GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ-2

DATE: 08-Mar-24 COURSE: ECE-2026

NAME: _____ gt Account: _____

LAST, FIRST (ex: gburdell21)

Instructions: READ and SIGN your name below

failing to write your name, gtAccount, and sign below may result in a 5% penalty

- You have **50 minutes** (students with accommodations will have the appropriate time extension automatically added to their time) to complete this exam.
- Graphics calculators are permitted. One sheet of paper is permitted. You can write on both sides hand-written notes.
- WRITE ANY RADIAN ANSWERS AS A MULTIPLE OF π. (i.e., write 0.4286π or $3\pi/7$ instead of 1.3464). ALL RADIAN ANSWERS MUST BE IN THE RANGE$(-\pi, \pi]$ FOR CREDIT.

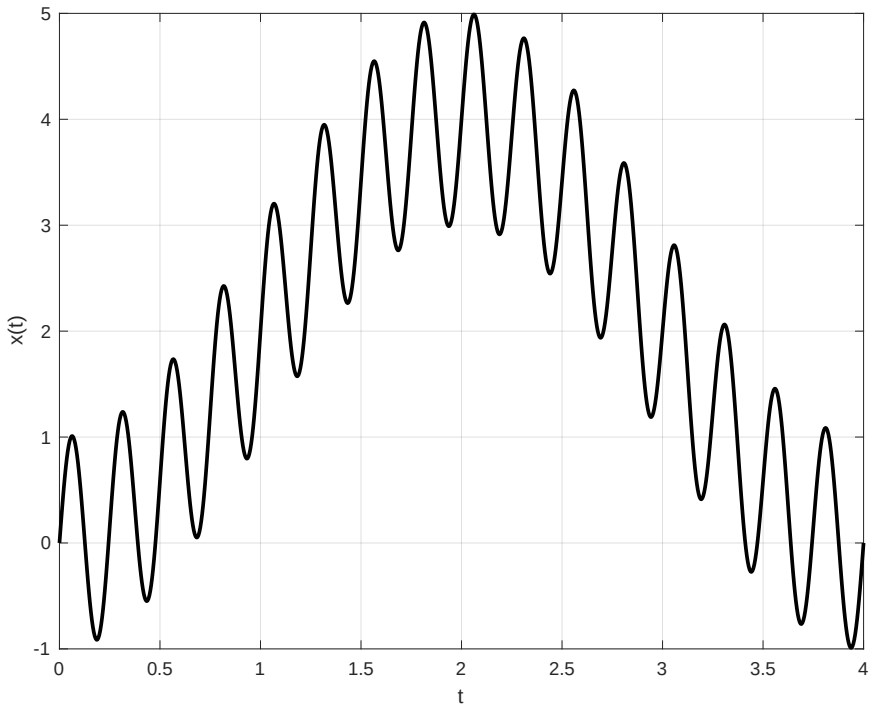
The Academic Honor Code will be strictly enforced. Forgeries and plagiarism are violations of the Georgia Tech honor code and will be referred to the Dean of Students for disciplinary action. You are not to discuss exam content or to share any written, electronic, or any other form of exam information with anyone during or after the exam until the solutions have been posted. By submitting this exam, you affirm that you have neither given nor received inappropriate assistance during this exam. I have read the instructions above and affirm that I will abide by the guidelines provided.

_____

Sign your name on the line above

PROBLEM SP-24-Q.2.1:

The figure shows one period of a periodical signal, $x(t)$, where $x(t) = x(t+4)$, $x(0) = x(4) = 0$, and $x(2) = 4$. The signal is a combination of sinusoids and a constant. Answer the following questions.



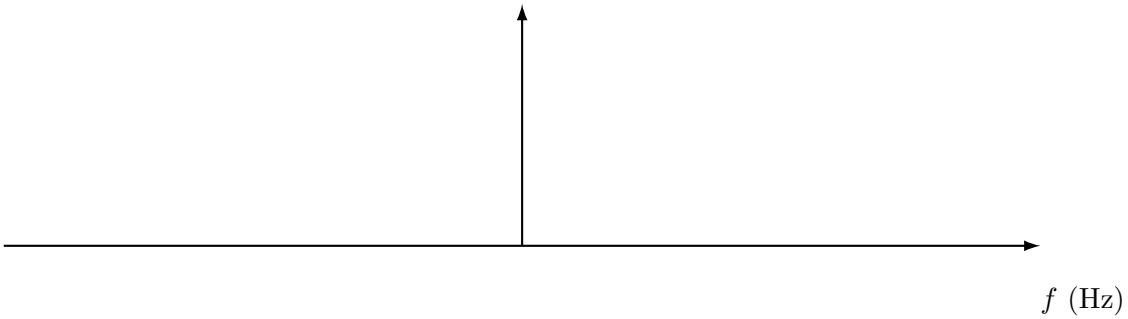
(a) (2 points) Determine the fundamental frequency of the signal, $x(t)$.

$f_o =$ _____ Hz

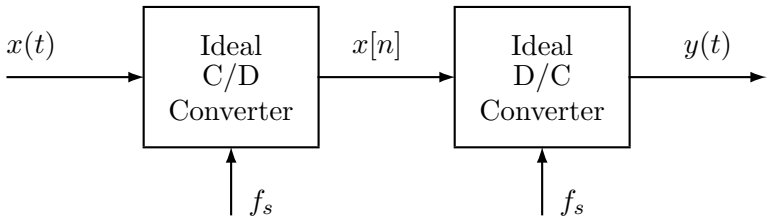
(b) (3 points) Determine the DC component, a_0.

$a_0 =$ _____

(c) (20 points) Determine *all* Fourier Series coefficients of $x(t)$ by sketching, in the space below, as accurately as possible the two-sided spectrum for $x(t)$. *Hint*: there are a total of five lines. Label the frequency and the amplitude for each line.



(d) (5 points) Given the system depicted in the figure below with $f_s = 40$ Hz, determine the output, $y(t)$, when the input is $x(t)$.

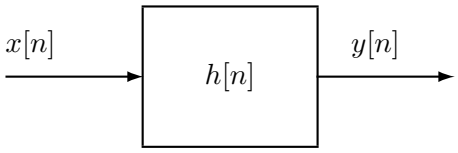


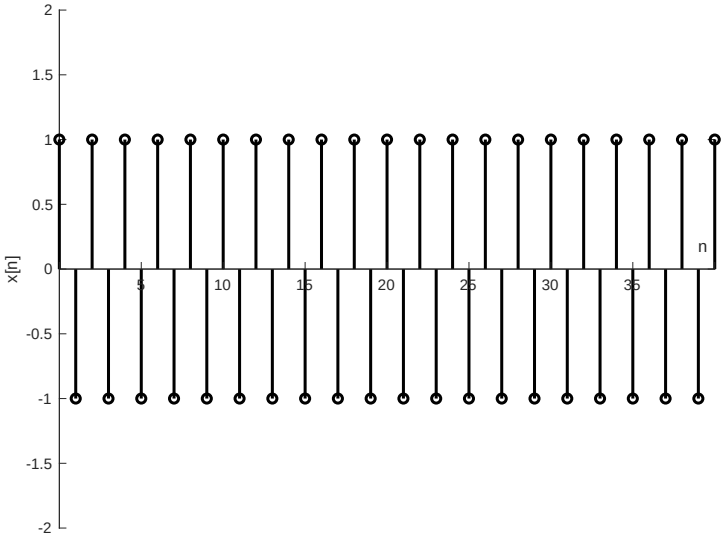
PROBLEM SP-24-Q.2.2:

Answer each of the following questions.

(a) (12 points) Part of the signal, $x[n] = A\cos(\hat{\omega}n)$, is shown in the stem plot. From the figure, one can determine the values for A, the period, and the frequency $\hat{\omega}$. Suppose that the signal is an input to an FIR system with the following impulse response: $h[n] = \delta[n-1]$. Determine $y[n]$ and write it in the form, $y[n] = K\cos(Gn)$. Specify the numbers K and G. Simplify as much as you can.

$K =$ _____ $G =$ _____





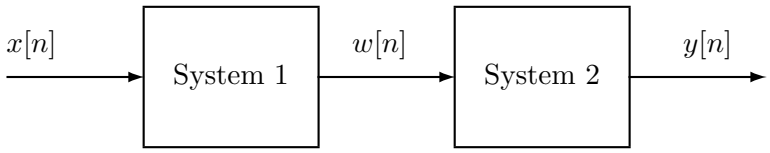
(b) (12 points) Suppose you evaluate the following code and find $y[0] = 1$, $y[2] = 4$, and $y[5] = 2$.

```
y = conv( [1 0 1 0 1], [A B C]) .
```

Determine the values A, B, and C.

$A =$ _____ $B =$ _____ $C =$ _____

(c) (12 points) Suppose we have a cascade of two systems as shown in the figure.

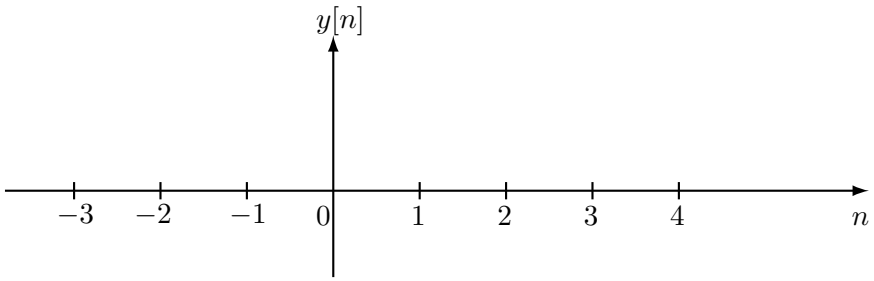


Let $w[n] = 7(x[n-1]-1)^2$ and $y[n] = nw[n]$.

Specify whether the two systems are **LTI** or **non-LTI**. Why?

Is system #1 LTI or non-LTI? Why?	Is system #2 LTI or non-LTI? Why?

Suppose that $x[n] = u[n]$. Find $y[n]$ by sketching it on the following stem plot. Label your sketch.



PROBLEM SP-24-Q.2.3:

Answer each of the following questions.

(a) (12 points) Suppose the following MATLAB code generates a spectrogram plot.

```
tt = 0:1/fs:1;
xx=cos(2*pi*A*tt - pi*B*(tt.^2));
plotspec(xx,fs);
```

Let $f_s = 12000$, $A = 6000$ and $B = -6000$.

Predict what the resulting spectrogram plot will look like by carefully sketching it in the space below. Be sure to label both the x axis (in units of seconds) and the y axis (in units of kHz).

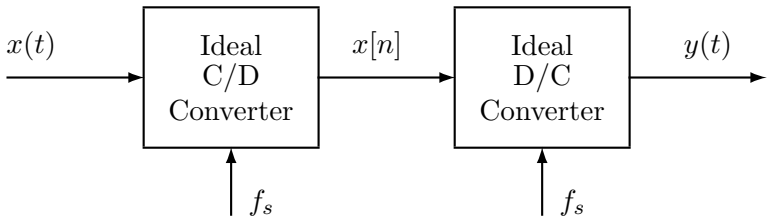
(b) (10 points) Suppose we run the following MATLAB code.

```
fs=8000;
tt = 0:1/fs:1000;
xx=cos(2*pi*7000*tt + pi/3);
soundsc(xx,fsamp);
```

Determine the value of `fsamp` that should be used to play the vector `xx` as a 3000 Hz tone. $fsamp$ can be different from f_s.

$f_{samp} =$ _____

(c) (12 points) This question concerns ideal sampling and reconstruction as shown in the following block diagram.



$$x(t) = \cos(800\pi t) + \cos(1800\pi t + \pi/4)$$

Determine the largest value of f_s, in Hz, such that $y(t) = A\cos(800\pi t + \varphi)$. Determine the value for φ. Make sure to write the phase as a multiple of π, e.g., 0.634π.

$f_s =$ _____	$\phi =$ _____

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ-2

DATE: 08-Mar-24 COURSE: ECE-2026

NAME: **ANSWER KEY** gt Account: **VERSION #1**

LAST, FIRST (ex: gburdell21)

Instructions: READ and SIGN your name below

failing to write your name, gtAccount, and sign below may result in a 5% penalty

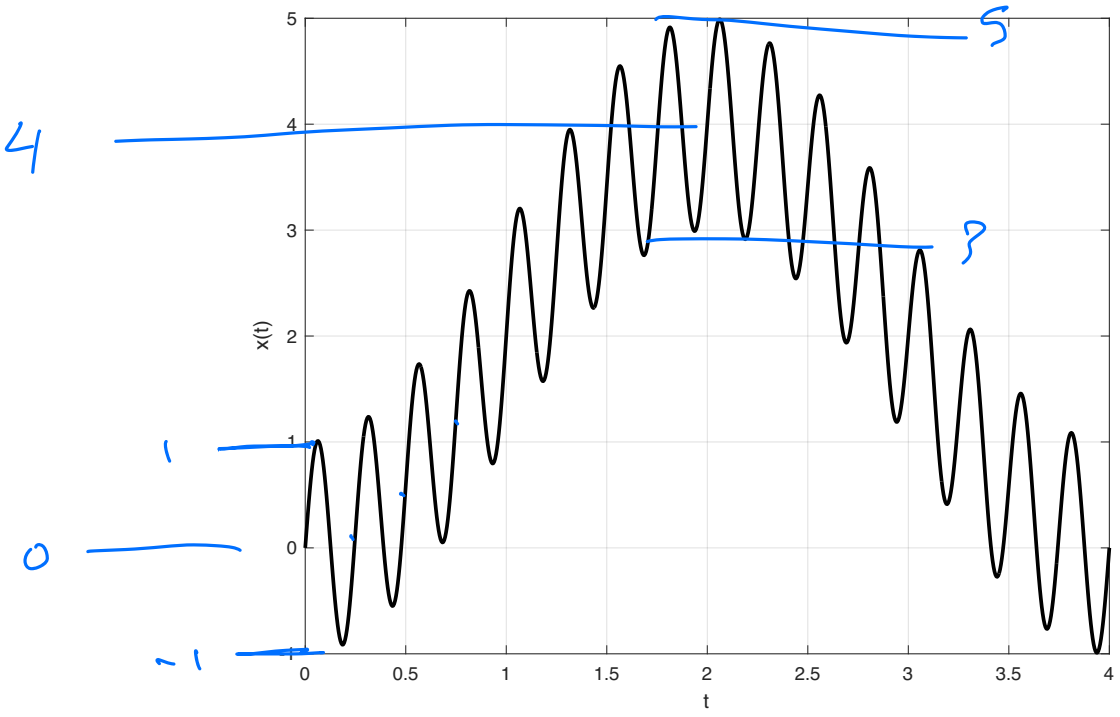
- You have **50 minutes** (students with accommodations will have the appropriate time extension automatically added to their time) to complete this exam.
- Graphics calculators are permitted. One sheet of paper is permitted. You can write on both sides hand-written notes.
- WRITE ANY RADIAN ANSWERS AS A MULTIPLE OF π. (i.e., write 0.4286π or $3\pi/7$ instead of 1.3464). ALL RADIAN ANSWERS MUST BE IN THE RANGE$(-\pi, \pi]$ FOR CREDIT.

The Academic Honor Code will be strictly enforced. Forgeries and plagiarism are violations of the Georgia Tech honor code and will be referred to the Dean of Students for disciplinary action. You are not to discuss exam content or to share any written, electronic, or any other form of exam information with anyone during or after the exam until the solutions have been posted. By submitting this exam, you affirm that you have neither given nor received inappropriate assistance during this exam. I have read the instructions above and affirm that I will abide by the guidelines provided.

Sign your name on the line above

PROBLEM SP-24-Q.2.1:

The figure shows one period of a periodical signal, $x(t)$, where $x(t) = x(t+4)$, $x(0) = x(4) = 0$, and $x(2) = 4$. The signal is a combination of sinusoids and a constant. Answer the following questions.



(a) (2 points) Determine the fundamental frequency of the signal, $x(t)$.

$f_o =$ 0.25 Hz

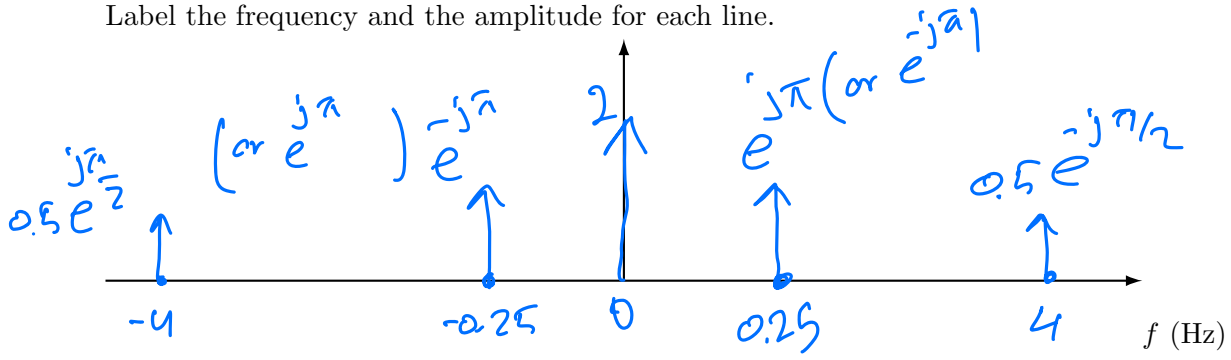
$T_0 = 4$ sec. $\Rightarrow f_o = \frac{1}{4} = 0.25$ Hz

(b) (3 points) Determine the DC component, a_0.

$a_0 =$ 2

Recognizing the sinusoids and one is shifted by 2

(c) (20 points) Determine *all* Fourier Series coefficients of $x(t)$ by sketching, in the space below, as accurately as possible the two-sided spectrum for $x(t)$. *Hint*: there are a total of five lines. Label the frequency and the amplitude for each line.



By inspection, there are two sinusoids and a constant.

One sinusoid has a frequency of $\frac{1}{4}$ and amplitude of 2. But this sinusoid has a shift of π

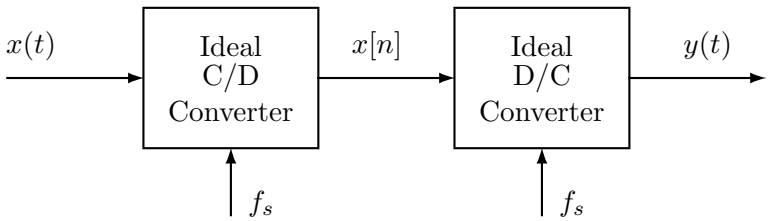
$$\Rightarrow -2\cos\left(2\pi\left(\tfrac{1}{4}\right)t\right)$$

The second smaller sinusoid ha a frequency of 4 Hz. One can count 4 periods within one second. The amplitude of this sinusoid is 1. The shift is $+\pi/2$.

$$\text{Thus} \Rightarrow \cos(2\pi(4)t - \pi/2) = \sin(2\pi(4)t)$$

$$\Rightarrow x(t) = 2 + \sin(2\pi(4)t) - 2\cos\left(2\pi\left(\tfrac{1}{4}\right)t\right)$$

(d) (5 points) Given the system depicted in the figure below with $f_s = 40$ Hz, determine the output, $y(t)$, when the input is $x(t)$.



The highest frequency in $x(t)$ is 4 Hz.

$f_s = 40\,Hz > 2 \times 4$. Thus, $y(t) = x(t)$

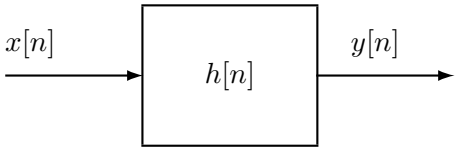
$y(t) = 2 + \sin(2\pi(4)t) - 2\cos(2\pi(\frac{1}{4})t)$

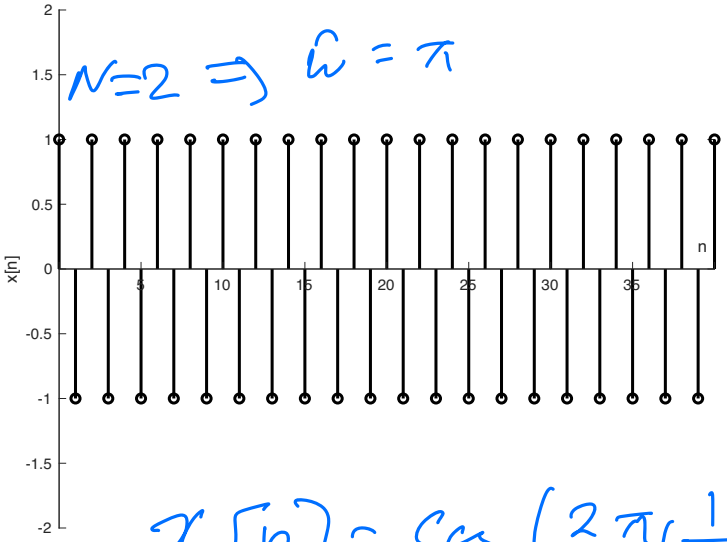
PROBLEM SP-24-Q.2.2:

Answer each of the following questions.

(a) (12 points) Part of the signal, $x[n] = A\cos(\hat{\omega}n)$, is shown in the stem plot. From the figure, one can determine the values for A, the period, and the frequency $\hat{\omega}$. Suppose that the signal is an input to an FIR system with the following impulse response: $h[n] = \delta[n-1]$. Determine $y[n]$ and write it in the form, $y[n] = K\cos(Gn)$. Specify the numbers K and G. Simplify as much as you can.

$K = e^{-j\pi}$ $\quad$ $G = \pi$





$N=2 \Rightarrow \hat{\omega} = \pi$

$x[n] = \cos\left(2\pi\left(\frac{1}{2}\right)n\right)$

$y[n] = x[n-1] = \cos(\pi(n-1))$

$= \cos(\pi n - \pi)$

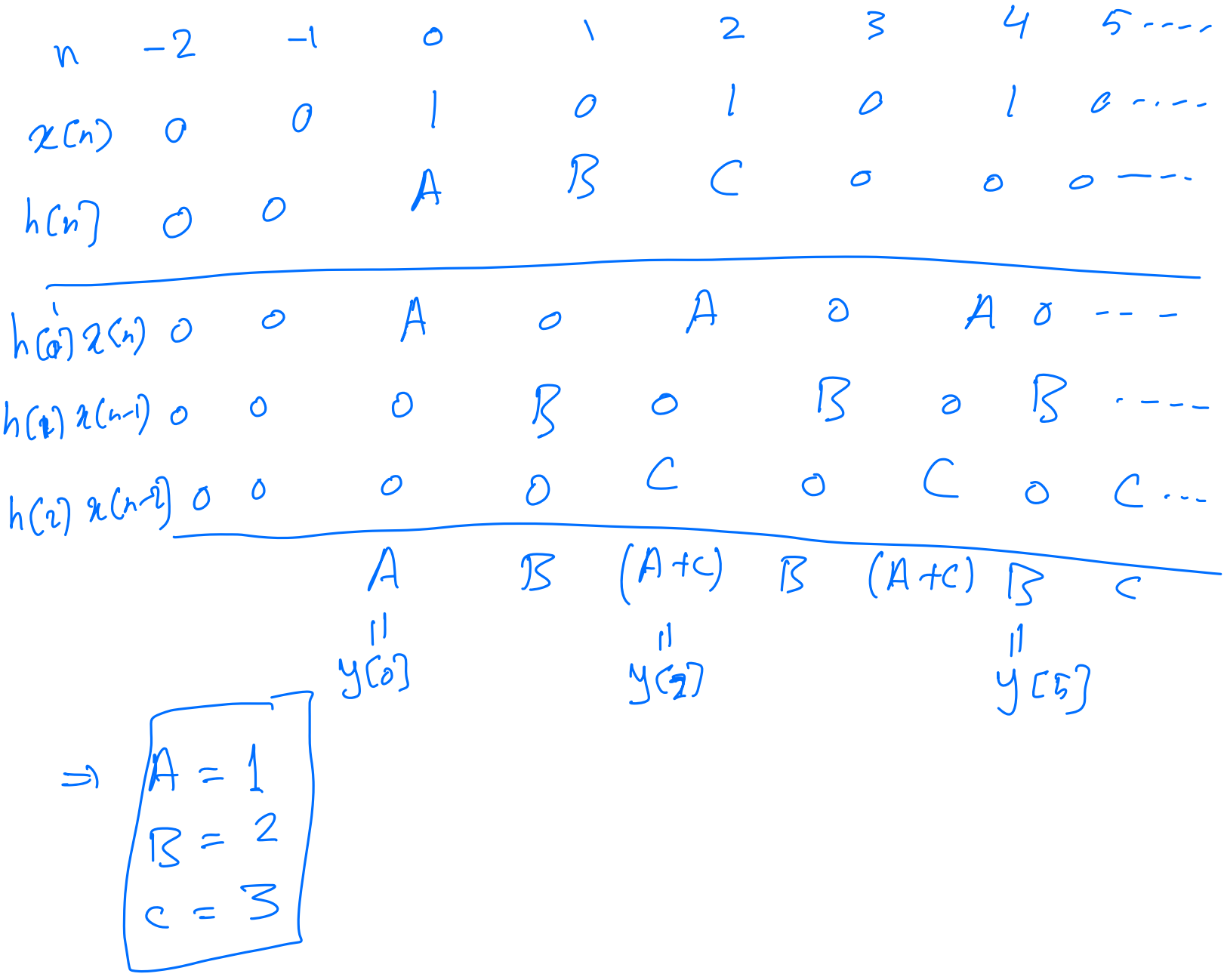
$= e^{-j\pi}\cos(\pi n)$

(b) (12 points) Suppose you evaluate the following code and find $y[0] = 1$, $y[2] = 4$, and $y[5] = 2$.

`y = conv( [1 0 1 0 1], [A B C])` .

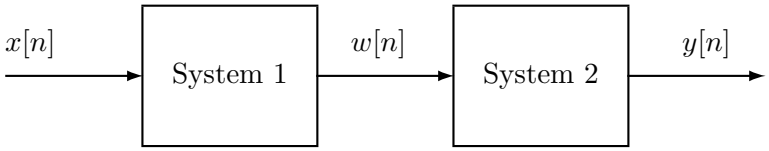
Determine the values A, B, and C.

$A =$ 1	$B =$ 2	$C =$ 3



$\Rightarrow$ $A = 1$, $B = 2$, $C = 3$

(c) (12 points) Suppose we have a cascade of two systems as shown in the figure.



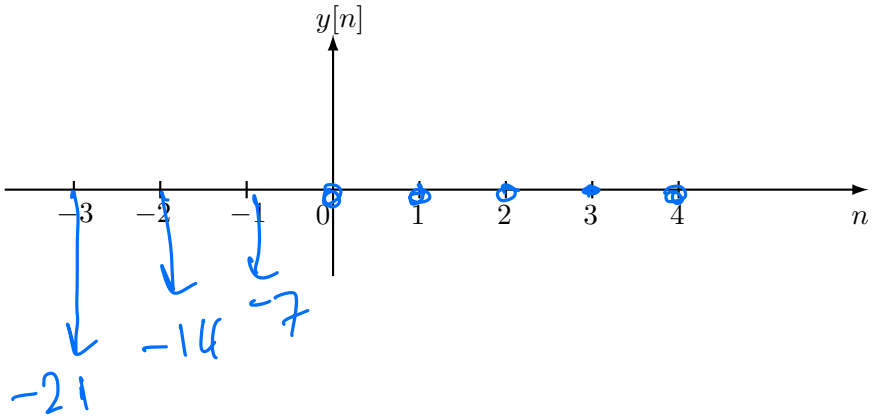
Let $w[n] = 7(x[n-1]-1)^2$ and $y[n] = nw[n]$.

Specify whether the two systems are **LTI** or **non-LTI**. Why?

Is system #1 LTI or non-LTI? Why?

Is system #2 LTI or non-LTI? Why?

Suppose that $x[n] = u[n]$. Find $y[n]$ by sketching it on the following stem plot. Label your sketch.



n	-3	-2	-1	0	1	2	3	4	5	6
$u(n)$	0	0	0	1	1	1	1	1	1	1
$w(n)$	7	7	7	7	0	0	0	0	0	0
$y(n)$	-21	-14	-7	0	0	0	0	0	0	0

PROBLEM SP-24-Q.2.3:

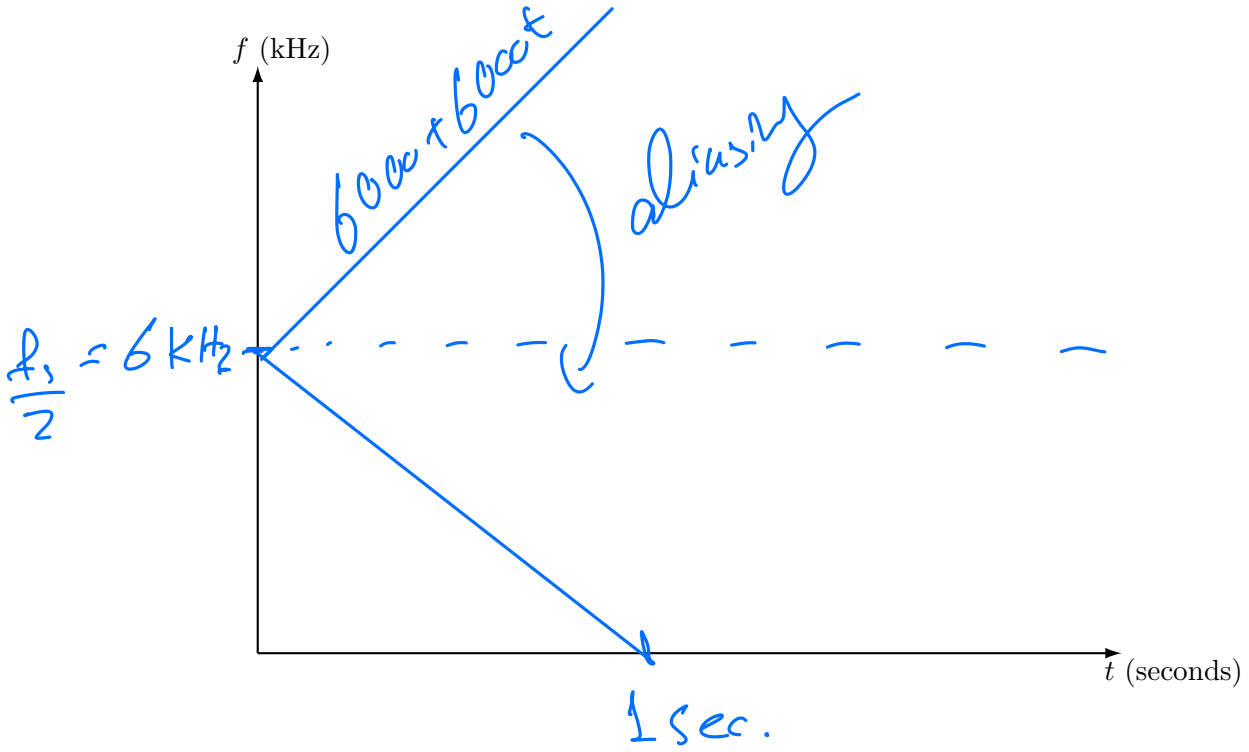
Answer each of the following questions.

(a) (12 points) Suppose the following MATLAB code generates a spectrogram plot.

```
tt = 0:1/fs:1;
xx=cos(2*pi*A*tt - pi*B*(tt.^2));
plotspec(xx,fs);
```

Let $f_s = 12000$, $A = 6000$ and $B = -6000$.

Predict what the resulting spectrogram plot will look like by carefully sketching it in the space below. Be sure to label both the x axis (in units of seconds) and the y axis (in units of kHz).



(b) (10 points) Suppose we run the following MATLAB code.

```
fs=8000;
tt = 0:1/fs:1000;
xx=cos(2*pi*7000*tt + pi/3);
soundsc(xx,fsamp);
```

Determine the value of `fsamp` that should be used to play the vector `xx` as a 3000 Hz tone. *fsamp* can be different from f_s.

$\hat{\omega} = \frac{2\pi(7000)}{8000} = 2\pi\left(\frac{7}{8}\right) = 1.75\pi$

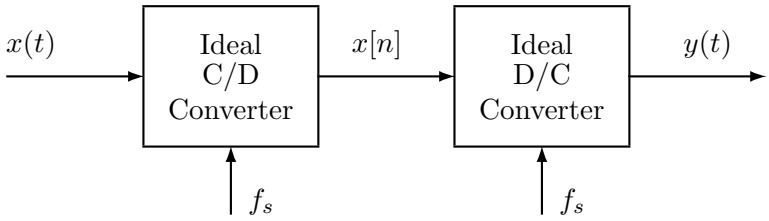
$8000 = f_s \neq 2 \times 7000 \Rightarrow$ aliasing

$-1.75\pi + 2\pi = 0.25\pi$

$\Rightarrow \frac{0.25\pi}{2\pi} \times f_{samp} = 3000$

$\Rightarrow \boxed{f_{samp} = 24{,}000 \text{ Hz}}$

(c) (12 points) This question concerns ideal sampling and reconstruction as shown in the following block diagram.



$$x(t) = \cos(800\pi t) + \cos(1800\pi t + \pi/4)$$

Determine the largest value of f_s, in Hz, such that $y(t) = A\cos(800\pi t + \varphi)$. Determine the value for φ. Make sure to write the phase as a multiple of π, e.g., 0.634π.

The $\cos(1800\pi t + \pi/4)$ will fold into 800π. Thus, $f_s = 900 + 400 = 1300$ Hz

phasor addition: $1 + e^{-j\pi/4}$

$= 2\cos\left(\frac{\pi}{8}\right) e^{-j\pi/8}$

$f_s = 1{,}300$ Hz

$\varphi = -0.125\pi$