GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 4-Mar-05 COURSE: ECE-2025

NAME: ______________________ GT #: ______________

LAST, FIRST (ex: gtz123q)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Chang)		L06:Thur-Noon (Ingram)
	L07:Tues-1:30pm (Chang)		L08:Thurs-1:30pm (Zhou)
L01:M-3pm (Williams)	L09:Tues-3pm (Casinovi)	L02:W-3pm (Juang)	L10:Thur-3pm (Zhou)
L03:M-4:30pm (Casinovi)	L11:Tues-4:30pm (Casinovi)	L04:W-4:30pm (Juang)	GTSav: (Moore)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
No Rec	–3	

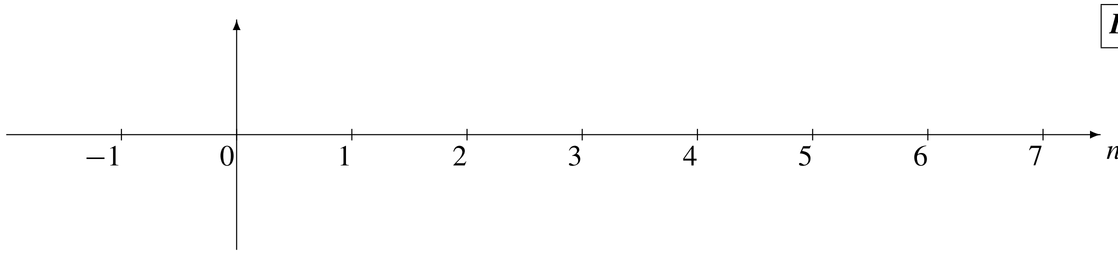
PROBLEM sp-05-Q.2.1:

(a) Determine the impulse response of the system:

$$y[n] = -3x[n-2] + 2x[n-4] - 2x[n-5]$$

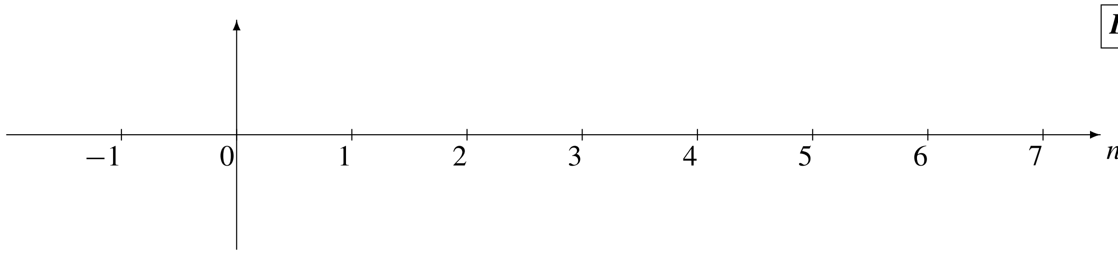
system. Give your answer as a ***stem plot.***

Label all points

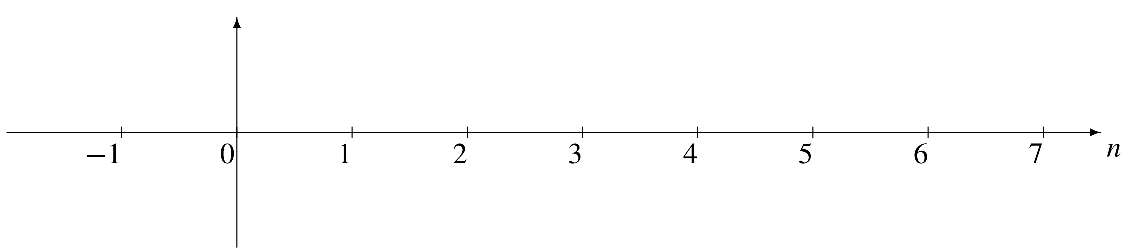


(b) Evaluate the convolution: `yn = conv( [1 0 1 0 1], [1 2 3] );`
Give your answer as a ***stem plot.***

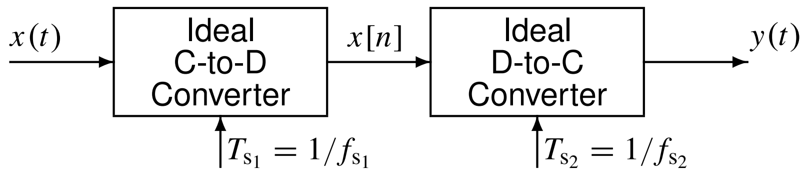
Label all points



(c) Make a ***stem plot*** of the signal $s[n] = u[n-2]$.



PROBLEM sp-05-Q.2.2:



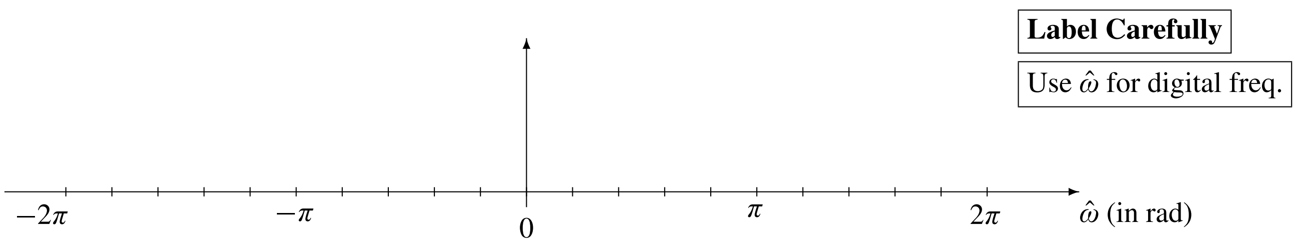
In all parts below, the sampling rates of the C/D and D/C converters are **NOT necessarily equal,** and the input to the ideal C/D converter is

$$x(t) = 2e^{j(44\pi t)} + 3e^{j(32\pi t + 3\pi/4)}$$

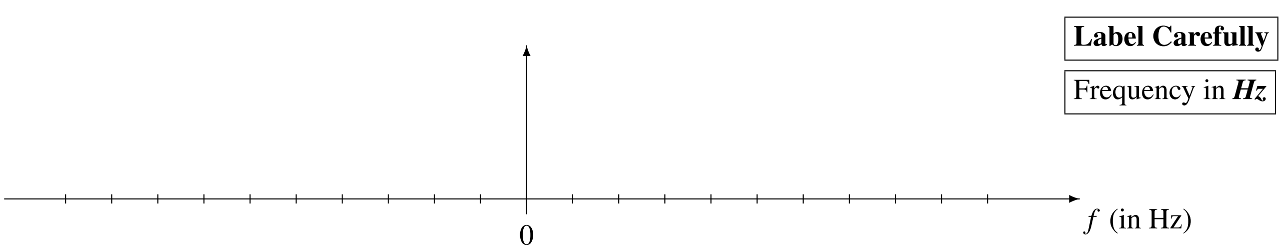
(a) Determine the Nyquist rate (in hertz) for sampling the real-valued signal $s(t) = \Re e\{x(t)\}$. Explain.

$f_{\text{Nyquist}} =$ [   ] Hz

(b) If the sampling rate of the C-to-D converter is $f_{s_1} = 20$ samples/sec, make a plot of the spectrum of the discrete-time signal $x[n]$ over the range of frequencies $-2\pi \le \hat{\omega} \le 2\pi$. Make sure to show <u>**all spectrum lines**</u> and label the frequency, amplitude and phase of each spectral component.



(c) If the sampling rate of the the ideal D-to-C converter is $f_{s_2} = 100$ samples/sec, draw the spectrum for the continuous-time output signal, $y(t)$. Use $x[n]$ from the previous part.



PROBLEM sp-05-Q.2.3:

A periodic signal $x(t)$ is represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} \left(10\delta[k^2 - 1] + 7\cos(\pi k/2)\right) e^{j10\pi kt}$$

(a) Determine the fundamental **period** of the signal $x(t)$, i.e., the minimum period. Explain.

$T_0 =$	sec. (Give a numerical answer.)

(b) Determine the DC value of $x(t)$. Give your answer as a number. Show your work.

$DC =$

(c) Define a new signal by adding a sinusoid to $x(t)$

$$y(t) = 12\cos(30\pi t - \pi/2) + x(t)$$

The new signal, $y(t)$ can be expressed in the following Fourier Series with new coefficients $\{b_k\}$:

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{j10\pi kt}$$

Fill in the following tables, giving *numerical values* for each $\{a_k\}$ and $\{b_k\}$ in polar form:.
Hint: Find a simple relationship between $\{b_k\}$ and $\{a_k\}$, where $\{a_k\}$ denotes the Fourier coefficients of the original signal $x(t)$.

Signal: $x(t)$

a_k	Mag	Phase
a_3		
a_2		
a_1		
a_0		
a_{-1}		
a_{-2}		
a_{-3}		

Signal: $y(t)$

b_k	Mag	Phase
b_3		
b_2		
b_1		
b_0	***SAME***	
b_{-1}		
b_{-2}		
b_{-3}		

Note: Whenever a b_k coefficient is equal the corresponding a_k coefficient, just write ***SAME*** in the b_k table.

PROBLEM sp-05-Q.2.4:

(a) The intent of the following MATLAB code is to play a 1200-Hz sinusoidal signal:

```
nTs = 0:(1/2000):10.08;
xn = real( (17+13i)*exp(j*2400*pi*nTs) );
soundsc( xn, 2000 );
```

However, when played out via the `soundsc` function, a different frequency is heard. Determine the actual output (in Hz). Take into account sampling and aliasing. Explain.

$f_{\text{out}} =$ Hz

(b) The following MATLAB code produces a frequency modulated signal and plots its spectrogram:

```
tt = 0:(1/10000):0.2;
xx = real( exp(j*(300*pi*tt + 10*cos(15*pi*tt)) ) );
plotspec( xx, 10000 );
```

Using the values in the MATLAB code and the signal defined by `xx` draw a sketch of the instantaneous frequency that will be seen in the spectrogram produced by `plotspec`, as time goes from $t = 0$ to $t = 0.2$ sec. ***Label the frequency axis in hertz.*** *Note:* there is no aliasing.

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 4-Mar-05 COURSE: ECE-2025

NAME: Answer Key (LAST, FIRST) GT #: Version - 1 (ex: gtz123q)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Chang)		L06:Thur-Noon (Ingram)
	L07:Tues-1:30pm (Chang)		L08:Thurs-1:30pm (Zhou)
L01:M-3pm (Williams)	L09:Tues-3pm (Casinovi)	L02:W-3pm (Juang)	L10:Thur-3pm (Zhou)
L03:M-4:30pm (Casinovi)	L11:Tues-4:30pm (Casinovi)	L04:W-4:30pm (Juang)	GTSav: (Moore)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
No Rec	−3	

PROBLEM sp-05-Q.2.1:

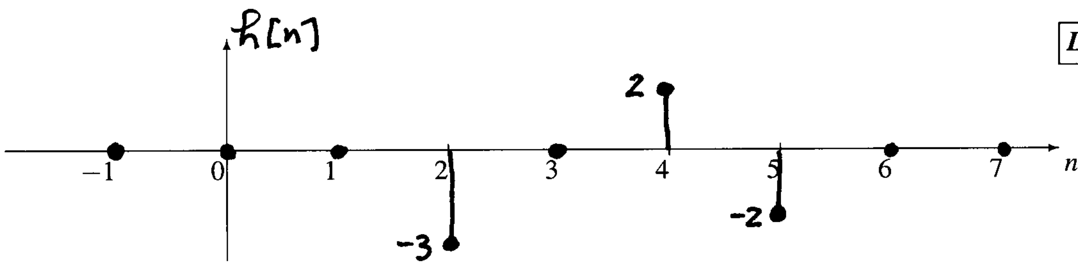
(a) Determine the impulse response of the system:

$$y[n] = -3x[n-2] + 2x[n-4] - 2x[n-5]$$

$h[n] = -3\delta[n-2] + 2\delta[n-4] - 2\delta[n-5]$

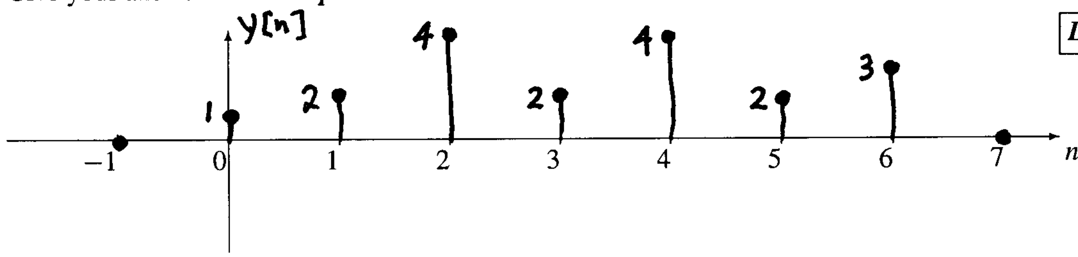
system. Give your answer as a ***stem plot.***

Label all points



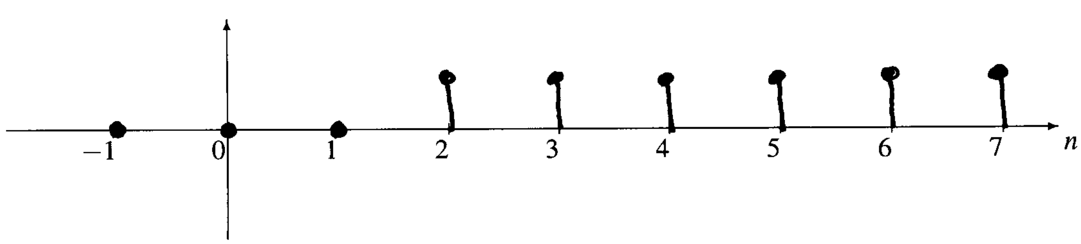
(b) Evaluate the convolution: `yn = conv( [1 0 1 0 1], [1 2 3] );`
Give your answer as a ***stem plot.***

Label all points

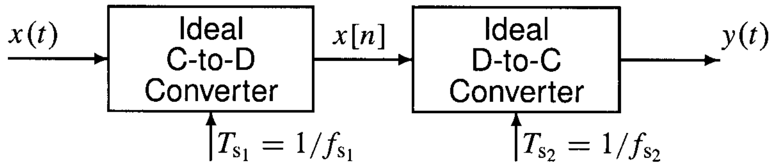



```
1 0 1 0 1
1 2 3
---------
1 0 1 0 1
  2 0 2 0 2
    3 0 3 0 3
-------------
1 2 4 2 4 2 3
↑     ↑     ↑
n=0   n=3   n=6
```

(c) Make a ***stem plot*** of the signal $s[n] = u[n-2]$. $= \begin{cases} 0, & n<2 \\ 1 & n\geq 2 \end{cases}$



PROBLEM sp-05-Q.2.2:



In all parts below, the sampling rates of the C/D and D/C converters are **NOT necessarily equal,** and the input to the ideal C/D converter is

$$x(t) = 2e^{j(44\pi t)} + 3e^{j(32\pi t + 3\pi/4)}$$

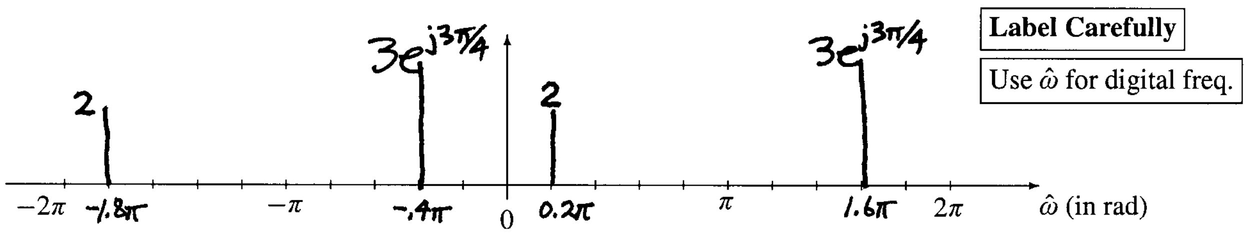
(a) Determine the Nyquist rate (in hertz) for sampling the real-valued signal $s(t) = \Re e\{x(t)\}$. Explain.

$f_{\text{Nyquist}} =$ 44 Hz

$s(t) = 2\cos(44\pi t) + 3\cos(32\pi t + 3\pi/4)$

$f_{MAX} = 44\pi/2\pi = 22\text{Hz}$ $\quad f_{NY} = 2 f_{MAX} = 44\text{ Hz}$

(b) If the sampling rate of the C-to-D converter is $f_{s_1} = 20$ samples/sec, make a plot of the spectrum of the discrete-time signal $x[n]$ over the range of frequencies $-2\pi \le \hat{\omega} \le 2\pi$. Make sure to show **all spectrum lines** and label the frequency, amplitude and phase of each spectral component.



$x[n] = x(n/f_s) = x(n/20)$

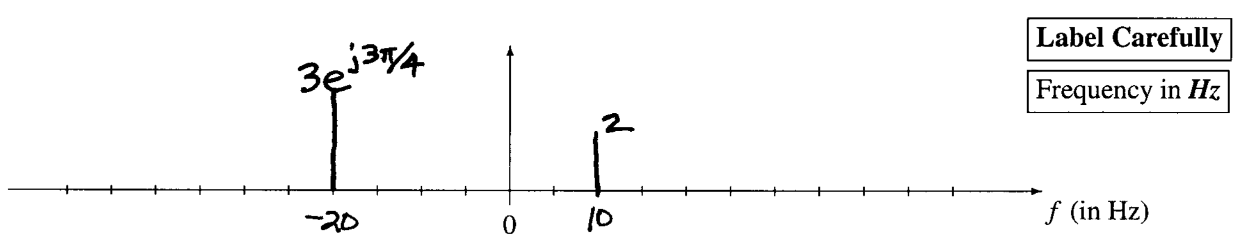
$= 2e^{j44\pi n/20} + 3e^{j(32\pi n/20 + 3\pi/4)}$

$= 2e^{j2.2\pi n} + 3e^{j1.6\pi n}e^{j3\pi/4}$

$\hat{\omega} = 2.2\pi$ aliases to 0.2π & -1.8π

$\hat{\omega} = 1.6\pi$ aliases to -0.4π

(c) If the sampling rate of the the ideal D-to-C converter is $f_{s_2} = 100$ samples/sec, draw the spectrum for the continuous-time output signal, $y(t)$. Use $x[n]$ from the previous part.



$f_{OUT} = \frac{\hat{\omega}}{2\pi} f_s$

$\hat{\omega} = 0.2\pi \rightarrow f_{OUT} = \frac{0.2\pi}{2\pi} \times 100 = 10\text{ Hz}$

$\hat{\omega} = -0.4\pi \rightarrow f_{OUT} = \frac{-0.4\pi}{2\pi} \times 100 = -20\text{Hz}$

PROBLEM sp-05-Q.2.3:
A periodic signal $x(t)$ is represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} \left(10\delta[k^2-1] + 7\cos(\pi k/2)\right) e^{j10\pi kt}$$

$a_k = 10\delta[k^2-1] + 7\cos(\pi k/2)$

(a) Determine the fundamental **period** of the signal $x(t)$, i.e., the minimum period. Explain.

$T_0 = 0.2$ sec. (Give a numerical answer.) $\omega_0 = 10\pi$ rad/s

$T_0 = 2\pi/\omega_0 = 1/5$ sec

(b) Determine the DC value of $x(t)$. Give your answer as a number. Show your work.

$DC = 7$ $\quad DC = a_0 = 10\delta[0-1] + 7\cos(0) = 7$

(c) Define a new signal by adding a sinusoid to $x(t)$

$y(t) = 12\cos(30\pi t - \pi/2) + x(t) = 6e^{-j\pi/2}e^{j30\pi t} + 6e^{j\pi/2}e^{-j30\pi t} + x(t)$

The new signal, $y(t)$ can be expressed in the following Fourier Series with new coefficients $\{b_k\}$:

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{j10\pi kt}$$

$30\pi = 3 \times 10\pi \Rightarrow$ 3rd harmonic

$\therefore a_3 + 6e^{-j\pi/2} = b_3$ & $b_{-3} = a_{-3} + 6e^{j\pi/2}$

Fill in the following tables, giving *numerical values* for each $\{a_k\}$ and $\{b_k\}$ in polar form:.
Hint: Find a simple relationship between $\{b_k\}$ and $\{a_k\}$, where $\{a_k\}$ denotes the Fourier coefficients of the original signal $x(t)$.

$a_1 = 10\delta[1-1] + 7\cos(\pi/2) = 10$

$a_2 = 10\delta[4-1] + 7\cos(\pi) = -7$

$a_3 = 10\delta[9-1] + 7\cos(3\pi/2) = 0$

$b_3 = 6e^{-j\pi/2}$

$b_{-3} = 6e^{j\pi/2}$

Signal: $x(t)$

a_k	Mag	Phase
a_3	0	anything
a_2	7	π
a_1	10	0
a_0	7	0
a_{-1}	10	0
a_{-2}	7	$-\pi$
a_{-3}	0	anything

Signal: $y(t)$

b_k	Mag	Phase
b_3	6	$-\pi/2$
b_2	Same	
b_1	Same	
b_0	***SAME***	
b_{-1}	Same	
b_{-2}	Same	
b_{-3}	6	$\pi/2$

Note: Whenever a b_k coefficient is equal the corresponding a_k coefficient, just write ***SAME*** in the b_k table.

PROBLEM sp-05-Q.2.4:

(a) The intent of the following MATLAB code is to play a 1200-Hz sinusoidal signal:

```
nTs = 0:(1/2000):10.08;
xn = real( (17+13i)*exp(j*2400*pi*nTs) );
soundsc( xn, 2000 );
```

However, when played out via the `soundsc` function, a different frequency is heard. Determine the actual output (in Hz). Take into account sampling and aliasing. Explain.

$f_{\text{out}} = 800$ Hz

xn is a sampled signal whose freq is $\hat{\omega} = \frac{2400\pi}{2000}$

$\hat{\omega} = 1.2\pi$ aliases to -0.8π rad

But a cosine has both positive & negative freq. components

i.e, $\hat{\omega} = \pm 0.8\pi$

$f_{out} = \frac{\hat{\omega}}{2\pi} f_s = \frac{0.8\pi}{2\pi} \times 2000 = 800$ Hz

(b) The following MATLAB code produces a frequency modulated signal and plots its spectrogram:

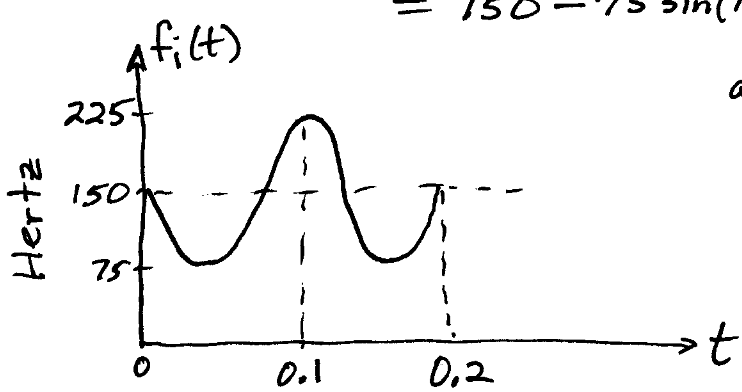
```
tt = 0:(1/10000):0.2;
xx = real( exp(j*(300*pi*tt + 10*cos(15*pi*tt)) ) );
plotspec( xx, 10000 );
```

Using the values in the MATLAB code and the signal defined by `xx` draw a sketch of the instantaneous frequency that will be seen in the spectrogram produced by `plotspec`, as time goes from $t = 0$ to $t = 0.2$ sec. ***Label the frequency axis in hertz.*** *Note:* there is no aliasing.

$\psi(t) = 300\pi t + 10\cos(15\pi t)$

$f_i(t) = \frac{1}{2\pi} \frac{d}{dt} \psi(t) = \frac{1}{2\pi} \left(300\pi - 150\pi \sin(15\pi t)\right)$

$= 150 - 75\sin(15\pi t)$ Hz



at $t = 0.2$ sec

$f_i(t) = 150 - 75\sin(3\pi)$

$= 150$ Hz