GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

ECE2026 – Fall 2025
Quiz #2
October 17, 2025

NAME: ___________________________ GT Username: _____________________
LAST FIRST (e.g., gtinit0101)

Important Notes:

- Write your name on each page.
- Closed book, but a calculator is permitted.
- One page (8.5"x11") of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Partial credit for incorrect answers may be granted ONLY when you JUSTIFY your correct reasoning CLEARLY.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers or write them in the boxes provided. If more space is needed for scratch work, use the backs of previous pages.

PROBLEM Fall-25-Q.2.1:
Consider the following MATLAB code:

```
tt = -0.6:1/1e4:1;
xx = real(abs(sqrt(3)+j)*exp(j*(4000*pi*tt - 1000*pi*tt.^3 + 21*2*pi)));
soundsc(xx,1e4);
```

If you run this code you will hear some sound.

(a) The variable **xx** represents a continuous-time signal $x(t)$. Express this signal in standard mathematical form. (7 pts.)

$$x(t) = 2\,cos(\,4000\pi t - 1000\pi t^3)$$

(b) Given the duration of the signal, the sound output of **soundsc** has a peak instantaneous frequency that a listener can hear. Find this highest instantaneous frequency (in Hz) and the time it reaches this frequency in terms of the index of **tt**, i.e., the value of **k** such that at **tt(k)** the highest instantaneous frequency is played? (7 pts.)

$$\varphi(t) = 4000\pi t - 1000\pi t^3$$

$$\omega_{inst}(t) = \frac{d}{dt}\varphi(t) = 4000\pi - 3000\pi t^2$$

The instantaneous frequency remains in $[1000\pi, 4000\pi]$ throughout the duration of the signal $-0.6 \leq t \leq 1$. This makes the problem straightforward – no concern of potential aliasing as the sampling rate is set at 10000 samples/s. To find the maximum,

$$\frac{d}{dt}\omega_{inst}(t) = -6000\pi t = 0 \rightarrow t = 0; \quad \frac{d^2}{dt^2}\omega_{inst}(t) = -6000\pi < 0$$

$$\omega_{inst}(t)|_{t=0} = 4000\pi \;\rightarrow\; f_{max} = 2000,$$

and with **fs=10000**, **tt(1)=-0.6,** we find **tt(k)=0** @ **k=6001**

$f_{max} = 2000Hz \,@\, t = 0$; **tt(6001)=0, k=6001**

(c) Suppose the signal lasted beyond the time limit in the code; i.e., the size of the **tt** array is indefinitely expanded. What would be the instantaneous frequency (in Hz) played at $t = 2$ (or equivalently at **tt(26001)** if the duration were expanded accordingly)? (6 pts)

$$\omega_{inst}(t)|_{t=2} = 4000\pi - 12000\pi = -8000\pi$$

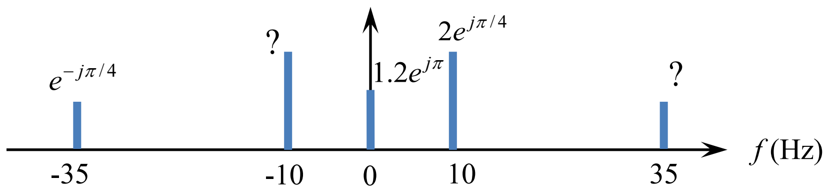
But, a folded frequency is going to be heard, i.e.,

$$\omega_{inst}(t)|_{t=2} = -8000\pi \text{ is folded to } 8000\pi$$

which corresponds to 4000Hz.

$f_{inst} = 4000\;Hz$

PROBLEM Fall-25-Q.2.2:



The spectrum of a real signal $x(t)$ is shown as above.

(a) Write the signal $x(t)$ as sum of sinusoids in standard form:

$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \, cos(\omega_k t + \phi_k)$. (6 pts)

$$x(t) = -1.2 + 4\,cos(20\pi t + \pi/4) + 2\,cos(70\pi t + \pi/4)$$

(b) Determine if the signal $x(t)$ is a harmonic signal by finding the existence and the value of the fundamental frequency in Hz. (6 pts.)

The largest common factor in (10, 35) is 5. So, $f_0 = 5Hz$.

$$f_0 = 5Hz$$

(c) Consider another signal $y(t)$, $y(t) = -2x(t - 0.1) + 4\cos(70\pi t - \frac{3\pi}{4})$ where $x(t)$ is given in (a). This signal can also be represented by a Fourier series: $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$. Write ***all*** non-zero Fourier coefficients of $y(t)$ in the table below; make sure to include the fundamental frequency ω_0 in rad/s and the correct harmonic number(s). (7 pts)

$\omega_0 = 20\pi$	
k	b_k
-1	$4e^{j3\pi/4}$
0	2.4
1	$4e^{-j3\pi/4}$

$$-2x(t-0.1) = 2.4 - 8\cos\left(20\pi(t-0.1) + \frac{\pi}{4}\right) - 4\cos\left(70\pi(t-0.1) + \frac{\pi}{4}\right)$$

$$= 2.4 - 8\cos\left(20\pi t - 2\pi + \frac{\pi}{4}\right) - 8\cos\left(70\pi t - 7\pi + \frac{\pi}{4}\right)$$

$$= 2.4 - 8\cos\left(20\pi t + \frac{\pi}{4}\right) + 4\cos\left(70\pi t + \frac{\pi}{4}\right)$$

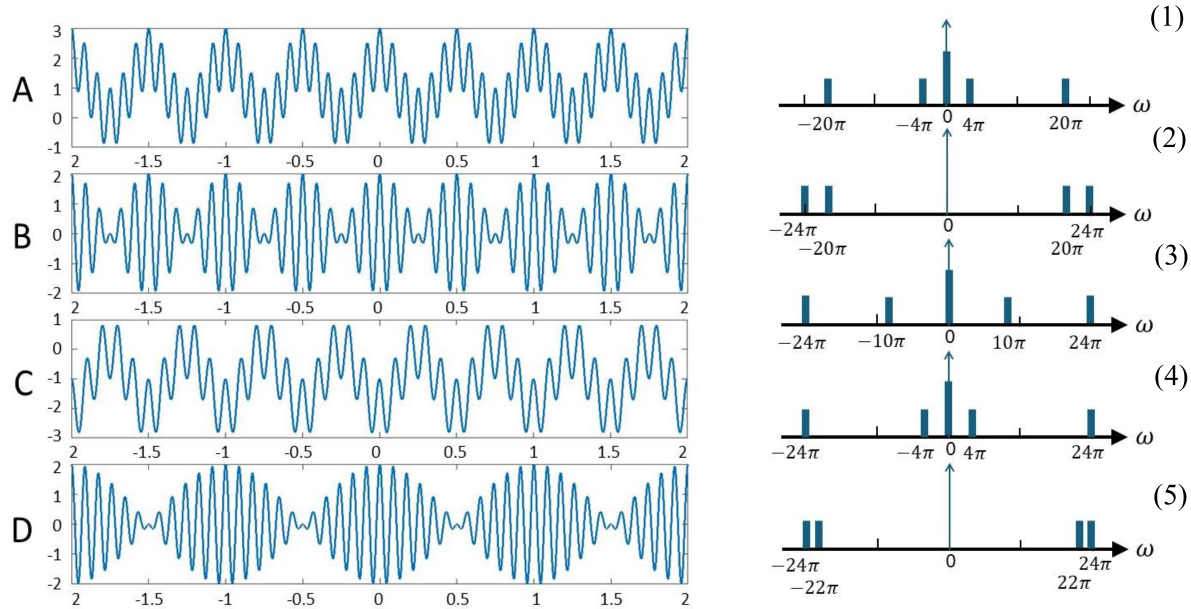
$$y(t) = 2.4 - 8\cos\left(20\pi t + \frac{\pi}{4}\right) + 4\cos\left(70\pi t + \frac{\pi}{4}\right) + 4\cos\left(70\pi t - \frac{3\pi}{4}\right) =$$

$$2.4 + 8\,cos(20\pi t - \frac{3\pi}{4}) = 2.4 + 4e^{-j3\pi/4}e^{j20\pi t} + 4e^{j3\pi/4}e^{-j20\pi t}$$

With only one sinusoid and DC, the fundamental frequency is same as its frequency,i.e. 20π, and the signal has only one "harmonic" .

PROBLEM Fall-25-Q.2.3:

The waveforms of four real signals are plotted in the following panels on the left, marked A, B, C, and D. Their (magnitude) spectra are shown on the right among five possible diagrams, indexed from 1 to 5. Find the corresponding spectrum for each of the signals on the left. State your reason below each answer box. (Note: The phase is not specified in the spectral plots and the line at $\omega = 0$ may have a phase value of π.) (20 pts; 5 each)



$A \leftrightarrow$ 4	$B \leftrightarrow$ 2	C $\leftrightarrow$ 1	D $\leftrightarrow$ 5