**GEORGIA INSTITUTE OF TECHNOLOGY**

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

# QUIZ #2

DATE: 9-Mar-09 COURSE: ECE-2025

NAME: ______________________________ GT username: ____________________

LAST, FIRST (ex: gtbuzz7)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

| | | | |
|---|---|---|---|
| | L05:Tues-Noon (Bhatti) | | L06:Thur-Noon (Barry) |
| | L07:Tues-1:30pm (Bhatti) | | L08:Thur-1:30pm (Barry) |
| L01:M-3pm (Chang) | L09:Tues-3pm (Lee) | L02:W-3pm (Fekri) | |
| | L11:Tues-4:30pm (Lee) | L04:W-4:30pm (Fekri) | |

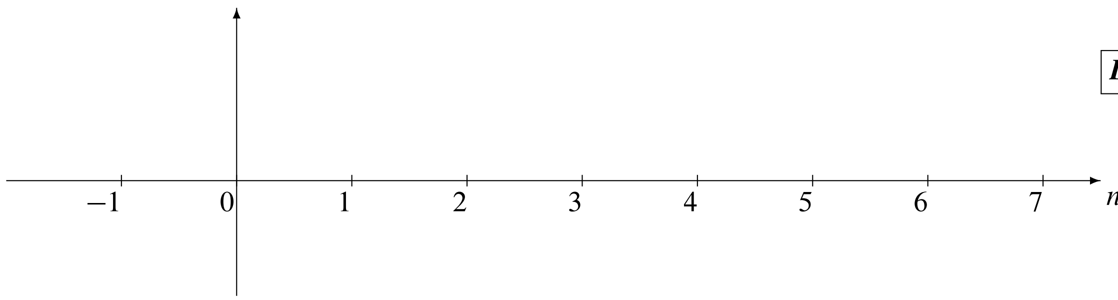
- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **Justify** your reasoning clearly to receive partial credit.
  Explanations are also REQUIRED to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

| *Problem* | *Value* | *Score* |
|---|---|---|
| 1 | 40 | |
| 2 | 30 | |
| 3 | 30 | |
| No/Wrong Rec | $-3$ | |

**PROBLEM sp-09-Q.2.1:**

(a) For the system described by the difference equation: $y[n] = 3x[n] + 2x[n-1] + x[n-2]$,

determine the output when the input is $x[n] = 100\delta[n] + 100\delta[n-2] + 100\delta[n-4]$.
Give your answer as a ***stem plot.***




***Label all points***

(b) When the MATLAB command `soundsc(xx,12000)` is used, the highest frequency that can be heard is __________ Hz.

(c) The signal $x(t)$ is bandlimited to 2000 Hz, i.e., it has no frequency components for $f > 2000$ Hz.

The Nyquist rate for sampling $x(t)$ is __________ samples/sec.

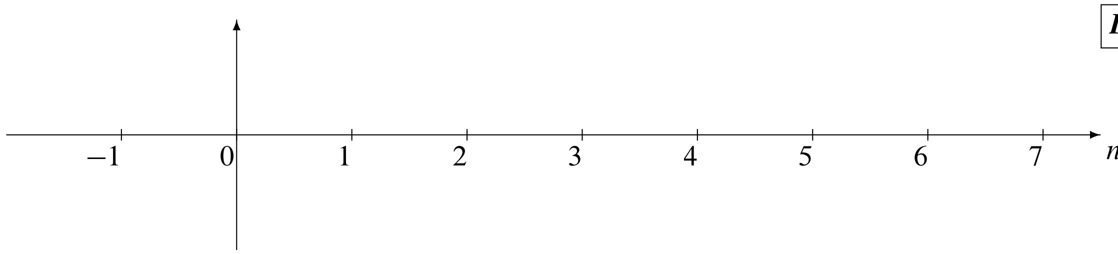
(d) Determine the impulse response of the FIR filter defined[1] in MATLAB via:

```
yy = firfilt( [0,3,-5,0,0,2], xx );
```

Express your answer as a ***sum of shifted unit-impulse signals.***

(e) Make a ***stem plot*** of the signal $s[n] = -\pi\,(u[n-2] - u[n-5])$, where $u[n]$ is the unit-step signal.

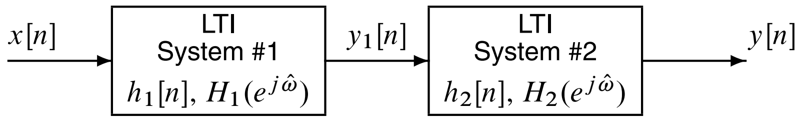
***Label all points***




[1]In MATLAB, the functions `firfilt` and `conv` are equivalent; they produce the same result.

**PROBLEM sp-09-Q.2.2:**

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.




Suppose that System #1 is a filter described by its impulse response: $h_1[n] = \frac{1}{2}\delta[n-1] - \delta[n-4]$

and System #2 is described by its frequency response: $H_2(e^{j\hat{\omega}}) = \dfrac{\sin(3.5\hat{\omega})}{\sin(0.5\hat{\omega})}\, e^{-j4\hat{\omega}}$

(a) Determine an expression for the frequency response, $H_1(e^{j\hat{\omega}})$, of the ***first*** system. No simplification is necessary.

(b) When the input to the ***second*** system is $y_1[n] = 7\cos(2n + 5.5)$, for all $n$, determine the output of the ***second*** system, $y[n]$, over the range $-\infty < n < \infty$. ***Explain your work to receive credit.***

(c) When the input to the ***first*** system is $x[n] = 20$, for $-\infty < n < \infty$, determine the ***overall*** output, $y[n]$, over the range $-\infty < n < \infty$. ***Explain your work to receive credit.***

**PROBLEM sp-09-Q.2.3:**

Two questions that involve common operations done in the Lab. Beware of folding or aliasing!

(a) Suppose that a student enters the following MATLAB code:

```
nn = 0:3480099;
xx = (5/pi) * cos(2*pi*0.75*nn + 2);
soundsc(xx,20000)
```

Determine the analog frequency (in Hz) that will be heard.

| FREQ = | Hz |
|---|---|

(b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
tt = 0:1/12000:13;
xx = sin(2*pi*300*tt+pi/3);
soundsc(xx,fsamp);
```

Although the sinusoid was not written to have a frequency of 400 Hz, it is possible to play out the vector `xx` so that it sounds like a 400 Hz tone. Determine the value of `fsamp` (in Hz) that should be used to play the vector `xx` as a 400 Hz tone. Write your answer as an integer.

| fsamp = | Hz |
|---|---|

(c) Consider the following piece of MATLAB code:

```
tt = 0:(1/16000):36;
xx = cos(2*pi*17000*tt);
soundsc(xx,8000);
```

Determine the ***duration (in secs)*** of the final played tone.

| DURATION = | sec. |
|---|---|

**GEORGIA INSTITUTE OF TECHNOLOGY**

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 9-Mar-09 COURSE: ECE-2025

NAME: **ANSWER KEY** GT username: **VERSION #1**

LAST, FIRST (ex: gtbuzz7)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L05:Tues-Noon (Bhatti) L06:Thur-Noon (Barry)

L07:Tues-1:30pm (Bhatti) L08:Thur-1:30pm (Barry)

L01:M-3pm (Chang) L09:Tues-3pm (Lee) L02:W-3pm (Fekri)

L11:Tues-4:30pm (Lee) L04:W-4:30pm (Fekri)

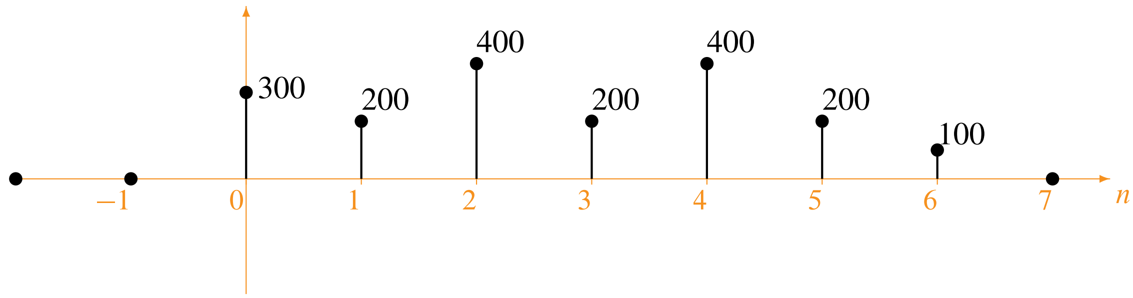
- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **Justify** your reasoning clearly to receive partial credit.
  Explanations are also REQUIRED to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

| *Problem* | *Value* | *Score* |
|---|---|---|
| 1 | 40 | |
| 2 | 30 | |
| 3 | 30 | |
| No/Wrong Rec | −3 | |

**PROBLEM sp-09-Q.2.1:**

(a) For the system described by the difference equation: $y[n] = 3x[n] + 2x[n-1] + x[n-2]$,

determine the output when the input is $x[n] = 100\delta[n] + 100\delta[n-2] + 100\delta[n-4]$.
Give your answer as a ***stem plot.***




The convolution table is:

```
yn = [ 300    0  300    0  300    0    0 ]
   + [   0  200    0  200    0  200    0 ]
   + [   0    0  100    0  100    0  100 ]
yn = [ 300  200  400  200  400  200  100 ]
```

(b) When the MATLAB command `soundsc(xx,12000)` is used, the highest frequency that can be heard is **6000** Hz.

`soundsc(xx,fs)` is actually a D-to-A converter. The highest frequency that will be produced from a D-to-A converter is $\frac{1}{2} f_s = \frac{1}{2}(12000) = 6000$.

(c) The signal $x(t)$ is bandlimited to 2000 Hz, i.e., it has no frequency components for $f > 2000$ Hz. The Nyquist rate for sampling $x(t)$ is **4000** samples/sec.

The Nyquist rate is twice the highest frequency in the signal, i.e., $2 \times 2000 = 4000$ samples/s.

(d) Determine the impulse response of the FIR filter defined[1] in MATLAB via:

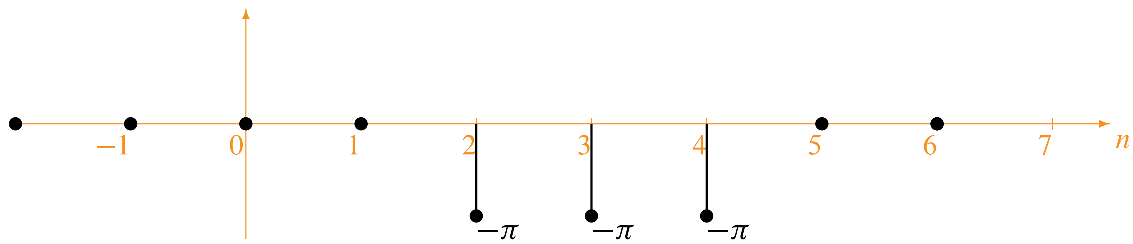
```
yy = firfilt( [0,3,-5,0,0,2], xx );
```

Express your answer as a ***sum of shifted unit-impulse signals.***

When $x[n] = \delta[n]$, you get the impulse response, $h[n]$. The values of the impulse response are the same as the filter coefficients. Thus,

$$h[n] = 3\delta[n-1] - 5\delta[n-2] + 2\delta[n-5]$$

(e) Make a ***stem plot*** of the signal $s[n] = -\pi\,(u[n-2] - u[n-5])$, where $u[n]$ is the unit-step signal.



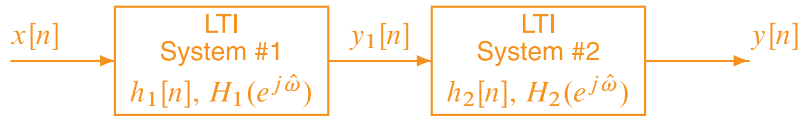

The difference of two unit-step signals is a pulse. The unit-step $u[n-2]$ goes "up" at $n = 2$; the unit-step $-u[n-5]$ goes "down" at $n = 2$, so the final result has two cases:

$$-\pi\,(u[n-2] - u[n-5]) = \begin{cases} -\pi & 2 \le n < 5 \\ 0 & \text{elsewhere} \end{cases}$$

[1] In MATLAB, the functions `firfilt` and `conv` are equivalent; they produce the same result.

**PROBLEM sp-09-Q.2.2:**

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.




Suppose that System #1 is a filter described by its impulse response: $h_1[n] = \frac{1}{2}\delta[n-1] - \delta[n-4]$

and System #2 is described by its frequency response: $H_2(e^{j\hat{\omega}}) = \dfrac{\sin(3.5\hat{\omega})}{\sin(0.5\hat{\omega})}\, e^{-j4\hat{\omega}}$

(a) Determine an expression for the frequency response, $H_1(e^{j\hat{\omega}})$, of the ***first*** system. No simplification is necessary.

$$H_1(e^{j\hat{\omega}}) = \tfrac{1}{2}e^{-j\hat{\omega}} - e^{-j4\hat{\omega}}$$

(b) When the input to the ***second*** system is $y_1[n] = 7\cos(2n + 5.5)$, for all $n$, determine the output of the ***second*** system, $y[n]$, over the range $-\infty < n < \infty$. ***Explain your work to receive credit.***

The input sinusoid has a frequency of $\hat{\omega} = 2$. Thus, we need to evaluate $H_2(e^{j\hat{\omega}})$ at $\hat{\omega} = 2$.

$$H_2(e^{j2}) = \frac{\sin(3.5(2))}{\sin(0.5(2))}\, e^{-j(4)(2)} = 0.781 e^{-j8}$$

Multiply the magnitudes and add the phases:

$$A_{\text{out}} = (0.781)(7) = 5.465 \qquad \phi_{\text{out}} = 5.5 - 8 = -2.5$$

The output sinusoid is $y[n] = 5.465\cos(2n - 2.5)$.

(c) When the input to the ***first*** system is $x[n] = 20$, for $-\infty < n < \infty$, determine the ***overall*** output, $y[n]$, over the range $-\infty < n < \infty$. ***Explain your work to receive credit.***

Since the input is DC, i.e., a sinusoid with $\hat{\omega} = 0$, the output will also be a zero-frequency sinusoid. The overall frequency response is the product $H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}})$, but we only need to evaluate at $\hat{\omega} = 0$.

Evaluating $H_1(e^{j\hat{\omega}})$ at $\hat{\omega} = 0$ requires the sum of the coefficients, i.e., $H_1(e^{j0}) = -0.5$.

Evaluating $H_2(e^{j\hat{\omega}})$ at $\hat{\omega} = 0$ can be done with L'Hôpital's rule, and we get $H_2(e^{j0}) = 7$.

Thus,

$$y[n] = H_1(e^{j0})H_2(e^{j0})(20) = (-0.5)(7)(20) = -70$$

## PROBLEM sp-09-Q.2.3:

Two questions that involve common operations done in the Lab. Beware of folding or aliasing!

(a) Suppose that a student enters the following MATLAB code:

```
nn = 0:3480099;
xx = (5/pi) * cos(2*pi*0.75*nn + 2);
soundsc(xx,20000)
```

Determine the analog frequency (in Hz) that will be heard.

FREQ = **5,000** Hz

The frequency $\hat{\omega} = \pm 2\pi(0.75) = \pm 1.5\pi$ is the same as $\hat{\omega} = \mp 0.5\pi$, i.e., it is a *folded* alias. Thus the output frequency will be $(\hat{\omega}/2\pi) f_s = (0.5\pi/2\pi)(20,000) = 5,000$ Hz.

(b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
tt = 0:1/12000:13;
xx = sin(2*pi*300*tt+pi/3);
soundsc(xx,fsamp);
```

Although the sinusoid was not written to have a frequency of 400 Hz, it is possible to play out the vector xx so that it sounds like a 400 Hz tone. Determine the value of fsamp (in Hz) that should be used to play the vector xx as a 400 Hz tone. Write your answer as an integer.

fsamp = **16,000** Hz

The frequency of xx is $\hat{\omega} = \pm 2\pi(300)/12000 = \pm 0.05\pi$. There is *no aliasing.*

The output frequency is $f_{\text{out}} = (\hat{\omega}/2\pi) f_s$, so

$$f_s = 2\pi(f_{\text{out}})/\hat{\omega} = 2\pi(400)/0.05\pi = 16,000 \text{ samples/sec.}$$

(c) Consider the following piece of MATLAB code:

```
tt = 0:(1/16000):36;
xx = cos(2*pi*17000*tt);
soundsc(xx,8000);
```

Determine the ***duration (in secs)*** of the final played tone.

DURATION = **72** sec.

The number of samples in xx is $N = 36 f_{s_{\text{in}}} = (36)(16,000)$.

The output duration is $N/f_{s_{\text{out}}} = N/8000 = (36)(16,000)/8000 = 72$ secs.

The output sampling rate is less than the input sampling rate, so the signal is played slower and its duration is longer.