GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 2026 – Fall 2024
Exam #2

NAME: Solution ______________________ GTemail: ______________________
FIRST LAST ex: gpburDELL@gatech.edu

- Write your name at the top of EACH PAGE.
- DO NOT unstaple the test.
- Closed book, except for one two-sided page $(8.5'' \times 11'')$ of hand-written notes permitted.
- Calculators are allowed, but no smartphones/readers/watches/tablets/laptops/etc.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- Express all angles as a fraction of π. (i.e., write 0.4π or $\frac{2\pi}{5}$ instead of 1.257)
- All angles/phase must be expressed in the range $(-\pi, \pi]$ for full credit.
- You must show your work in the space provided on the exam paper itself. Only these answers with shown work can received credit. Write your answers in the **boxes/spaces** provided. DO NOT write on the backs of the pages.
- All exams will be collected and uploaded to gradescope for grading.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
Total		

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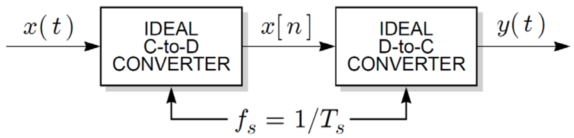
PROBLEM 1:

Parts a and b (10 points each) can be solved independently of each other.

(a) Let $x(t) = 3\cos(84\pi t - \pi/4) + 8\cos(72\pi t + 3\pi/5)\cos(504\pi t)$ in the ideal C-D/D-C system shown below.



(i) Find the condition on f_s (in Hz) to avoid aliasing in $y(t)$.

$$x(t) = 3\cos\left(84\pi t - \frac{\pi}{4}\right) + 4\cos\left(432\pi t - \frac{3\pi}{5}\right) + 4\cos\left(576\pi t + \frac{3\pi}{5}\right)$$

Max Frequency is: 288 Hz therefore $f_s > 2 * 288 = 576$

$f_s >$ $576\ Hz$

(ii) Independent of your answer in part (i), now assume $f_s = 500\ Hz$ and find $y(t)$ as a sum of sinusoids in the standard form below (where N represents the total number of sinusoids)

$$y(t) = A_0 + A_1\cos(\omega_1 t + \varphi_1) + A_2\cos(\omega_2 t + \varphi_2) + \cdots + A_N\cos(\omega_N t + \varphi_N)$$

For $f_s = 500$, only the 288 Hz cosine will alias and fold.

$$y(t) = 3\cos\left(84\pi t - \frac{\pi}{4}\right) + 4\cos\left(424\pi t - \frac{3\pi}{5}\right) + 4\cos\left(432\pi t - \frac{3\pi}{5}\right)$$

$y(t) =$

(b) Let $x(t) = -9 + 5\cos\left(72\pi t - \frac{\pi}{9}\right) + 8\cos\left(120\pi t + \frac{\pi}{7}\right) + 2\cos\left(264\pi t + \frac{\pi}{10}\right)$

Find f_0

$f_0 =$ $12\ Hz$

The Fourier Series (FS) representation of $x(t)$ is $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$. List the **non-zero** FS Coefficients (a_k) along with their k-index in the table below in *increasing index order* (i.e. for all **non-zero** FS Coefficients start with the smallest **negative** k-index in the first column and end with the largest **positive** k-index in the last column).

k	-11	-5	-3	0	3	5	11
a_k	$e^{-\frac{j\pi}{10}}$	$4e^{-\frac{j\pi}{7}}$	$2.5e^{\frac{j\pi}{9}}$	$9e^{j\pi}$	$2.5e^{-\frac{j\pi}{9}}$	$4e^{\frac{j\pi}{7}}$	$e^{\frac{j\pi}{10}}$

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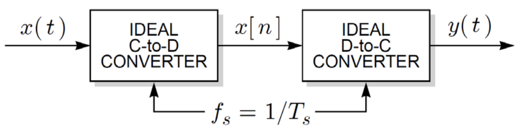
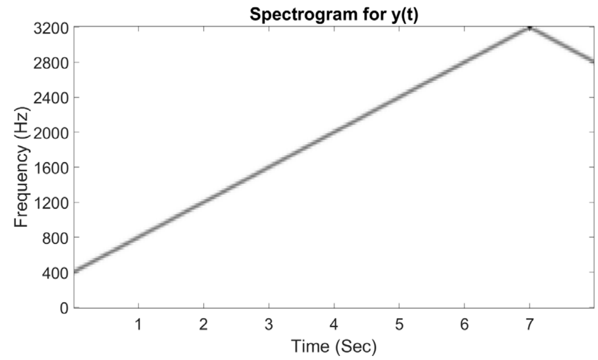
PROBLEM 2:

Parts a and b (10 points each) can be solved independently of each other.

(a) For the ideal C-D/D-C system below, assume $x(t) = \cos(A\pi t^2 + B\pi t + 0.2\pi)$, $f_s = 6400$ Hz, and the spectrogram for $y(t)$ is as shown in the figure. Let $f_{i,y}(t)$ represent the instantaneous frequency of $y(t)$.





Find: (i) $A > 0$ and (ii) $B > 0$ and (iii) $f_{i,y}(38)$

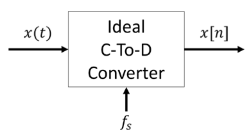
$$f_{i,x}(t) = At + \frac{B}{2}$$
$$f_{i,x}(0) = \frac{B}{2} = 400 \rightarrow B = 800$$
$$f_{i,x}(7) = A(7) + 400 = 3200 \rightarrow A = 400$$
$$f_{i,y}(38) = 400(38) + 400 = 15600 - 2 * 6400 = 2800\ Hz$$

$A =$ 400

$B =$ 800

$f_{i,y}(38) =$ $2800\ Hz$

(b) For the ideal C-To-D converter below assume that $x(t) = 10\cos\left(210\pi t + \frac{\pi}{10}\right) + 6\cos\left(330\pi t - \frac{\pi}{8}\right)$



(i) Find the fundamental frequency, f_0, for $x(t)$ in Hz.

$f_0 =$ $15\ Hz$

(ii) We wish to set the sampling rate f_s such that $x[n] = x[n + N_0]$ (i.e., $x[n]$ is periodic with a period of N_0 samples). Find the <u>*smallest sampling rate*</u> f_s that avoids aliasing and sets $N_0 = 67$ samples.

Highest frequency is: 165 Hz so $f_s > 330\ Hz$

$$\frac{N_0}{M} = \frac{f_s}{f_0} \rightarrow f_s = \frac{N_0}{M} f_0 > 330\ Hz$$
$$f_s = \frac{67}{M}(15) > 330 \rightarrow M < \frac{67 * 15}{330} = 3.05$$

The smallest f_s is equal to $f_s = \frac{67}{3}(15) = 335$

$f_s =$ $335\ Hz$

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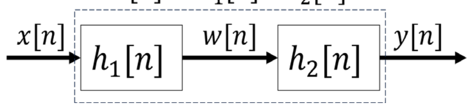
PROBLEM 3:

Parts a and b (10 points each) can be solved independently of each other.

(a) Consider the cascaded LTI system below

$$h[n] = h_1[n] * h_2[n]$$



where $h_1[n] = \sum_{k=1}^{2} k\delta[n-2k]$ and $y[n] = 6w[n-1] - 9w[n-3]$.

(i) Find the overall impulse response $h[n]$.

$$h_1[n] = \delta[n-2] + 2\delta[n-4]$$
$$h_2[n] = 6\delta[n-1] - 9\delta[n-3]$$
$$h[n] = h_1[n] * h_2[n] = \delta[n-2] * (6\delta[n-1] - 9\delta[n-3]) + 2\delta[n-4] * (6\delta[n-1] - 9\delta[n-3])$$
$$h[n] = (6\delta[n-3] - 9\delta[n-5]) + (12\delta[n-5] - 18\,\delta[n-7])$$
$$h[n] = 6\delta[n-3] + 3\delta[n-5] - 18\delta[n-7]$$

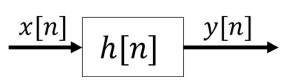
$h[n] =$

(ii) Find $y[6]$ when $x[n] = u[n]$ (i.e., the unit step function)

$$y[n] = 6u[n-3] + 3u[n-5] - 18u[n-7]$$
$$y[6] = 6u[3] + 3u[1] - 18u[-1] = 6*1 + 3*1 - 18*0 = 9$$

$y[6] =$ 9

(b) Consider the LTI system shown below.



We know the following information:

$$h[n] = -2\delta[n-A] + 3\delta[n-B]$$
$$x[n] = 3\delta[n] - 2\delta[n-1]$$
$$y[n] = -6\delta[n-2] + 4\delta[n-3] + 9\delta[n-4] - 6\delta[n-5]$$

Find A and B

Let $n_{\{x,h,y\},F}$ and $n_{\{x,h,y\},L}$ represent the First and Last non-zero values for the signals $x[n], h[n]$ and $y[n]$, respectively. Then:

$$n_{\{x\},F} + n_{\{h\},F} = n_{\{y\},F} \rightarrow 0 + A = 2 \rightarrow A = 2$$
$$n_{\{x\},L} + n_{\{h\},L} = n_{\{y\},L} \rightarrow 1 + B = 5 \rightarrow B = 4$$

$A =$

$B =$