GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 Spring 2022
Quiz #2

March 11, 2022

NAME: ______________ ________________ GT username: ________________
(FIRST) (LAST) (e.g., gtxyz123)

To earn 2 points, circle your recitation section:

L01 (Tai)	L07 (Tai)	L09 (Hessler)	L11 (Hessler)
L02 (Duan)	L08 (Sadiq)	L10 (Sadiq)	L12 (Duan)

Important Notes:

- Do not unstaple the test.
- One two-sided page (8.5” × 11”) of hand-written notes permitted.
- Calculators are allowed, but no smartphones/tablets/readers/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. For example, write 0.1π as opposed to 18° or 0.3142 radians.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Write your answers in the provided answer boxes.
  If more space is needed for scratch work, use the backs of the previous pages.

Problem	Value	Score
1	32	
2	30	
3	36	
RECITATION	2	
Total		

PROB. Sp22-Q2.1. (32 points, 1 point for each answer in part (b), 3 points for each remaining answer)

Let $x(t)$ be the following sum of one sinusoid plus the *square* of another:

$$x(t) = 2\cos(72\pi t + \varphi) + 4\cos^2(2\pi f_c t),$$

where the φ and f_c parameters are unspecified. They can be different in each part below.

(a) TRUE ☐ FALSE ☐ There exist values of f_c for which $x(t)$ is *not* periodic.

(b) When $\varphi = 0$ and $f_c = 12$ Hz, we can write this signal as $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi f_0 t}$, where:

$a_0 =$ ☐ , $a_1 =$ ☐ , $a_2 =$ ☐ , $a_3 =$ ☐ , $a_4 =$ ☐ .

(c) Specify an $f_c > 0$ and a $\varphi \in (-\pi, \pi]$ so that $x(t)$ is a *constant* (independent of time t):

$f_c =$ ☐ Hz and $\varphi =$ ☐ .

(d) Assume $\varphi = 0$. Find the fundamental frequency of $x(t)$ for each value of the f_c parameter listed below:
(*Hint:* Restrict your answers to the set $\{2, 4, 6, 12, 18, 36\}$, each of these answers should appear once in the boxes below!)

(i) $f_c = 18$ Hz $\Rightarrow$ $f_0 =$ ☐ Hz.

(ii) $f_c = 21$ Hz $\Rightarrow$ $f_0 =$ ☐ Hz.

(iii) $f_c = 22$ Hz $\Rightarrow$ $f_0 =$ ☐ Hz.

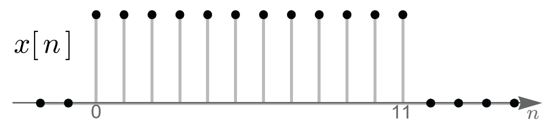
(iv) $f_c = 24$ Hz $\Rightarrow$ $f_0 =$ ☐ Hz.

(v) $f_c = 27$ Hz $\Rightarrow$ $f_0 =$ ☐ Hz.

(vi) $f_c = 31$ Hz $\Rightarrow$ $f_0 =$ ☐ Hz.

PROB. Sp22-Q2.2. (30 points, 5 points per answer)

Suppose that the rectangular sequence $x[n] = u[n] - u[n-12]$ shown here:



is the input to an FIR filter whose difference equation is

$$y[n] = x[n] + b_1 x[n-1] + x[n-2].$$

Shown below are six different filter outputs $y[n]$, labeled K through P, that result from six different values of the filter coefficient b_1. (The time axis is labeled and identical for all six plots, but the y-axis scales are unlabeled and could be different for each plot.) Match each value for b_1 below to the corresponding output $y[n]$. Indicate answers by writing a letter (from {K, ...P}) into each answer box.

(i) $b_1 = -3$ [ ].

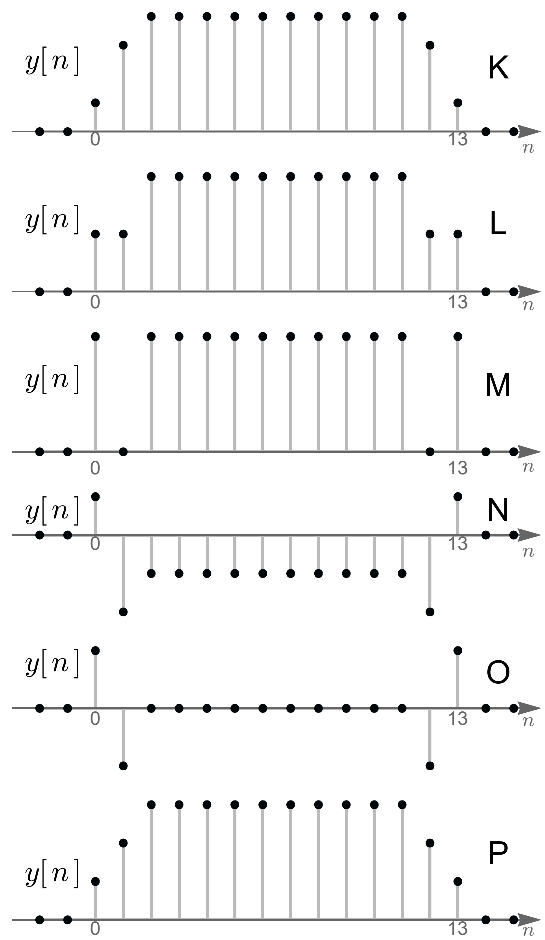
(ii) $b_1 = -2$ [ ].

(iii) $b_1 = -1$ [ ].

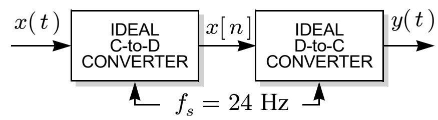
(iv) $b_1 = 0$ [ ].

(v) $b_1 = 1$ [ ].

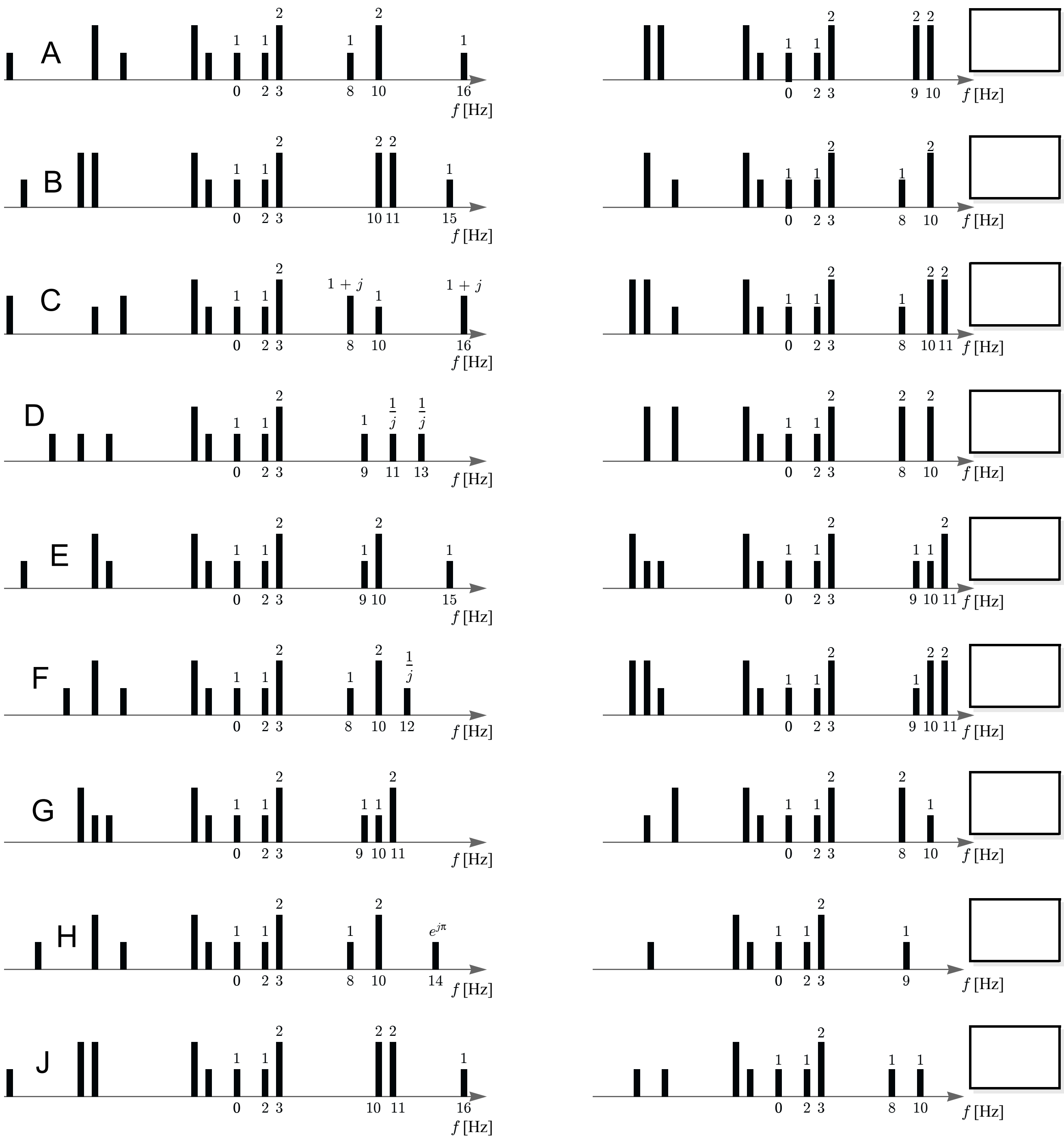
(vi) $b_1 = 2$ [ ].



PROB. Sp22-Q2.3. (36 points, 4 points per answer)



Shown below on the *left* are nine different spectra (labeled A through J) for the input $x(t)$ to the pictured C-to-D converter. Shown below on the *right* are the nine different spectra for the output $y(t)$ of the pictured D-to-C converter, but in a scrambled order. Match the spectrum for $x(t)$ to the corresponding spectrum for $y(t)$. Indicate your answers by writing a letter (from $\{A, ... J\}$) into each answer box. In all cases the sample rate for both the C-D and D-C is $f_s = 24$ Hz.



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To earn 2 points, circle your recitation section:

L01 (Tai)	L07 (Tai)	L09 (Hessler)	L11 (Hessler)
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PROB. Sp22-Q2.1. (32 points, 1 point for each answer in part (b), 3 points for each remaining answer)

Let $x(t)$ be the following sum of one sinusoid plus the *square* of another:

$$x(t) = 2\cos(72\pi t + \varphi) + 4\cos^2(2\pi f_c t)$$
$$= 2\cos(72\pi t + \varphi) + 2 + 2\cos(4\pi f_c t)$$

where the φ and f_c parameters are unspecified. They can be different in each part below.

(a) TRUE [X] FALSE [ ] There exist values of f_c for which $x(t)$ is *not* periodic.

Not periodic when f_c is *irrational*

(b) When $\varphi = 0$ and $f_c = 12$ Hz, we can write this signal as $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi f_0 t}$, where:

$a_0 =$ [2], $a_1 =$ [0], $a_2 =$ [1], $a_3 =$ [1], $a_4 =$ [0].

$$x(t) = 2\cos(72\pi t) + 2 + 2\cos(48\pi t) \quad \Rightarrow \quad f_0 = \gcd(36, 24) = 12 \text{ Hz}$$

$$\text{Euler} \Rightarrow x(t) = e^{j72\pi t} + e^{-j72\pi t} + 2 + e^{j48\pi t} + e^{-j48\pi t}$$
$$= a_3 e^{j2\pi(3)f_0 t} + a_{-3} e^{j2\pi(-3)f_0 t} + a_0 + a_2 e^{j2\pi(2)f_0 t} + a_{-2} e^{j2\pi(-2)f_0 t}$$

(c) Specify an $f_c > 0$ and a $\varphi \in (-\pi, \pi]$ so that $x(t)$ is a *constant* (independent of time t):

$f_c =$ [18] Hz and $\varphi =$ [π].

(d) Assume $\varphi = 0$. Find the fundamental frequency of $x(t)$ for each value of the f_c parameter listed below:
(*Hint:* Restrict your answers to the set {2, 4, 6, 12, 18, 36}, each of these answers should appear once in the boxes below!)

(i) $f_c = 18$ Hz $\Rightarrow$ $f_0 =$ [36] Hz.

(ii) $f_c = 21$ Hz $\Rightarrow$ $f_0 =$ [6] Hz.

(iii) $f_c = 22$ Hz $\Rightarrow$ $f_0 =$ [4] Hz.

(iv) $f_c = 24$ Hz $\Rightarrow$ $f_0 =$ [12] Hz.

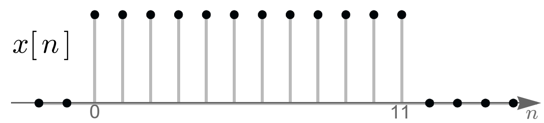
(v) $f_c = 27$ Hz $\Rightarrow$ $f_0 =$ [18] Hz.

(vi) $f_c = 31$ Hz $\Rightarrow$ $f_0 =$ [2] Hz.

In all cases: $f_0 = \gcd(36, 2f_c)$

PROB. Sp22-Q2.2. (30 points, 5 points per answer)

Suppose that the rectangular sequence $x[n] = u[n] - u[n-12]$ shown here:



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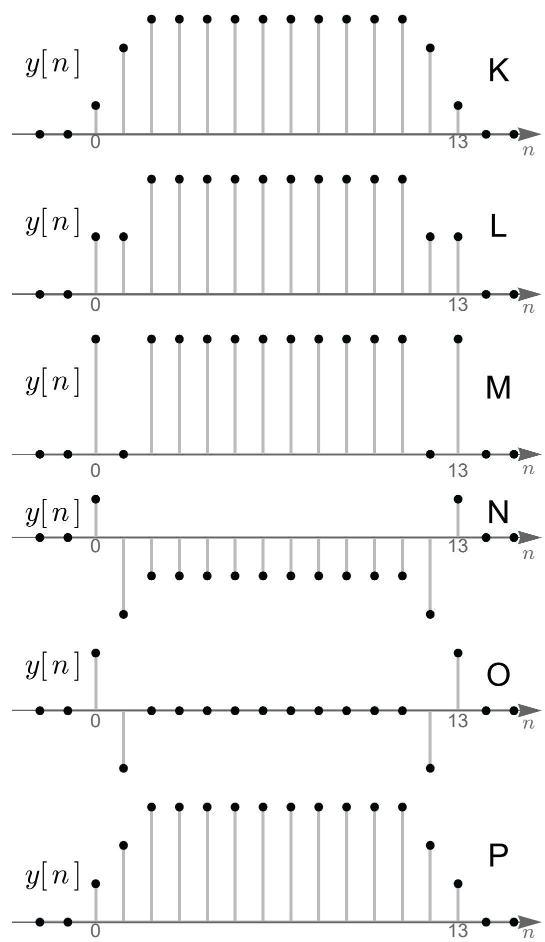
		Answer	$y[0]$	$y[1]$	$y[2]$
(i)	$b_1 = -3$	N	1	−2	−1
(ii)	$b_1 = -2$	O	1	−1	0
(iii)	$b_1 = -1$	M	1	0	1
(iv)	$b_1 = 0$	L	1	1	2
(v)	$b_1 = 1$	P	1	2	3
(vi)	$b_1 = 2$	K	1	3	4

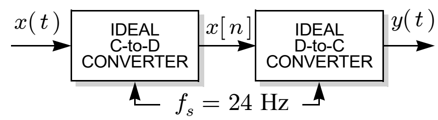
$y[0] = x[0] = 1$

$y[1] = x[1] + b_1 x[0] = 1 + b_1$

$y[2] = x[2] + b_1 x[1] + x[0] = 2 + b_1$



PROB. Sp22-Q2.3. (36 points, 4 points per answer)



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