$ $\boxed{0}$, $b_2 =$ $\boxed{\frac{1}{2}}$, and $b_3 =$ $\boxed{0}$.

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} \cos(\hat{\omega})$$

$$= e^{-j\hat{\omega}}\left(\tfrac{1}{2}e^{j\hat{\omega}} + \tfrac{1}{2}e^{-j\hat{\omega}}\right)$$

$$= \tfrac{1}{2} + \tfrac{1}{2}e^{-j2\hat{\omega}}$$

PROB. Su13-Q2.2. A sequence $x[n]$ is passed through a 5-point running averager, resulting in $y_1[n]$. The same sequence is also passed through a 10-point averager, resulting in $y_2[n]$. Finally, the results of the two filters are combined by *adding*, yielding an overall system output of $y[n] = y_1[n] + y_2[n]$, as sketched below:



The *overall* system is the one that maps $x[n]$ to $y[n]$.

5

(a) The overall output in response to a constant $x[n] = 0.3$ (for all n) is $y[n] =$ 0.6.

8

(b) The overall output in response to $x[n] = \cos(0.2\pi n)$ is $y[n] = A\cos(0.2\pi n + \theta)$, where:

$H_1(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}} \frac{\sin(2.5\hat{\omega})}{5\sin(0.5\hat{\omega})}$

@ $0.2\pi \rightarrow = e^{-j0.4\pi} \frac{\sin(0.5\pi)}{5\sin(0.1\pi)}$

$= \frac{1}{5\sin(0.1\pi)} e^{-j0.4\pi} = 0.647 e^{-j0.4\pi}$

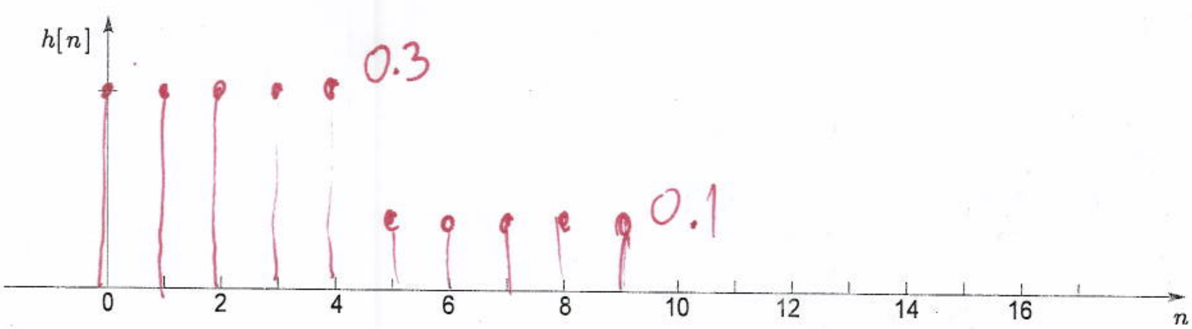
$A =$ 0.647

$\theta =$ -0.4π = 1.26 rad

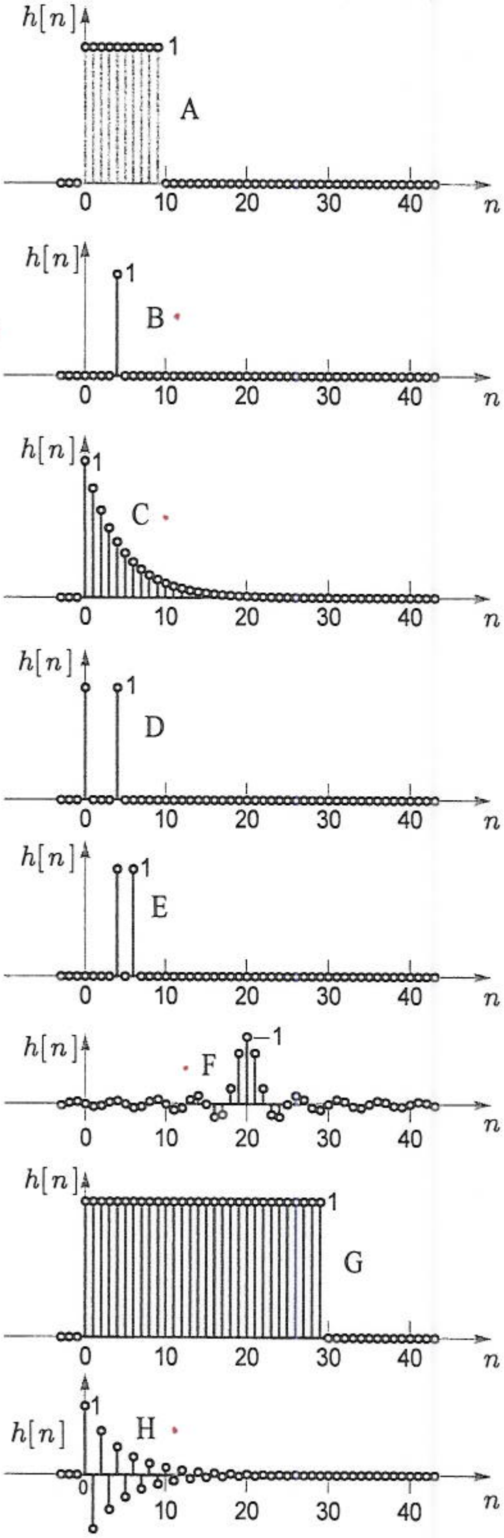
answers are numbers, not equations

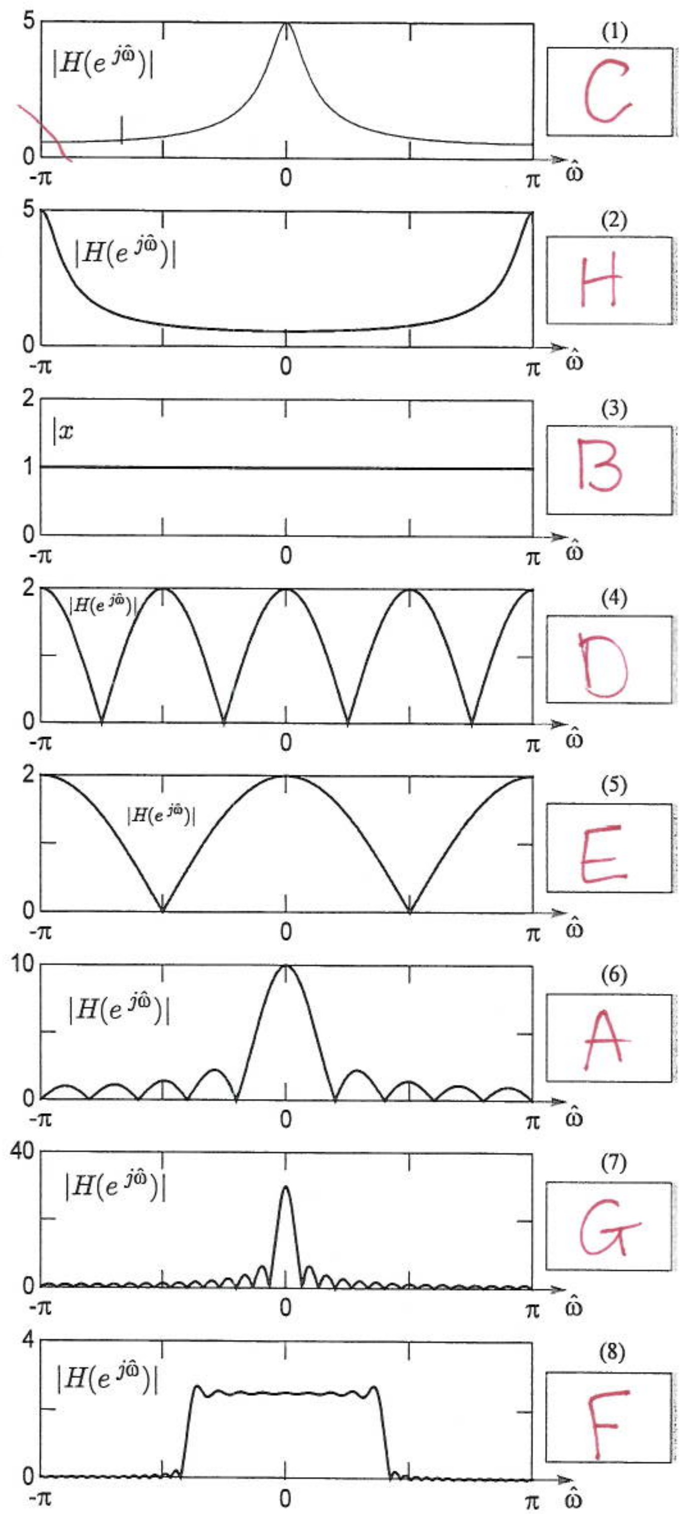
7

(c) Carefully sketch a stem plot of the impulse response $h[n]$ of the overall system:



PROB. Su13-Q2.3. The impulse responses of several LTI systems are sketched below. (Values not shown are zero.) Also shown below are the corresponding frequency responses, in a scrambled order. For each impulse response, identify its corresponding magnitude response by writing a letter {A, B, ... H} into each answer box below.





A

−2 ea

PROB. Su13-Q2.4. (Answer True or False, or provide a short numerical answer.)

(a) T [X] F [ ] The convolution of a discrete-time sinc function with a discrete-time sinc function is always a discrete-time sinc function.

(b) T [X] F [ ] The output of a linear system in response to a zero input is always a zero output.

(c) T [X] F [ ] The system $y[n] = 2^n x[n]$ is linear.

(d) T [X] F [ ] The DTFT of the unit step does not exist.

(e) T [X] F [ ] The system $y[n] = (x[n])^2$ is time-invariant.

(f) T [X] F [ ] The 3-point inverse DFT of $[X[0]\ X[1]\ X[2]] = [0\ 1\ 1]$ is real-valued.

(g) T [X] F [ ] If the only nonzero N-point DFT coefficient is $X[0]$, then all values of $x[n]$ are the same.

(h) T [X] F [ ] Summing $x[n]$ always yields the zeroth DFT coefficient: $X[0] = x[0] + ... + x[N-1]$.

(i) If the unit step $u[n]$ is fed as an input to a 10-point running average filter, the output $y[n]$ at time 8 will be $y[8] =$ [ ]. 0.9

(j) If the frequency response is $H(e^{j\hat{\omega}}) = 2\cos(\hat{\omega})$, the impulse response is $h[n] =$ [$\delta[n \pm 1]$].

$= e^{j\hat{\omega}} + e^{-j\hat{\omega}}$

PROB. Su13-Q2.5. Find the 8-point inverse DFT $\{x[0],\ x[1],\ \ldots\ x[7]\}$ when the 8-point DFT coefficients are:

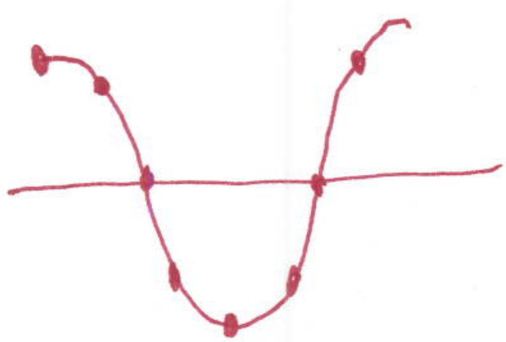
$$X[0] = 0$$
$$X[1] = 4$$
$$X[2] = 0$$
$$X[3] = 0$$
$$X[4] = 0$$
$$X[5] = 0$$
$$X[6] = 0$$
$$X[7] = 4$$

$$x[n] = \frac{1}{8}\left(0 + 4e^{j(1)2\pi n/8} + 0 + 0 + \ldots + 0 + 4e^{j(7)2\pi n/8}\right)$$

$$= \tfrac{1}{2}e^{j\pi n/4} + \tfrac{1}{2}e^{j7\pi n/4}$$

$$= \tfrac{1}{2}e^{j\pi n/4} + \tfrac{1}{2}e^{-j\pi n/4}$$

$$= \cos(\pi n/4)$$



$x[0] =$	1
$x[1] =$	0.707
$x[2] =$	0
$x[3] =$	-0.707
$x[4] =$	-1
$x[5] =$	-0.707
$x[6] =$	0
$x[7] =$	0.707

Hint: Answers are real numbers.