GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #2 | SOLUTION Version A |

DATE: 07-MAR-16 | SOLUTION Version A | COURSE: ECE 2026A,B

NAME: _____   STUDENT #: _____

LAST,   FIRST

| 2 points | | 2 points | | 2 points |

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L00:Tues-9:30am (Yang)   L01:Mon-3:00pm (Odom)

L03:Mon-4:30pm (Odom)   L05:Tues-12:00pm (Yang)

L06:Thur-12:00pm (Zajic)   L07:Tues-1:30pm (Stüber)

L08:Thur-1:30pm (Zajic)   L09:Tues-3:00pm (Stüber)

L10:Thur-3:00pm (Yeredor)   L12:Thur-4:30pm (Yeredor)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.

- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.

- Unless stated otherwise, JUSTIFY your reasoning clearly to receive any partial credit. Explanations are also required to receive full credit for any answer.

- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	Value	Score
1	35	
2	35	
3	28	
Rec	2	
Total	100	

NOTE: **THERE ARE NO INTEGRALS THAT NEED TO BE COMPUTED ANYWHERE ON THIS QUIZ. Nothing requires tedious calculations, either.**
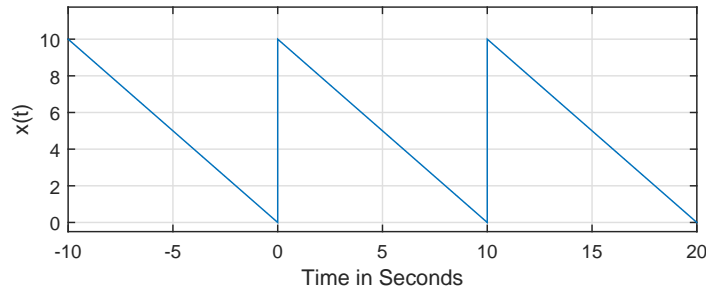
Problem Q2.1:

(a) (7 pts) Suppose the periodic signal $x(t)$ is given by

$$x(t) = \sum_{k=-\infty}^{\infty} \left\{ \frac{1}{10} \int_0^{10} (10 - u) \exp[-j(0.2)\pi ku] du \right\} \exp[j(0.2)\pi kt]$$

Sketch $x(t)$ for t between -10 and 20. Explain your reasoning. (No integration is required).

Answer: Just look at the integrand. $x(t) = 10 - t$ over $[0, 10)$ with period 10.

(b) (7 pts) Find the nonzero Fourier coefficients a_k of

$$x(t) = 4 - 2\cos(300\pi t + \pi/9) + \sin(700\pi t)$$

Be sure the values of k are specified explicitly.

Answer: $\omega_0 = 100\pi$. $a_0 = 4$; $a_3 = e^{-j8\pi/9}$; $a_{-3} = e^{j8\pi/9}$; $a_7 = 0.5e^{-j\pi/2}$; $a_{-7} = 0.5e^{j\pi/2}$.

For parts (c), (d), and (e), consider a periodic signal $x(t)$ with a fundamental period 0.01 seconds where $x(t)$ for $0 \le t \le 0.01$, is given by

$$x(t) = \begin{cases} 1 & \text{for } 0 \le t < 0.005 \\ 0 & \text{for } 0.005 \le t < 0.01 \end{cases}$$

The Fourier series coefficients were computed and found to be a_k.

(c) (7 pts) Let $v(t)$ have the same FS coefficients as $x(t)$ except for index $k = 0$; i.e., $b_k = a_k$ for $k \neq 0$ and $b_0 = 1.0$, where b_k represents the kth FS coefficient of $v(t)$. Find and sketch $v(t)$ over one period.

Answer: The original DC value is $1/2$. To make it 1, you need to add $1/2$ to $x(t)$.
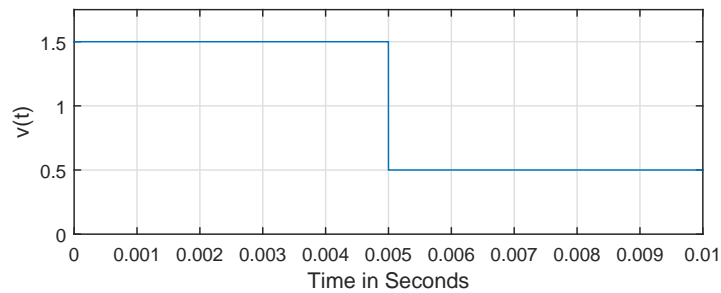
(d) (7 pts) Let $w(t)$ have FS coefficients c_k where $c_k = a_k(j)^k$. Find and sketch $w(t)$ over one period.

Answer: Multiplying by j^k is the same as multiplying by $e^{jk\pi/2}$ which is an advance by $1/4$ of a period.
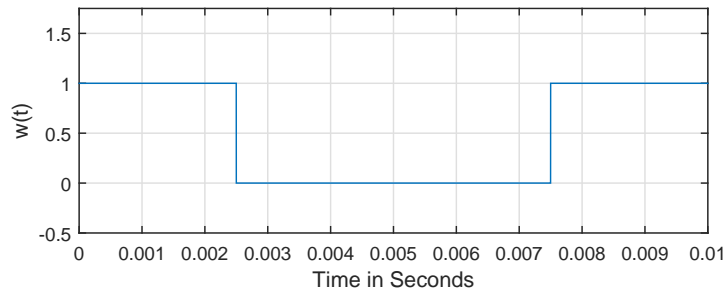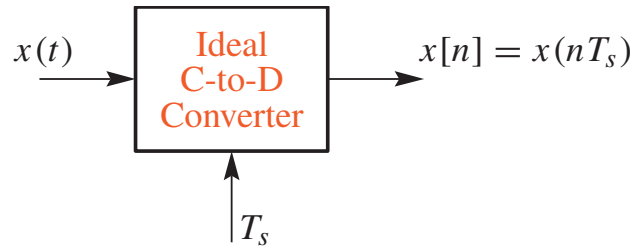

(e) (7 pts) Let $z(t)$ be a time-stretched version of $x(t)$ where the stretching factor is 10, i.e., $z(t) = x(t/10)$. Specify d_k, the FS coefficients of $z(t)$ in terms of a_k.

Answer: Time scaling by a positive value only affects the fundamental frequency, not the FS coefficients. Therefore $d_k = a_k$.

Problem Q2.2:

Consider the ideal sampling system shown below.

(a) (7 pts) **Fill in the blank** with the largest possible number that makes this a true statemet:

A radar system contains C/D converter cards with sampling rate 105 *megasamples*/sec. An engineer can safely input frequencies up to (but not including) _____ megahertz without aliasing.

Answer: According to the sampling theorem, you can have frequencies up to, but not including, half the sampling rate $= 52.5$ megahertz.

(b) (7 pts) Suppose $x[n]$ is the output of the continuous-to-discrete converter operating at a sampling rate of $f_s = 2000$ samples/sec. Let $x[n] = 8\cos(0.25\pi n + 5\pi/7)$. Specify **three** possible input signals of the typical cosine form $x(t) = A\cos(\omega t + \phi)$ that could have given that output. **Restrict your frequencies** ω to be in the range -7000π rad/sec $< \omega < 7000\pi$ rad/sec.

Answer: $0.25\pi = \pi/4$ corresponds to $1/8$ the sampling rate. The signal that has no aliasing would be at $2000/8 = 250$ Hz. We would also have an aliased signal at $250 + f_s = 2250$ Hz, plus a folded signal at $|250 - f_s| = 1750$ Hz. Folding inverts the phase.

(**1**) $8\cos(2\pi 250t + 5\pi/7)$

(**2**) $8\cos(2\pi 2250t + 5\pi/7)$

(**3**) $8\cos(2\pi 1750t - 5\pi/7)$

(c) (7 pts) Consider the square wave from the previous problem.

Does there exist a minimum f_s for which the square wave can be reconstructed without aliasing? If there is, give the f_s and explain how you obtained it. If no such f_s exists, explain why.

Answer: The square wave has components that go to ∞ in frequency. Therefore, no such sampling frequency exists.

(d) (7 pts) Consider the following piece of MATLAB code:

```
tt = 1:(1/8000):3;
xx = cos(2*pi*(100 tt^2 + 50*tt));
soundsc(xx,fs_playback);
```

What choice of `fs_playback` would cause a 12 second long sound to be played?

Answer: Six times the duration would mean scaling the sampling rate for playback by 1/6, which implies `fs_playback` should be $8000/6 = 1333.33$.

(e) (7 pts) Suppose we make a digital recording using a sampling rate of 48 kHz of a musician playing a particular note, which has a frequency of 440 Hz, on a clarinet. We can make it sound like other notes by changing the playback sampling rate, which will change the frequency. What playback sampling rate should we use to make it sound like a 330 Hz tone?

Answer: We are slowing the signal down by $330/440$. Therefore `fs_playback` should be $48,000(330/440)=36,000$.

Problem Q2.3:

(a) (7 pts) Suppose $x(t) = 6\cos(5\pi t + \pi/3)\cos(5\pi t + \phi)$. Determine *all* possible ϕ, $-\pi \le \phi < \pi$ for which the D.C. component of $x(t)$ is zero.

Answer: We get zero DC when the two sinusoids are $\pi/2$ out of phase (easy to show). There are two values for ϕ: $\pi/3 + \pi/2 = 5\pi/6$ and $\pi/3 - \pi/2 = -\pi/6$.

(b) (7 pts) Suppose $x(t) = 15\cos(400[\sin(20\pi t) + 20\pi t] + 3.92)$. Find $f_i(t)$, the instantaneous frequency of $x(t)$ in Hertz.

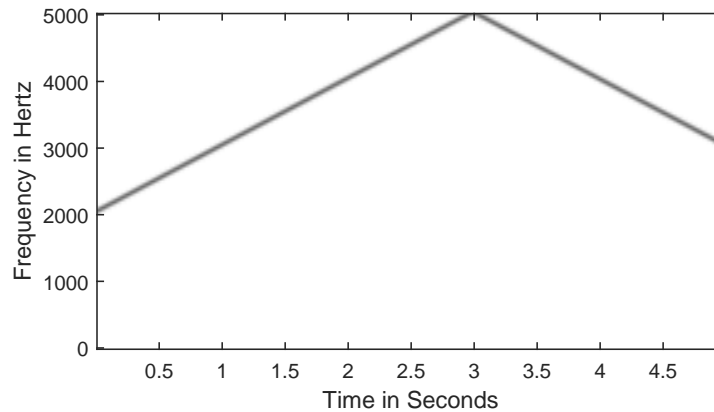**Answer:** We just need to find the derivative of the argument:
$\omega_{inst} = 8000\pi\cos(20\pi t) + 8000\pi$ rads/sec
$f_i(t) = 4000\cos(20\pi t) + 4000$ Hz

(c) (9 pts) The following piece of MATLAB code

```
fs = houdini;
tt = 0:(1/fs):5;
xx = 3.2 * cos(2*pi*(penn * tt .^ 2 + teller * tt) + 2.413);
plotspec(xx,fs);
ylabel('Frequency in Hertz'); xlabel('Time in Seconds')
```

created this plot

Find the values of the variables houdini, penn and teller.
Answer: houdini=10000; penn=500; teller = 2000.

(d) (5 pts) Suppose line 4 of the code above is changed to:

```
plotspec(xx+.000001*j,fs);
```

Assuming nothing else is changed, sketch the resulting plot.

Answer: It adds a mirror image.