GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 03-MAR-14 COURSE: ECE 2026A,B

NAME: ______________________________ STUDENT #: ______________

LAST, FIRST

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L01:Mon-3:00pm (Causey)	L02:Wed-3:00pm (Romberg)
L03:Mon-4:30pm (Causey)	L04:Wed-4:30pm (Romberg)
L05:Tues-Noon (Fekri)	L06:Thur-Noon (Chang)
L07:Tues-1:30pm (Fekri)	L08:Thur-1:30pm (Stüber)
L10:Thur-3:00pm (Stüber)	L12:Thur-4:30pm (Chang)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Unless stated otherwise, JUSTIFY your reasoning clearly to receive any partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	33	
2	32	
3	32	
Rec	3	
Total	100	

Problem Q2.1:

(a) (8 points) Sketch the spectrum of $\Re e\{3e^{j7\pi(t-1/5)}\}$ in radians/sec.

(b) (8 points) Express $x(t) = \cos(1028\pi t)\cos(256\pi t + \pi/8)$ as a sum of two cosines.

$$x(t) = A_1 \cos(\omega_1 t + \phi_1) + A_2 \cos(\omega_2 t + \phi_2)$$

$A_1 =$ __________. $\phi_1 =$_____________. $\omega_1 =$______________. (in radians/sec)

$A_2 =$ __________. $\phi_2 =$ _____________. $\omega_2 =$______________. (in radians/sec)

(c) (9 points) Suppose $\psi(t) = 120\pi t - \cos(2\pi 30t)$ and $x(t) = 2\cos(\psi(t) + 0.35)$.

(6)(i) Find $f_{xi}(t)$, the instantaneous frequency of $x(t)$ in Hz.

(3)(ii) Find the fundamental period of $f_{xi}(t)$.

(d) (8 points) $y(t)$ has instantaneous frequency $f_{yi}(t) = 3t^2 + 5t$ in Hz. Express $y(t)$ as $\cos(\zeta(t))$ where $\zeta(t)$ is to be determined.

Problem Q2.2:

(a) (8 pts) Suppose $x(t)$ is given by

$$x(t) = \sum_{k=-\infty}^{\infty} \left\{ \frac{1}{10} \int_0^{10} (u) \exp[-j(0.2)\pi k u] du \right\} \exp[j(0.2)\pi k t]$$

Sketch $x(t)$ for t between -10 and 20. Explain your reasoning. (Hint: Do not try to integrate; the form of the equation should be enough.)

(b) (8 pts) Find the nonzero Fourier series coefficients a_k of

$$x(t) = -3 + 2\cos(300\pi t + \pi/9) + 3\sin(700\pi t)$$

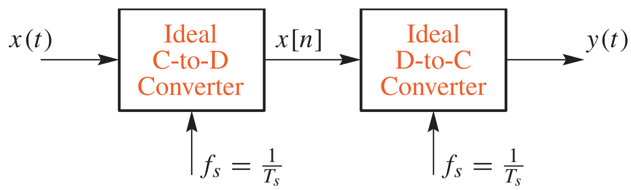
(c) A real signal, $w(t)$, has fundamental period 0.001. We also know some of its Fourier series coefficients: $a_0 = 1.5$, $a_1 = 2e^{j\pi/2}$, and $a_5 = 3e^{-j\pi/5}$.

(i) (8 points) Find a_{-1} and a_{-5}.

(ii) (8 points) Assuming all other FS coeffients are zero, $w(t)$ can be expressed as a sum of cosines, each with specific amplitude, phase, and frequency. Specify this sum.

Problem Q2.3:

Consider the ideal sampling and reconstruction system shown below for parts (a) and (b).



(a) (8 pts) Suppose that the discrete-time signal $x[n]$ in the above figure is given by the formula

$$x[n] = 5\cos(0.2\pi n - \pi/4)$$

If the sampling rate of the C-to-D converter is $f_s = 8000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs **with frequency between 0 and 8000 Hz**; i.e., find $x_1(t) = A_1\cos(\omega_1 t + \phi_1)$ and $x_2(t) = A_2\cos(\omega_2 t + \phi_2)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 1/8000$ secs.

(b) (8 pts) In this part, for the system depicted in the figure, we do not know f_s, but we can measure $x(t)$ and $y(t)$. We discover that when $x(t) = \cos(2200\pi t - \pi/3)$, $y(t) = \cos(1800\pi t + \pi/3)$. Find a possible value for f_s. Explain if it is unique or not.

(c) (8 pts) Consider the following piece of MATLAB code:

```
tt = 0:(1/8000):3;
xx = cos(2*pi*(100 tt^2 + 50 tt));
soundsc(xx,fs_playback);
```

What choice of `fs_playback` would cause a 12 second long sound to be played?

(d) (8 pts) Suppose we make a digital recording using a sampling rate of 48,000 Hz of a pure tone with a frequency of 400 Hz. We can make it sound like other notes by changing the playback sampling rate, which will change the frequency. What playback sampling rate should we use to make it sound like a 300 Hz tone?

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QUIZ #2 SOLUTION Version A

DATE: 03-MAR-14 COURSE: ECE 2026A,B

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LAST, FIRST

3 points 3 points 3 points

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Problem	*Value*	*Score*
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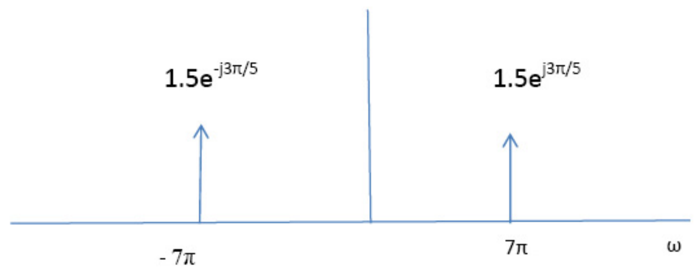
3	32	
Rec	3	
Total	100	

Problem Q2.1:

(a) (8 points) Sketch the spectrum of $\Re e\{3e^{j7\pi(t-1/5)}\}$ in radians/sec.

$3\cos(7\pi t - 7\pi/5) = 3\cos(7\pi t + 3\pi/5) = \frac{3}{2}e^{j\frac{3\pi}{5}}e^{j7\pi t} + \frac{3}{2}e^{-j\frac{3\pi}{5}}e^{-j7\pi t}$



(b) (8 points) Express $x(t) = \cos(1028\pi t)\cos(256\pi t + \pi/8)$ as a sum of two cosines.

$$x(t) = A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2)$$

$A_1 = \underline{\frac{1}{2}}$. $\phi_1 = \underline{\frac{\pi}{8}}$. $\omega_1 = \underline{1284\pi}$. (in radians/sec)

$A_2 = \underline{\frac{1}{2}}$. $\phi_2 = \underline{-\frac{\pi}{8}}$. $\omega_2 = \underline{772\pi}$. (in radians/sec)

$\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$

(c) (9 points) Suppose $\psi(t) = 120\pi t - \cos(2\pi 30t)$ and $x(t) = 2\cos(\psi(t) + 0.35)$.

(6)(i) Find $f_{xi}(t)$, the instantaneous frequency of $x(t)$ in Hz.

$\frac{d\psi(t)}{dt} = \omega_{xi}(t) = 120\pi + 60\pi\sin(60\pi t)$
$f_{xi}(t) = 60 + 30\sin(60\pi t)$

---

(3)(ii) Find the fundamental period of $f_{xi}(t)$.

This is a DC plus a sine wave of frequency 30 Hz. The period is $\frac{2\pi}{60\pi} = \frac{1}{30}$

(d) (8 points) $y(t)$ has instantaneous frequency $f_{yi}(t) = 3t^2 + 5t$ in Hz. Express $y(t)$ as $\cos(\zeta(t))$ where $\zeta(t)$ is to be determined.

$\omega_{yi}(t) = 6\pi t^2 + 10\pi t$
$\zeta(t) = 2\pi t^3 + 5\pi t^2$

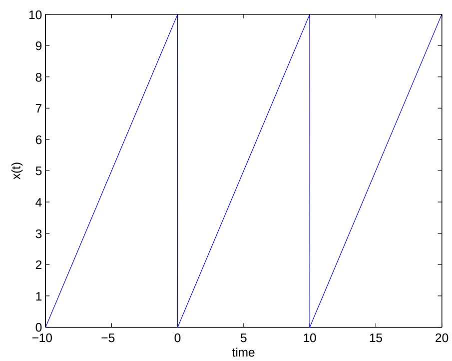
Problem Q2.2:

(a) (8 pts) Suppose $x(t)$ is given by

$$x(t) = \sum_{k=-\infty}^{\infty} \left\{ \frac{1}{10} \int_0^{10} (u) \exp[-j(0.2)\pi k u] du \right\} \exp[j(0.2)\pi k t]$$

Sketch $x(t)$ for t between -10 and 20. Explain your reasoning. (Hint: Do not try to integrate; the form of the equation should be enough.)

The term in the braces must be a_k. Therefore $x(t) = t$ over $[0, 10)$ and repeating every 10.



(b) (8 pts) Find the nonzero Fourier series coefficients a_k of

$$x(t) = -3 + 2\cos(300\pi t + \pi/9) + 3\sin(700\pi t)$$

$\omega_0 = 100\pi$ (LCD). We therefore have DC and the 3rd and 7th harmonics. $a_0 = 3e^{j\pi}$; $a_3 = e^{j\pi/9}$; $a_7 = \frac{3}{2}e^{-j\pi/2}$; $a_{-3} = e^{-j\pi/9}$; $a_{-7} = \frac{3}{2}e^{j\pi/2}$

(c) A real signal, $w(t)$, has fundamental period 0.001. We also know some of its Fourier series coefficients: $a_0 = 1.5$, $a_1 = 2e^{j\pi/2}$, and $a_5 = 3e^{-j\pi/5}$.

(i) (8 points) Find a_{-1} and a_{-5}.

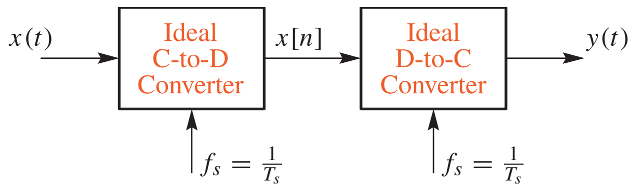
$a_{-1} = a_1^* = 2e^{-j\pi/2}$
$a_{-5} = a_5^* = 3e^{j\pi/5}$

(ii) (8 points) Assuming all other FS coeffients are zero, $w(t)$ can be expressed as a sum of cosines, each with specific amplitude, phase, and frequency. Specify this sum.

$w(t) = 1.5\cos(0t) + 4\cos(2000\pi t + \pi/2) + 6\cos(10000\pi t - \pi/5)$

Problem Q2.3:

Consider the ideal sampling and reconstruction system shown below for parts (a) and (b).



(a) (8 pts) Suppose that the discrete-time signal $x[n]$ in the above figure is given by the formula

$$x[n] = 5\cos(0.2\pi n - \pi/4)$$

If the sampling rate of the C-to-D converter is $f_s = 8000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs **with frequency between 0 and 8000 Hz**; i.e., find $x_1(t) = A_1\cos(\omega_1 t + \phi_1)$ and $x_2(t) = A_2\cos(\omega_2 t + \phi_2)$ such that $x[n] = x_1(nT_\mathrm{s}) = x_2(nT_\mathrm{s})$ if $T_\mathrm{s} = 1/8000$ secs.

$x_1(t)$ is the one without aliasing: $x_1(t) = 5\cos(1600\pi t - \pi/4)$
$x_2(t)$ is above $f_s/2$ but below f_s implying folding: $x_2(t) = 5\cos(14400\pi t + \pi/4)$

(b) (8 pts) In this part, for the system depicted in the figure, we do not know f_s, but we can measure $x(t)$ and $y(t)$. We discover that when $x(t) = \cos(2200\pi t - \pi/3)$, $y(t) = \cos(1800\pi t + \pi/3)$. Find a possible value for f_s. Explain if it is unique or not.

Phase reversal means it was a fold. Therefore $f_s = 2000$ Hz. Yes it is unique.

(c) (8 pts) Consider the following piece of MATLAB code:

```
tt = 0:(1/8000):3;
xx = cos(2*pi*(100 tt^2 + 50 tt));
soundsc(xx,fs_playback);
```

What choice of `fs_playback` would cause a 12 second long sound to be played?

Stretch by a factor of 4 $\rightarrow$ reduce sampling rate by a factor of 4. `fs_playback` $= 2000$.

(d) (8 pts) Suppose we make a digital recording using a sampling rate of 48,000 Hz of a pure tone with a frequency of 400 Hz. We can make it sound like other notes by changing the playback sampling rate, which will change the frequency. What playback sampling rate should we use to make it sound like a 300 Hz tone?

If we want the frequency to be reduced by a factor of 3/4, we must make the playback sampling frequency be reduced by a factor of 3/4. Therefore, playback sampling frequency = 36,000.
