GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 — Spring 2023
Quiz #2

March 10, 2023

NAME: ______________ ________________ GT username: ________________
(FIRST) (LAST) (e.g., gtbuzz2026)

Circle your recitation section:

L01 (Chen)	L07 (Davenport)	L09 (Hessler)	L11 (Hessler)
L02 (Duan)	L08 (Duan)	L10 (Chen)	

Important Notes:

- Do not unstaple the test.
- Closed book, except for one two-sided page (8.5” $\times$ 11”) of hand-written notes.
- Calculators are allowed, but no other electronics (no smartphones/watches/readers/tablets/laptops/etc).
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score Earned
1	35	
2	35	
3	30	
Total		

PROB. Sp23-Q2.1. Let $y(t) = 2\cos(30\pi t + 0.3\pi) + x(t)\cos(45\pi t)$.

(a) If $x(t) = 12$ (a constant for all t),
then the Fourier series for $y(t)$ is $y(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$, where $f_0 =$ [> 0] Hz, and where:

a_{-5}	a_{-4}	a_{-3}	a_{-2}	a_{-1}	a_0	a_1	a_2	a_3	a_4	a_5

(Leave an answer box *empty* to signify that the corresponding a_k is zero, specify only the *nonzero* FS coefficients.)

(b) If $x(t) = 16\cos(5\pi t)$,
then the FS for $y(t)$ is $y(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$, where $f_0 =$ [> 0] Hz, and where:

a_{-5}	a_{-4}	a_{-3}	a_{-2}	a_{-1}	a_0	a_1	a_2	a_3	a_4	a_5

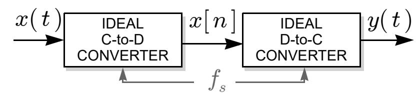
(Leave an answer box *empty* to signify that the corresponding a_k is zero, specify only the *nonzero* FS coefficients.)

(c) If $x(t) = A\cos(2\pi f_c t + \varphi)$ causes $y(t)$ to be periodic with fundamental frequency $f_0 = 30$ Hz,
then it must be that:

$A =$ [> 0], $f_c =$ [> 0] Hz, $\varphi =$ [$\in (-\pi, \pi]$] rads.

PROB. Sp23-Q2.2. Consider the signal $x(t)$ whose spectrum is shown below, where the constant $r > 0$ is real and nonnegative but otherwise unspecified:

Suppose we sample $x(t)$ with sampling rate f_s, and then feed the samples to an ideal D-to-C converter with the same f_s parameter, producing the continuous-time output $y(t)$, as shown here.
The parameters r and f_s may be different in each part below.



(a) In order for $y(t) = x(t)$, the sampling rate must satisfy $f_s >$ ______ Hz.

(b) Give three examples of sampling rates for which $y(t)$ is a *constant*, independent of time t:

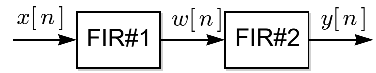
$f_s =$ ______ (> 0) Hz, $f_s =$ ______ (> 0) Hz, $f_s =$ ______ (> 0) Hz.

(c) If $f_s = 3$ Hz and $y(t) = 20 + A\cos(2\pi f_1 t + \varphi)$, then:

$A =$ ______ (> 0), $f_1 =$ ______ (> 0) Hz, $\varphi =$ ______ ($\in(-\pi, \pi]$) rads

(d) If $y(t) = 10\cos(64\pi t)$ then $f_s =$ ______ (> 0) Hz, and $r =$ ______ .

PROB. Sp23-Q2.3. As shown in the figure, suppose that two FIR filters are connected in cascade, so that an input $x[n]$ to the first system produces an intermediate signal $w[n]$, which becomes the input to the second, producing an overall output $y[n]$, where:



- FIR#1 has impulse response $h_1[n] = 3\delta[n] + \beta\delta[n-1]$,
- FIR#2 has impulse response $h_2[n] = \delta[n] - \delta[n-1]$.

The constant β is different in each part below.

(a) If $\beta = 15$ and $x[n] = \dfrac{1}{1+n^2}$ then $w[3] =$ ______ .

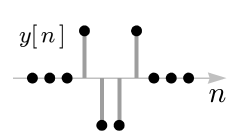
(b) If $\beta = 3$, the difference equation relating the overall output to input is $y[n] = \sum_k b_k x[n-k]$, where:

b_0	b_1	b_2	b_3	b_4

(c) If $\beta = 6$ and $x[n] = nu[n]$ (*i.e.*, $x[n] = 0$ for $n < 0$, and $x[n] = n$ for $n \geq 0$), specify numerical values for the following outputs:

$y[0]$	$y[1]$	$y[2]$	$y[3]$	$y[4]$

(d) If $x[n] = \delta[n] + \delta[n-1]$ results in an output stem plot with the following shape, then it must be that



$\beta =$ ______ .

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PROB. Sp23-Q2.1. Let $y(t) = 2\cos(30\pi t + 0.3\pi) + x(t)\cos(45\pi t)$.

(a) If $x(t) = 12$ (a constant for all t),
then the Fourier series for $y(t)$ is $y(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$, where $f_0 =$ [7.5] (> 0) Hz, and where:

$= \gcd(15, 22.5)$

a_{-5}	a_{-4}	a_{-3}	a_{-2}	a_{-1}	a_0	a_1	a_2	a_3	a_4	a_5
		6	$e^{-j0.3\pi}$				$e^{j0.3\pi}$	6		

(Leave an answer box *empty* to signify that the corresponding a_k is zero, specify only the *nonzero* FS coefficients.)

(b) If $x(t) = 16\cos(5\pi t)$,
then the FS for $y(t)$ is $y(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$, where $f_0 =$ [5] (> 0) Hz, and where:

$= \gcd(15, 20, 25)$

a_{-5}	a_{-4}	a_{-3}	a_{-2}	a_{-1}	a_0	a_1	a_2	a_3	a_4	a_5
4	4	$e^{-j0.3\pi}$						$e^{j0.3\pi}$	4	4

(Leave an answer box *empty* to signify that the corresponding a_k is zero, specify only the *nonzero* FS coefficients.)

(c) If $x(t) = A\cos(2\pi f_c t + \varphi)$ causes $y(t)$ to be periodic with fundamental frequency $f_0 = 30$ Hz,
then it must be that:

$A =$ [4] (> 0), $f_c =$ [7.5] (> 0) Hz, $\varphi =$ [0.7π] ($\in(-\pi, \pi]$) rads.

Frequencies for $y(t)$ at $\{15,\ 22.5 - f_c,\ 22.5 + f_c\}$,

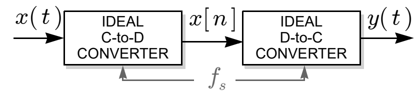
$\underbrace{22.5 + f_c}$: 30 Hz when $f_c = 7.5$ Hz

$$\Rightarrow y(t) = \underbrace{2\cos(30\pi t + 0.3\pi) + \tfrac{A}{2}\cos(30\pi t - \varphi)} + \tfrac{A}{2}\cos(60\pi t + \varphi)$$

cancel when $\varphi = 0.7\pi$
and $A = 4$

PROB. Sp23-Q2.2. Consider the signal $x(t)$ whose spectrum is shown below, where the constant $r > 0$ is real and nonnegative but otherwise unspecified:

Suppose we sample $x(t)$ with sampling rate f_s, and then feed the samples to an ideal D-to-C converter with the same f_s parameter, producing the continuous-time output $y(t)$, as shown here. The parameters r and f_s may be different in each part below.



(a) In order for $y(t) = x(t)$, the sampling rate must satisfy $f_s >$ **96** Hz.

(b) Give three examples of sampling rates for which $y(t)$ is a *constant*, independent of time t:

$f_0 = \gcd(32, 48) = 16$

$f_s =$ **16** (> 0) Hz, $f_s =$ **8** (> 0) Hz, $f_s =$ **4** (> 0) Hz.

$\Rightarrow f_s = \frac{16}{\ell}$ for any $\ell \in \{1, 2, 3, \ldots\}$

(c) If $f_s = 3$ Hz and $y(t) = 20 + A\cos(2\pi f_1 t + \varphi)$, then:

$A =$ **34.0** (> 0), $f_1 =$ **1** (> 0) Hz, $\varphi =$ **-0.3π** ($\in(-\pi, \pi]$) rads

$$x(t) = 2r\cos(2\pi(32)t + 0.3\pi) + 2r\cos(2\pi(48)t + 0.3\pi)$$

$$\Rightarrow y(t) = 2r\cos(2\pi(32 - (11)(3))t + 0.3\pi) + 2r\cos(2\pi(48 - (16)(3))t + 0.3\pi)$$

$$= \underbrace{2r}_{A}\cos(2\pi\underbrace{(1)}_{f_1}t \underbrace{- 0.3\pi}_{\varphi}) + \underbrace{2r\cos(0 + 0.3\pi)}_{20} \Rightarrow 2r = A = \frac{20}{\cos(0.3\pi)} \approx 34.0$$

(d) If $y(t) = 10\cos(64\pi t)$ then $f_s =$ **80** (> 0) Hz, and $r =$ **4.25**.

48 Hz aliases to 32 Hz when $f_s = 80$ Hz

$$\Rightarrow y(t) = 2r\cos(2\pi(32)t + 0.3\pi) + 2r\cos(2\pi(48 - 80)t + 0.3\pi)$$

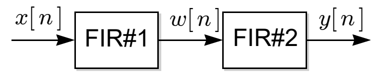
$$= 2r\cos(2\pi(32)t + 0.3\pi) + 2r\cos(2\pi(32)t - 0.3\pi)$$

$$= \underbrace{4r\cos(0.3\pi)}_{10}\cos(64\pi t)$$

phasor addition: $2re^{j0.3\pi} + 2re^{-j0.3\pi} = 4r\cos(0.3\pi)$

$$\Rightarrow r = \frac{10}{4\cos(0.3\pi)} \approx 4.25$$

PROB. Sp23-Q2.3. As shown in the figure, suppose that two FIR filters are connected in cascade, so that an input $x[n]$ to the first system produces an intermediate signal $w[n]$, which becomes the input to the second, producing an overall output $y[n]$, where:



- FIR#1 has impulse response $h_1[n] = 3\delta[n] + \beta\delta[n-1]$,
- FIR#2 has impulse response $h_2[n] = \delta[n] - \delta[n-1]$.

The constant β is different in each part below.

(a) If $\beta = 15$ and $x[n] = \frac{1}{1+n^2}$ then $w[3] =$ **3.3**.

$= 3x[3] + 15x[2]$

$= \frac{3}{1+3^2} + \frac{15}{1+2^2} = 3.3$

(b) If $\beta = 3$, the difference equation relating the overall output to input is $y[n] = \sum_k b_k x[n-k]$, where:

b_0	b_1	b_2	b_3	b_4
3	0	−3	0	0

3	β	
	−3	−β
3	β−3	−β

(c) If $\beta = 6$ and $x[n] = nu[n]$ (*i.e.*, $x[n] = 0$ for $n < 0$, and $x[n] = n$ for $n \geq 0$), specify numerical values for the following outputs:

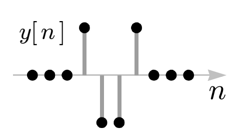
$y[0]$	$y[1]$	$y[2]$	$y[3]$	$y[4]$
0	3	9	9	9

3	6	
	−3	−6
3	3	−6

0 0 0 0 1 2 3 4

$x[0]$ $x[1]$ $x[2]$ $x[3]$ $x[4]$

(d) If $x[n] = \delta[n] + \delta[n-1]$ results in an output stem plot with the following shape, then it must be that



$\beta =$ **−3**.

3	β−3	−β	
	3	β−3	−β
3	β	−3	−β

$\Rightarrow y[n]$ matches when $\beta = -3$