GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Spring 2025
Quiz #2

March 7, 2025

NAME: ____________ ________________ GT username: ________________
(FIRST) (LAST) (e.g., gtxyz123)

Circle your recitation section:

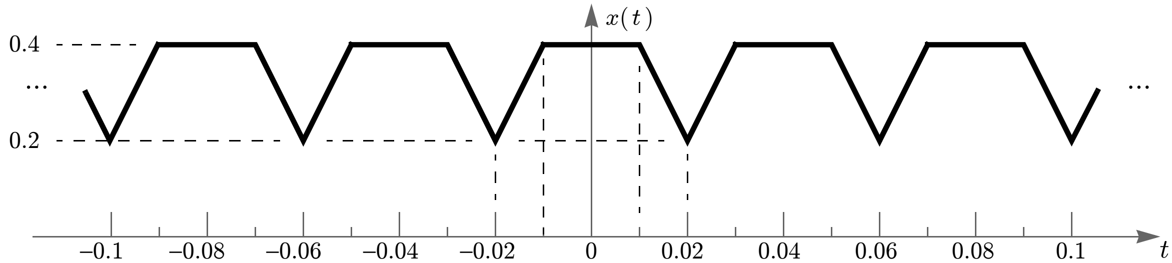
L01 (Daniela)	L05 (Chun-Wei)	L07 (Chun-Wei)	L09 (Daniela)
L02 (Greg)	L06 (Kennedy)	L08 (Kennedy)	L10 (Greg)

Important Notes:

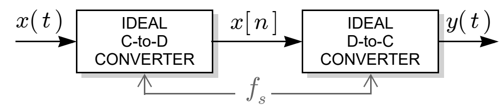
- Do not unstaple the test.
- Closed book, except for one two-sided page (8.5” × 11”) of hand-written notes.
- Calculators are allowed, but no other electronics (no smartphones/watches/readers/tablets/laptops/etc).
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	35	
2	30	
3	35	
Total		

PROB. Sp25-Q2.1. Consider the periodic signal $x(t)$ shown below:



Suppose this signal is input to the cascade of an ideal C-to-D and ideal D-C converter, both with the same (unspecified) sampling rate f_s, as shown here:



(a) In the Fourier series representation $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$, the fundamental frequency and zero-th coefficient are:

$f_0 =$ [ ] Hz (≥ 0),

$a_0 =$ [ ].

(b) If the sampling rate is $f_s = 12.5$ Hz, the D-C output can be written as $y(t) = B_1 + A_1\cos(25\pi t)$ where:

$A_1 =$ [ ] (≥ 0),

$B_1 =$ [ ].

(c) If the sampling rate is $f_s = 50$ Hz, the D-C output can be written as $y(t) = B + A\cos(2\pi f_0 t + \varphi)$ where:

$f_0 =$ [ ] Hz (≥ 0),

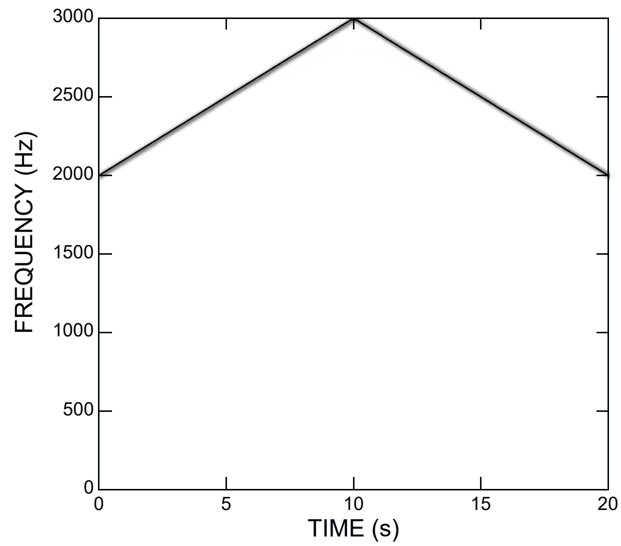
$\varphi =$ [ ] rads ($\in (-\pi, \pi]$)

$A =$ [ ] (≥ 0),

$B =$ [ ].

PROB. Sp25-Q2.2.

```
A =
B =
fsamp =
t2 =
t = 0:1/fsamp:t2;
x = sin(A*pi*(t + B).^2);
spectrogram(x,200,[],1e5,fsamp,'yaxis')
```



If running the shown code leads to the shown plot:

(a) then the unspecified variables `fsamp` and `t2` are:

`fsamp` = ________ (> 0),

`t2` = ________ (> 0).

(b) Specify one possible set of numerical values for the unspecified variables `A` and `B`, with the constraint that both are positive, satisfying: $0 <$ `A` < 300 and $0 <$ `B` < 300.

`A` = ________ (> 0),

`B` = ________ (> 0).

(c) Specify a second possible set of numerical values for the unspecified variables `A` and `B`, this time with the constraint that both are *negative*, satisfying: $-300 <$ `A` < 0 and $-300 <$ `B` < 0.

`A` = ________ (< 0),

`B` = ________ (< 0).

PROB. Sp25-Q2.3. All parts of this problem relate to an FIR filter whose impulse response has the form:

$$h[n] = 3\delta[n] + 5\delta[n-1] + \beta\delta[n-2],$$

where the value of the unspecified parameter β may be different in each part.

(a) If the filter output at time 3 is $y[3] = 3$ when the filter input is $x[n] = \dfrac{2}{1+n^2}$, then $\beta =$ ______ .

(b) Let $s[n]$ denote the *step response* of the filter (i.e., the filter output in response to the unit step input $u[n]$, where $u[n] = 0$ for $n < 0$ and $u[n] = 1$ for $n \geq 0$). If $s[n] = 0$ for all $n \notin \{0, 1, 2\}$, then it must be that:

$\beta =$ ______ ,

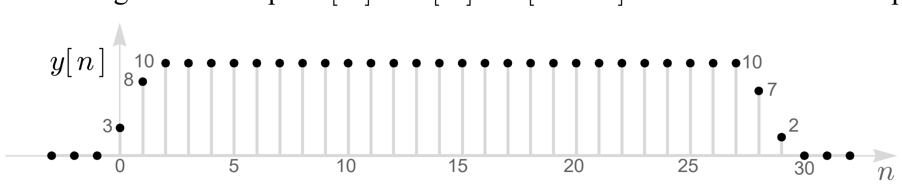
$s[0] =$ ______ ,

$s[1] =$ ______ ,

$s[2] =$ ______ .

(c) Let $z[n] = h[n] * h[n]$ denote the convolution of $h[n]$ with itself. If $z[2] = 27$ then $\beta =$ ______ .

(d) If the rectangular filter input $x[n] = u[n] - u[n-L]$ results in the filter output shown below, then:



$\beta =$ ______ ,

$L =$ ______ .

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NAME: ANSWER KEY (FIRST) (LAST)

GT username: VERSION A (e.g., gtxyz123)

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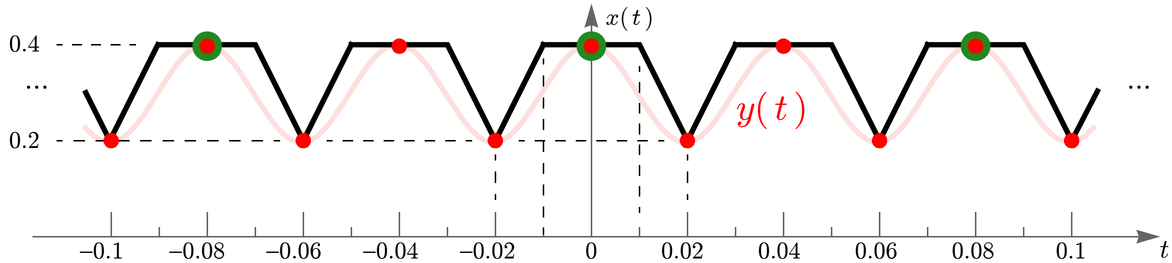
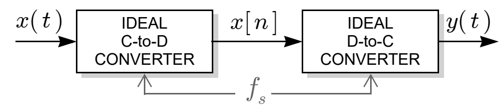
PROB. Sp25-Q2.1. Consider the periodic signal $x(t)$ shown below:



Suppose this signal is input to the cascade of an ideal C-to-D and ideal D-C converter, both with the same (unspecified) sampling rate f_s, as shown here:



(a) In the Fourier series representation $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$, the fundamental frequency and zero-th coefficient are:

$f_0 =$ 25 Hz (≥ 0),

$a_0 =$ 0.35 .

$$\text{Measure } T_0 = 0.04 \;\Rightarrow\; f_0 = \frac{1}{T_0} = 25 \text{ Hz}$$

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)dt = 25(0.01)(0.3 + 0.4 + 0.4 + 0.3) = 0.35$$

(b) If the sampling rate is $f_s = 12.5$ Hz, the D-C output can be written as $y(t) = B_1 + A_1\cos(25\pi t)$ where:

$$x[n] = x\left(\frac{n}{f_s}\right) = x(0.08n) = 0.4 \text{ (constant) for all } n$$

(see green dots in figure)

$A_1 =$ 0 (≥ 0),

$B_1 =$ 0.4 .

$$\Rightarrow y(t) = 0.4 \text{ (constant) for all } t$$

(c) If the sampling rate is $f_s = 50$ Hz, the D-C output can be written as $y(t) = B + A\cos(2\pi f_0 t + \varphi)$ where:

$f_0 =$ 25 Hz (≥ 0),

$$x[n] = x\left(\frac{n}{f_s}\right) = x(0.02n) = \text{red dots in the figure}$$

$$= 0.3 + 0.1\cos(\pi n)$$

$\varphi =$ 0 rads ($\in(-\pi, \pi]$)

$A =$ 0.1 (≥ 0),

$$\Rightarrow y(t) = x[n]\big|_{n = f_s t}$$

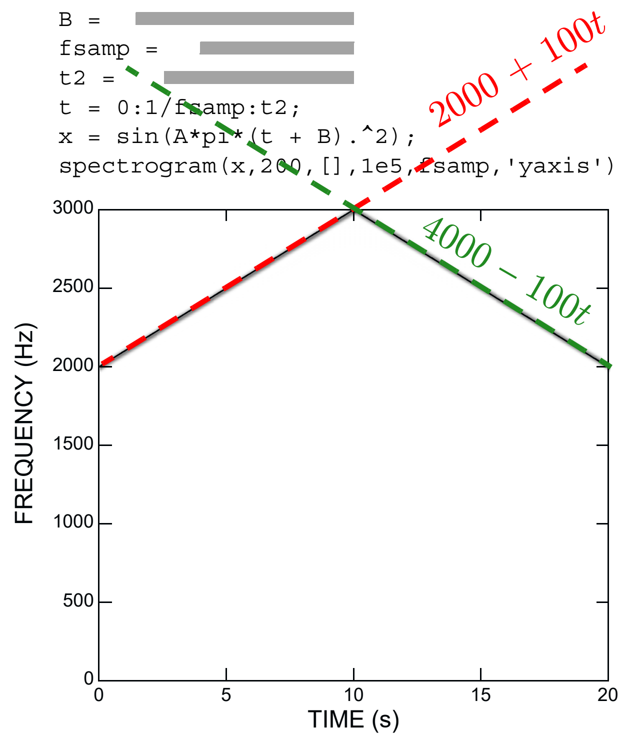
$$= 0.3 + 0.1\cos(50\pi t)$$

$$= \text{pink curve in figure}$$

$B =$ 0.3 .

PROB. Sp25-Q2.2.

```
A =
B =
fsamp =
t2 =
t = 0:1/fsamp:t2;
x = sin(A*pi*(t + B).^2);
spectrogram(x,200,[],1e5,fsamp,'yaxis')
```



If running the shown code leads to the shown plot:

(a) then the unspecified variables `fsamp` and `t2` are:

`fsamp` = $\boxed{6000}$ (> 0),

`t2` = $\boxed{20}$ (> 0).

(b) Specify one possible set of numerical values for the unspecified variables `A` and `B`, with the constraint that both are positive, satisfying: $0 <$ `A` < 300 and $0 <$ `B` < 300.

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}(A\pi(t+B)^2)$$

$$= AB + At$$

Equate this to equation for red dashed line, assuming aliasing happens after $t > 10$:

$$= \underbrace{2000}_{\Rightarrow B = 20} + \underbrace{100t}_{\Rightarrow A = 100}$$

`A` = $\boxed{100}$ (> 0),

`B` = $\boxed{20}$ (> 0).

(c) Specify a second possible set of numerical values for the unspecified variables `A` and `B`, this time with the constraint that both are *negative*, satisfying: $-300 <$ `A` < 0 and $-300 <$ `B` < 0.

Equate

$$f_i(t) = AB + At$$

to equation for green dashed line, assuming aliasing happens before $t < 10$:

$$= \underbrace{4000}_{\Rightarrow B = -40} - \underbrace{100t}_{\Rightarrow A = -100}$$

`A` = $\boxed{-100}$ (< 0),

`B` = $\boxed{-40}$ (< 0).

PROB. Sp25-Q2.3. All parts of this problem relate to an FIR filter whose impulse response has the form:

$$h[n] = 3\delta[n] + 5\delta[n-1] + \beta\delta[n-2],$$

where the value of the unspecified parameter β may be different in each part.

(a) If the filter output at time 3 is $y[3] = 3$ when the filter input is $x[n] = \frac{2}{1+n^2}$, then $\beta = \boxed{0.4}$.

$$\begin{aligned} y[3] &= 3x[3] + 5x[2] + \beta x[1] \\ &= 3\left(\frac{2}{1+9}\right) + 5\left(\frac{2}{1+4}\right) + \beta\left(\frac{2}{1+1}\right) \\ &= 2.6 + \beta \end{aligned}$$

(b) Let $s[n]$ denote the *step response* of the filter (i.e., the filter output in response to the unit step input $u[n]$, where $u[n] = 0$ for $n < 0$ and $u[n] = 1$ for $n \geq 0$). If $s[n] = 0$ for all $n \notin \{0, 1, 2\}$, then it must be that:

If input is unit step, output at time 3 is:

$$\begin{aligned} s[3] &= 3u[3] + 5u[2] + \beta u[1] \\ &= 3(1) + 5(1) + \beta(1) \\ &= 8 + \beta \Rightarrow \text{zero when } \beta = -8 \end{aligned}$$

$\beta = \boxed{-8}$,

$s[0] = \boxed{3}$,

$s[1] = \boxed{8}$,

$s[2] = \boxed{0}$.

$$\Rightarrow s[0] = 3u[0] + 5u[-1] - 8u[-2] = 3 + 0 + 0 = 3$$

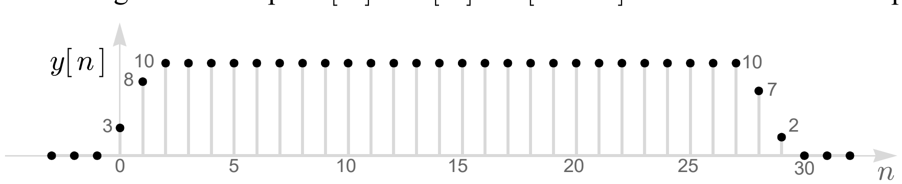
$$s[1] = 3u[1] + 5u[0] - 8u[-1] = 3 + 5 + 0 = 8$$

$$s[2] = 3u[2] + 5u[1] - 8u[0] = 3 + 5 - 8 = 0$$

(c) Let $z[n] = h[n] * h[n]$ denote the convolution of $h[n]$ with itself. If $z[2] = 27$ then $\beta = \boxed{\frac{1}{3}}$.

$$\begin{aligned} z[2] &= 3h[2] + 5h[1] + \beta h[0] \\ &= 3\beta + 5(5) + \beta(3) \\ &= 6\beta + 25 \end{aligned}$$

(d) If the rectangular filter input $x[n] = u[n] - u[n-L]$ results in the filter output shown below, then:



$\beta = \boxed{2}$,

$L = \boxed{28}$.

$$y[2] = 3u[2] + 5u[1] + \beta u[0] = 3 + 5 + \beta = 10 \Rightarrow \beta = 2$$

$$\text{Output length} = \text{input length} + \text{filter order} \Rightarrow 30 = L + 2$$