GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 5-Mar-18 COURSE: ECE-2025

NAME: Solutions GT ID: ____________

LAST, FIRST (ex: buzz1a)

3 points 3 points 3 points

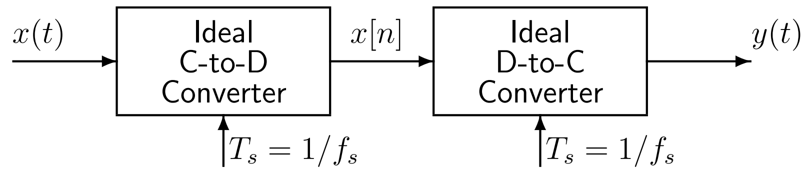
Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

			L06:Thur-Noon (Fekri)
			L08:Thurs-1:30pm (Fekri)
L01:M-3pm (Valenta)	L09:Tues-3pm (Rohling)	L02:W-3pm (Yang)	L10:Thur-3pm (Marenco)
L03:M-4:30pm (Valenta)	L11:Tues-4:30pm (Rohling)	L04:W-4:30pm (Yang)	L12:Thur-4:30pm (Marenco)

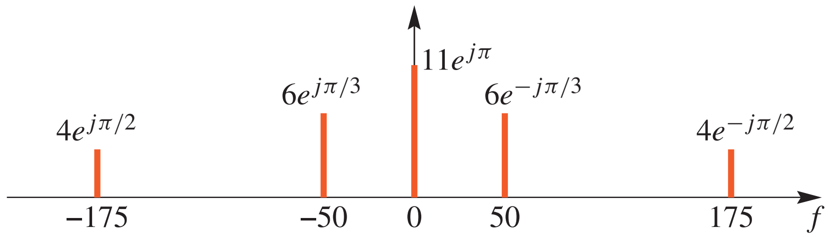
- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	36	
2	32	
3	32	
No/Wrong Rec	-3	

PROBLEM sp-18-Q.2.1:



In all parts below, the sampling rates of the C/D and D/C converters are **equal,** and the input to the Ideal C/D converter is a signal $x(t)$ whose spectrum is shown below, where the **frequency** f **is in hertz**.



(a) (6 pts) Determine the Nyquist rate (in hertz) for sampling $x(t)$. $f_{\text{Nyquist}} = \boxed{350}$

(b) (12 pts) If the sampling rate is $f_s = 150$ samples/sec., determine the discrete-time signal $x[n]$, and give an expression for $x[n]$ as a sum of cosines. *Make sure that all frequencies in your answer are positive and less than π radians.*

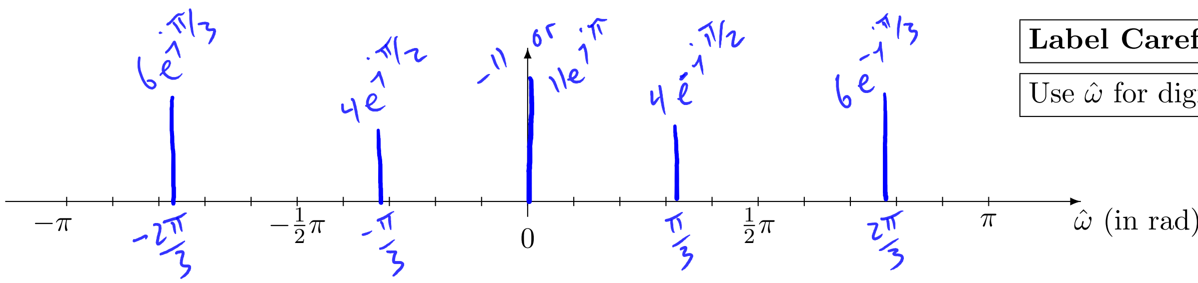
$$\boxed{x[n] = -11 + 12\cos\left(\frac{2\pi}{3}n - \frac{\pi}{3}\right) + 8\cos\left(\frac{\pi}{3}n - \frac{\pi}{2}\right)}$$

$$\hat{\omega} = 2\pi \frac{f}{f_s} \qquad \hat{\omega}_1 = \frac{50}{150} \cdot 2\pi = \frac{2\pi}{3}$$

$$\hat{\omega}_2 = \frac{175}{150} \cdot 2\pi = \frac{7\pi}{3} \Rightarrow \frac{7\pi}{3} - 2\pi = \frac{\pi}{3}$$

aliasing

(10 pts) Plot the spectrum of this signal over the range of frequencies $-\pi \leq \hat{\omega} \leq \pi$. Make sure to label the frequency, amplitude and phase of each spectral component.



Label Carefully

Use $\hat{\omega}$ for digital freq.

(c) (8 pts) If the sampling rate is $f_s = 175$, list ***all*** non-negative frequencies that will be present in the spectrum of the output signal, $y(t)$.

$f = 0$ Hz (DC) and $f = 50$ Hz

175 Hz aliases to 0 — no aliasing

PROBLEM sp-18-Q.2.2:

A signal $x(t)$ is periodic with period $T_0 = 9$ seconds. Therefore it can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/9)kt}$$

It is known that the Fourier series coefficients for this representation of a particular signal $x(t)$ are given by the integral

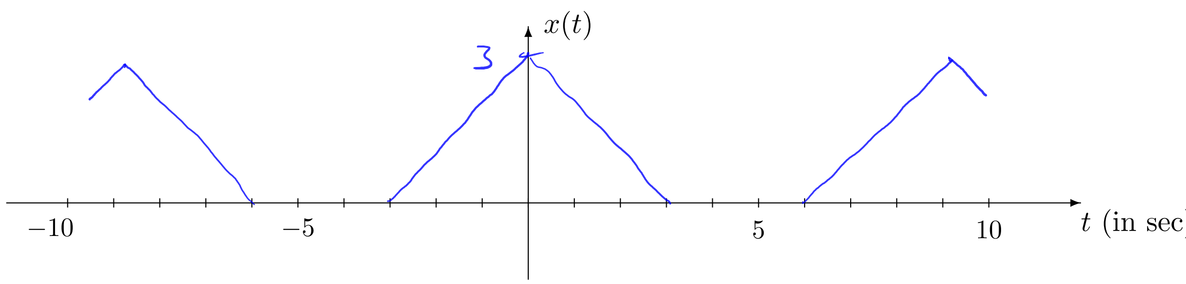
$$a_k = \frac{1}{9} \int_{-3}^{3} (3 - |t|)\, e^{-j(2\pi/9)kt} dt \tag{1}$$

NOTE: Parts (c) and (d) of this problem can be worked independently of parts (a) and (b).

(a) (8 pts) In the expression for a_k in Eq. (1) above, the integral and its limits define the signal $x(t)$. Write an equation for $x(t)$ that is valid over one period.

$$x(t) = \begin{cases} 3 - |t| & -3 \le t \le 3 \\ 0 & 3 < t < 6 \end{cases}$$

(b) (8 pts) Using your result from part (a), draw a plot of $x(t)$ over the range $-10 \le t \le 10$ seconds. Label it carefully.



(c) (8 pts) Which value of k in Eq. (1) gives the DC (or average) value of $x(t)$? $\boxed{k = 0}$

(d) (8 pts) Determine the DC value of $x(t)$. Give your answer as a number. $\boxed{DC = 1}$

$$\text{Area} = \tfrac{1}{2}(6 \cdot 3) = 9$$

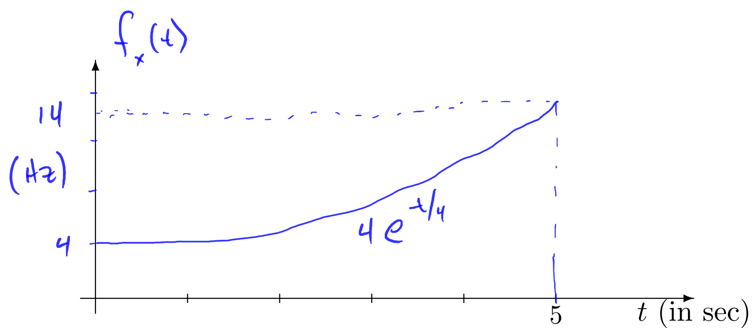
$$DC = \tfrac{1}{9}(\text{Area}) = 1$$

PROBLEM sp-18-Q.2.3:

(a) (16 pts) For the FM signal $x(t)$ defined as:

$$x(t) = \Re e \left\{ e^{j(Ce^{\lambda t})} \right\}$$

we will denote its instantaneous frequency (in Hz) as $f_x(t)$. (Pay attention to the equation for $x(t)$; there is a exponential in the exponent.) For $C = 32\pi$ and $\lambda = 0.25$, make a *carefully labeled* plot of the (positive) instantaneous frequency $f_x(t)$ over the time interval $0 \le t \le 5$ secs. *Note: the frequency should be in hertz (Hz).*



$x(t) = \cos(Ce^{\lambda t})$

$f_x(t) = \frac{1}{2\pi} \frac{\partial}{\partial t} (Ce^{\lambda t}) = \frac{C\lambda}{2\pi} e^{\lambda t}$

for $C = 32\pi$, $\lambda = 1/4$

$f_x(t) = 4e^{t/4}$

(b) (16 pts) Determine C and λ so that the instantaneous frequency of $x(t)$ is equal to 1000 Hz at $t = 0$ and 200 Hz at $t = 2$ secs.

at $t=0$

$\frac{C\lambda}{2\pi} = 1000$

at $t=2$

$\frac{C\lambda}{2\pi} e^{2\lambda} = 200$

$1000 e^{2\lambda} = 200$

$\boxed{\lambda = \frac{1}{2} \log 0.2 = -0.8047}$

$C = \frac{2\pi \cdot 1000}{\lambda}$

$\boxed{C = \frac{4000\pi}{\log 0.2} = -7808}$